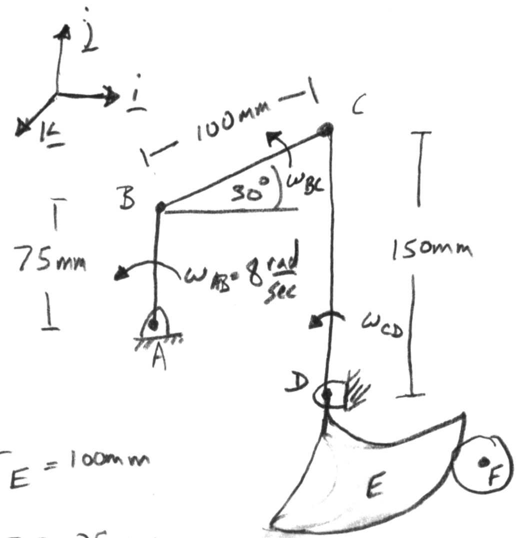
MAE 325 Fall 1999

HW 5 Solutions Due: 9/29/99

(1)



$r_E = 100mm$

$r_F = 25mm$

Find ω_F

$\underset{\sim}{V_B} = \underset{\sim}{\omega_{AB}} \times \underset{\sim}{r_{B/A}}$

$\underset{\sim}{V_B} = 8\underset{\sim}{k} \times 75\underset{\sim}{j}$

$\underset{\sim}{V_B} = -600\ \frac{mm}{sec}\ \underset{\sim}{i}$

$\underline{V_c} = \underline{V_c} = \underline{V_B} + \underline{V_{C/B}}$

$\underset{\sim}{\omega_{CD}} \times \underset{\sim}{r_{C/D}} = -600\underset{\sim}{i} + \underset{\sim}{\omega_{BC}} \times \underline{r_{C/B}}$

$\omega_{CD}\underline{k} \times 150\underset{\sim}{j} = -600\underset{\sim}{i} + \omega_{BC}\underline{k} \times 100(\cos 30\underline{i} + \sin 30\underset{\sim}{j})$

$\{-150\,\omega_{CD}\underset{\sim}{i} = -600\underset{\sim}{i} + 50\sqrt{3}\,\omega_{BC}\underset{\sim}{j} - 50\,\omega_{BC}\underline{i}\}$

$\{\ \}\cdot\underline{j} \Rightarrow \quad 0 = 50\sqrt{3}\,\omega_{BC} \longrightarrow \underline{\omega_{BC} = 0}$

$\{\ \}\cdot\underline{i} \Rightarrow \quad -150\,\omega_{CD} = -600 - 50(0)$

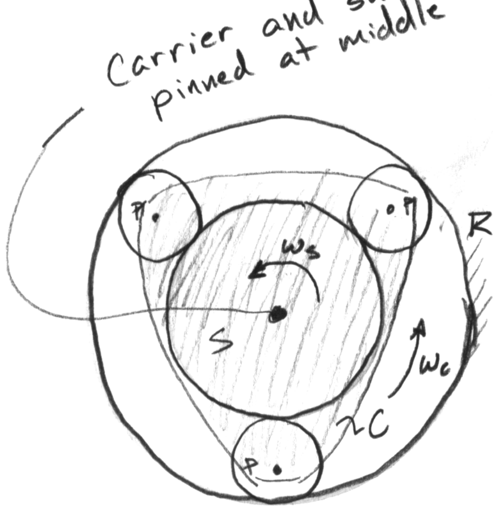
$$\boxed{\omega_{CD} = 4\ rad/sec = \omega_E}$$

$\omega_E r_E = \omega_F r_F$ since the speed of the two gears at the point of contact is equal

$\omega_F = \frac{\omega_E r_E}{r_F} = \frac{4(100)}{25}$

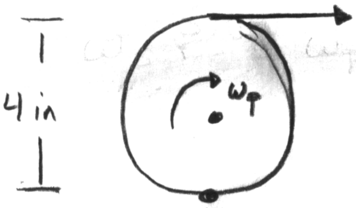
$$\boxed{\omega_F = 16\ rad/sec}$$

(2)



Find ω_c (angular velocity of the planet carrier)

Look at a planet gear first



$V = \omega_S r_S = (5)(4)$
$= 20$ in/sec

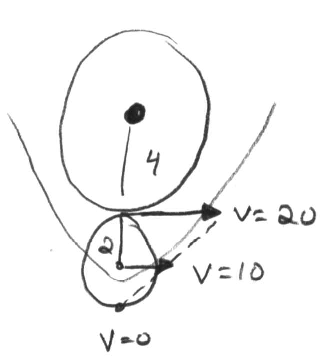
Point of contact w/ fixed ring gear $\Rightarrow V = 0$

$r_S = 4 in$ $\quad \omega_S = 5 \frac{rad}{sec}$
$r_P = 2 in$
$r_R = 8 in$

so $\omega_P = V / r = \frac{20 \text{ in/sec}}{4 \text{ in}}$ → (not 2 in because the center of the gear is not the point of zero velocity or center of rotation)

$\omega_P = 5 \frac{rad}{sec}$

look at the planet and sun:

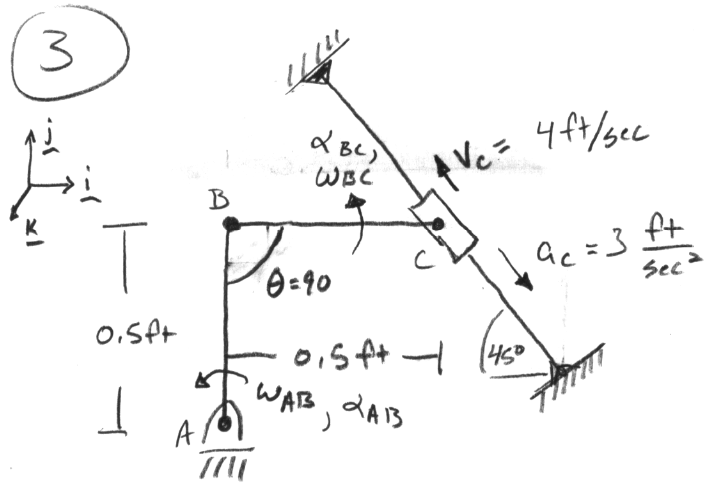


The velocity at the center of the planet is 10 in/sec
This is also the velocity of the carrier at a point 6 in from where it is pinned

so $\omega_c = V/r = \frac{10 \text{ in/sec}}{6 \text{ in}}$

$$\boxed{\omega_c = 1.667 \frac{rad}{sec}}$$

3



Find α_{BC} when $\theta = 90°$

$$\underset{\sim}{V_c} = \underset{\sim}{V_c} = \underset{\sim}{V_B} + \underset{\sim}{V_{c/B}}$$

$$4(-\cos 45\, \underset{\sim}{i} + \sin 45\, \underset{\sim}{j}) = \omega_{AB} \underset{\sim}{k} \times 0.5 \underset{\sim}{j} + \omega_{BC} \underset{\sim}{k} \times 0.5 \underset{\sim}{i}$$

$$\{-2\sqrt{2}\, \underset{\sim}{i} + 2\sqrt{2}\, \underset{\sim}{j} = -0.5\omega_{AB}\, \underset{\sim}{i} + 0.5\omega_{BC}\, \underset{\sim}{j}\}$$

$$\{\ \} \cdot \underset{\sim}{i} \Rightarrow \quad \omega_{AB} = 4\sqrt{2} = 5.657 \text{ rad/sec}$$

$$\{\ \} \cdot \underset{\sim}{j} \Rightarrow \quad \omega_{BC} = 4\sqrt{2} = 5.657 \text{ rad/sec}$$

$$\underset{\sim}{a_c} = \underset{\sim}{a_c} = \underset{\sim}{a_B} + \underset{\sim}{a_{c/B}}$$

$$3(\cos 45\, \underset{\sim}{i} - \sin 45\, \underset{\sim}{j}) = -\omega_{AB}^2\, 0.5 \underset{\sim}{j} + \alpha_{AB} \underset{\sim}{k} \times 0.5 \underset{\sim}{j}$$

$$- \omega_{BC}^2\, 0.5 \underset{\sim}{i} + \alpha_{BC} \underset{\sim}{k} \times 0.5 \underset{\sim}{i}$$

$$\left\{ \frac{3\sqrt{2}}{2} \underset{\sim}{i} - \frac{3\sqrt{2}}{2} \underset{\sim}{j} = -16 \underset{\sim}{j} - 0.5\alpha_{AB} \underset{\sim}{i} - 16 \underset{\sim}{i} + 0.5\alpha_{BC} \underset{\sim}{j} \right\}$$

$$\{\ \} \cdot \underset{\sim}{i} \Rightarrow \quad \frac{3\sqrt{2}}{2} = -16 - 0.5\alpha_{AB}$$

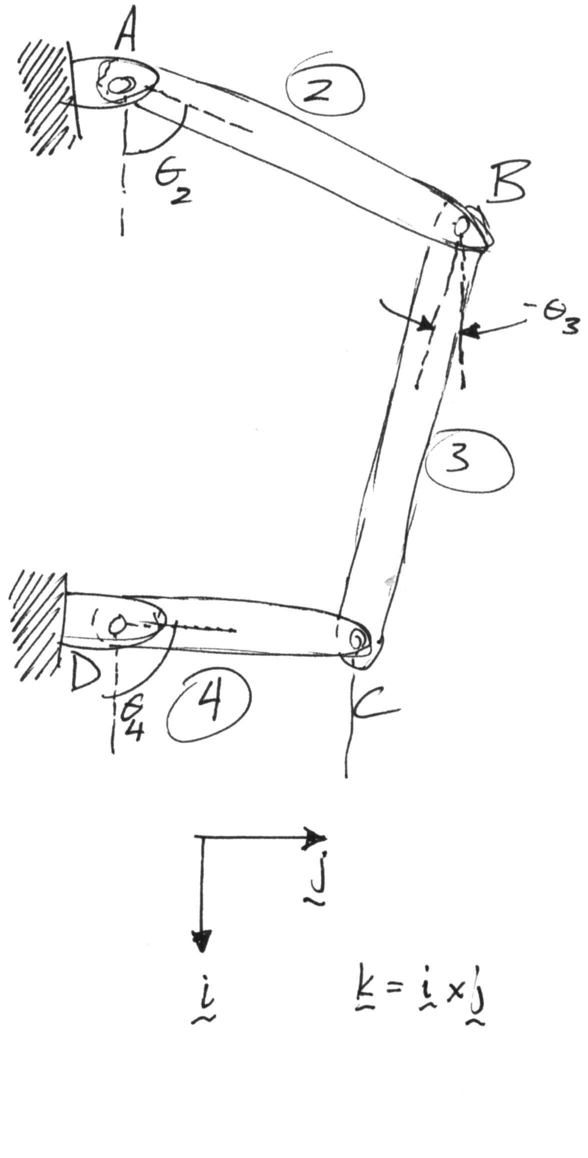
$$\alpha_{AB} = -36.243 \text{ rad/sec}^2$$

$$\{\ \} \cdot \underset{\sim}{j} \Rightarrow \quad -\frac{3\sqrt{2}}{2} = -16 + 0.5\alpha_{BC}$$

$$\boxed{\alpha_{BC} = 27.757 \text{ rad/sec}^2}$$

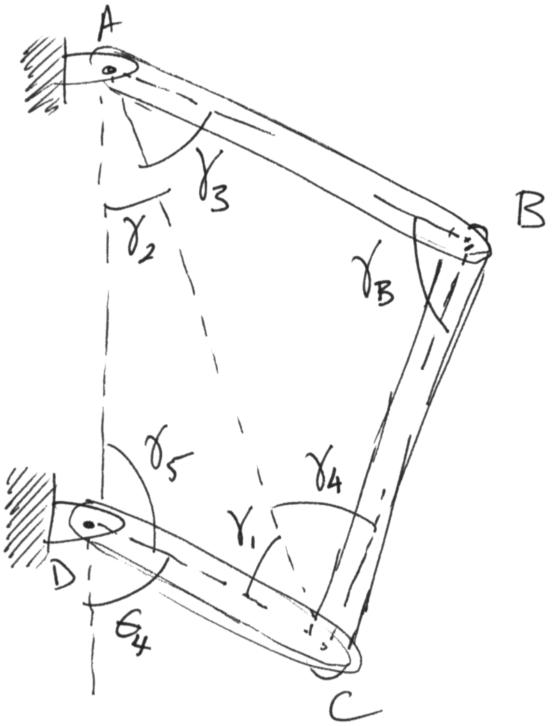
Problem 4 – bicycle rider's leg as part of a planar 4-bar linkage

- include gravity
- find positions for a full cycle, crank at constant speed
- for no muscular forces, find the moment at the crank over a cycle



given:
- r_{AD}, r_{AB}, r_{BC}, r_{CD},
- $\theta_4 \big|_{t=0}$
- masses m_2, m_3, m_4
- mass uniformly distributed
- $\underset{\sim}{\omega}_4 = -2\pi \underset{\sim}{k}$ ... constant

- first solve geometry problem — needed for initial configuration and values of angles and useful for checking matlab integration of angles. Rely heavily on law of cosines.



Note: • for this geometry, it is always try that $\gamma_B \leq \pi$, otherwise the cyclist has over extended his/her knee
• θ_4 may be negative, in which case $\gamma_2, \gamma_5, \gamma_1$ change signs as well; this case is handled in the matlab solution, but not explicitly handled here

$$\gamma_5 = \pi - \cos(\theta_4)$$

$$r_{AC}^2 = r_{AD}^2 + r_{DC}^2 - 2 r_{AD} r_{DC} \cos(\gamma_5) \Rightarrow r_{AC}$$

$$r_{AD}^2 = r_{CD}^2 + r_{AC}^2 - 2 r_{CD} r_{AC} \cos(\gamma_1) \Rightarrow \gamma_1$$

$$\gamma_2 = \pi - \gamma_1 - \gamma_5$$

$$\gamma_B \Leftarrow r_{AC}^2 = r_{BC}^2 + r_{AB}^2 - 2 r_{BC} r_{AB} \cos(\gamma_\beta)$$

$$\gamma_3 \Leftarrow r_{BC}^2 = r_{AC}^2 + r_{AB}^2 - 2 r_{AC} r_{BC} \cos(\gamma_3)$$

$$\gamma_4 = \pi - \gamma_3 - \gamma_\beta$$

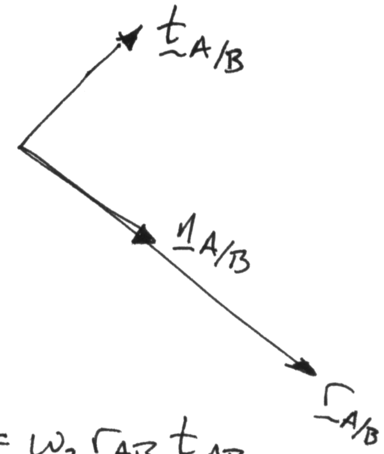
$$\left.\begin{aligned} \theta_2 &= \gamma_2 + \gamma_3 \\ \theta_3 &= \theta_2 - \pi + \gamma_B \end{aligned}\right\} \text{given } \theta_4 \text{, may find } \theta_2 \text{ and } \theta_3$$

Velocities and accelerations

- Find angular velocities for numerical integration of angles
- Find accelerations for use in force/moment evaluation

① $\underset{\sim}{V}_A = \underset{\sim}{0} = \underset{\sim}{V}_{C/D} + \underset{\sim}{V}_{B/C} + \underset{\sim}{V}_{A/B}$

$= \underset{\sim}{\omega}_4 \times \underset{\sim}{r}_{C/D} + \underset{\sim}{\omega}_3 \times \underset{\sim}{r}_{B/C} + \underset{\sim}{\omega}_2 \times \underset{\sim}{r}_{A/B}$



<u>define</u> $\underset{\sim}{r}_{A/B} = r_{AB}\,\underset{\sim}{n}_{A/B}$ etc.

$\underset{\sim}{t}_{A/B} = \underset{\sim}{k} \times \underset{\sim}{n}_{A/B}$

then $\underset{\sim}{\omega}_2 \times \underset{\sim}{r}_{A/B} = \omega_2 r_{AB}\,\underset{\sim}{k} \times \underset{\sim}{n}_{A/B} = \omega_2 r_{AB}\,\underset{\sim}{t}_{AB}$

note: $\|\underset{\sim}{n}_{A/B}\| = \|\underset{\sim}{t}_{A/B}\| = 1$

$\underset{\sim}{n}_{A/B} \cdot \underset{\sim}{t}_{A/B} = 0$

$\underset{\sim}{t}_{C/D} = -\sin\theta_4\,\underset{\sim}{i} + \cos\theta_4\,\underset{\sim}{j}$ $\quad \underset{\sim}{n}_{C/D} = \cos\theta_4\,\underset{\sim}{i} + \sin\theta_4\,\underset{\sim}{j}$

$\underset{\sim}{t}_{A/B} = \sin\theta_2\,\underset{\sim}{i} - \cos\theta_2\,\underset{\sim}{j}$ $\quad \underset{\sim}{n}_{A/B} = -\cos\theta_2\,\underset{\sim}{i} - \sin\theta_2\,\underset{\sim}{j}$

$\underset{\sim}{t}_{B/C} = \sin\theta_3\,\underset{\sim}{i} - \cos\theta_3\,\underset{\sim}{j}$ $\quad \underset{\sim}{n}_{B/C} = -\cos\theta_3\,\underset{\sim}{i} - \sin\theta_3\,\underset{\sim}{j}$

equation ① thus becomes

② $\underset{\sim}{0} = \omega_4 r_{CD}\,\underset{\sim}{t}_{C/D} + \omega_3 r_{BC}\,\underset{\sim}{t}_{B/C} + \omega_2 r_{AB}\,\underset{\sim}{t}_{A/B}$

② $\cdot\,\underset{\sim}{n}_{B/C} \Rightarrow 0 = \omega_4 r_{CD}\,\underset{\sim}{t}_{C/D} \cdot \underset{\sim}{n}_{B/C} + \omega_2 r_{AB}\,\underset{\sim}{t}_{A/B} \cdot \underset{\sim}{n}_{B/C}$ → solve for ω_2

② $\cdot\,\underset{\sim}{n}_{A/B} \Rightarrow 0 = \omega_4 r_{CD}\,\underset{\sim}{t}_{C/D} \cdot \underset{\sim}{n}_{A/B} + \omega_3 r_{BC}\,\underset{\sim}{t}_{B/C} \cdot \underset{\sim}{n}_{A/B}$ → solve for ω_3

So that given the current configuration $(\theta_2, \theta_3, \theta_4)$ and ω_4, we may compute ω_2 and ω_3.

$$\underset{\sim}{a}_A = \underset{\sim}{0} = \underset{\sim}{a}_{C/D} + \underset{\sim}{a}_{B/C} + \underset{\sim}{a}_{A/B}$$

$$\begin{aligned} = & -\underset{\sim}{n}_{C/D}\, \omega_4^2\, r_{CD} + \underset{\sim}{t}_{C/D}\, \alpha_4\, r_{CD} \\ & -\underset{\sim}{n}_{B/C}\, \omega_3^2\, r_{BC} + \underset{\sim}{t}_{B/C}\, \alpha_3\, r_{BC} \\ & -\underset{\sim}{n}_{A/B}\, \omega_2^2\, r_{AB} + \underset{\sim}{t}_{A/B}\, \alpha_2\, r_{AB} \end{aligned}$$

given $\alpha_4 = 0$, this is two equations in the unknowns α_2 and α_3. The solution is found in matlab ... see the .m-files for details ... the matlab code has be set up to save having to do some of the grunt work of writing the equations in $\underset{\sim}{i}, \underset{\sim}{j}$ components

For the acceleration of the centers of mass:

$$\underset{\sim}{a}_{G4} = -\underset{\sim}{n}_{C/D}\, \omega_4^2\, r_{G_4D} + \underset{\sim}{t}_{C/D}\, \alpha_4\, r_{G_4D}$$

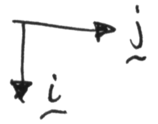
$$\begin{aligned} \underset{\sim}{a}_{G2} &= -\underset{\sim}{n}_{B/A}\, \omega_2^2\, r_{G_2A} + \underset{\sim}{t}_{B/A}\, \alpha_2\, r_{G_2A} \\ &= \underset{\sim}{n}_{A/B}\, \omega_2^2\, r_{G_2A} - \underset{\sim}{t}_{A/B}\, \alpha_2\, r_{G_2A} \end{aligned}$$

$$\begin{aligned} \underset{\sim}{a}_{G3} = \underset{\sim}{a}_C + \underset{\sim}{a}_{G3/C} = & -\underset{\sim}{n}_{C/D}\, \omega_4^2\, r_{CD} + \underset{\sim}{t}_{C/D}\, \alpha_4\, r_{CD} \\ & -\underset{\sim}{n}_{B/C}\, \omega_3^2\, r_{G_3C} + \underset{\sim}{t}_{B/C}\, \alpha_3\, r_{G_3C} \end{aligned}$$

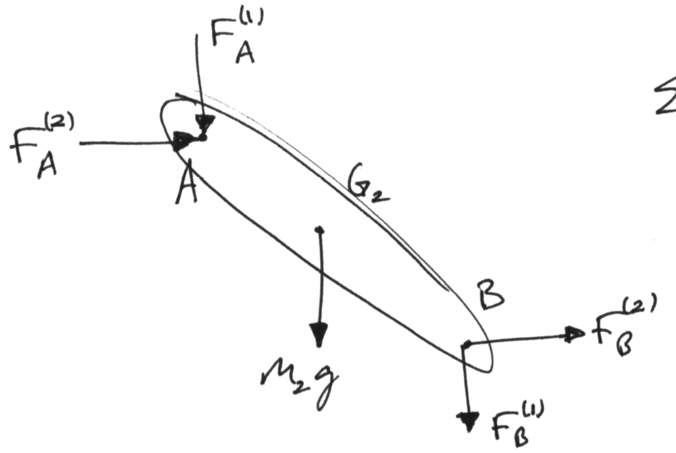
Force & Moment Analysis

set up F.B.D., B.L.M., B.A.M. for the three moving components — obtain a system of equations that may be solved for M_D, the moment at the crankshaft.

(2)



$$\sum \mathbf{F} = m_2 \mathbf{a}_{G2} = m_2 \mathbf{g} + F_A^{(1)}\mathbf{i} + F_A^{(2)}\mathbf{j} + \dots + F_B^{(1)}\mathbf{i} + F_B^{(2)}\mathbf{j}$$



$$\sum \mathbf{M}_{/A} = I_2 \boldsymbol{\alpha}_2 + m_2 \mathbf{r}_{G2/A} \times \mathbf{a}_{G2} = \dots$$

$$= \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{G2/A} \times (m_2 \mathbf{g})$$

$$= r_{G2A}\, m_2 (\mathbf{n}_{B/A} \times \mathbf{g}) + r_{AB} (\mathbf{n}_{B/A} \times \mathbf{F}_B)$$

$$\mathbf{n}_{B/A} \times \mathbf{F}_B = (\cos\theta_2\, \mathbf{i} + \sin\theta_2\, \mathbf{j}) \times (F_B^{(1)}\mathbf{i} + F_B^{(2)}\mathbf{j})$$

$$= (\cos\theta_2\, F_B^{(2)} - \sin\theta_2\, F_B^{(1)})\,\mathbf{k}$$

$$\therefore\ I_2 \alpha_2 + r_{G2A}\, m_2 (\mathbf{n}_{B/A} \times \mathbf{a}_{G2})\cdot\mathbf{k} = r_{G2A}\, m_2 (\mathbf{n}_{B/A} \times \mathbf{g})\cdot\mathbf{k} + \dots + r_{AB}(\cos\theta_2\, F_B^{(2)} - \sin\theta_2\, F_B^{(1)})$$

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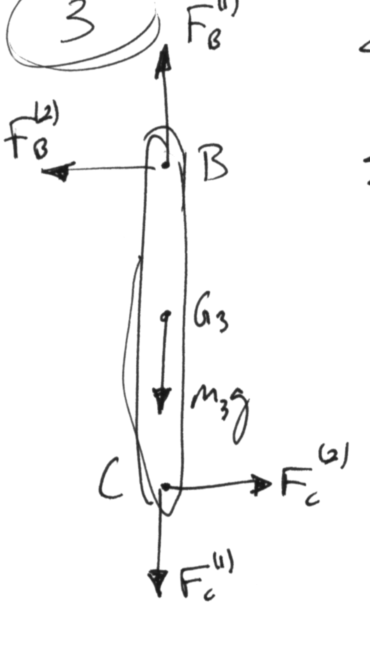
(3)



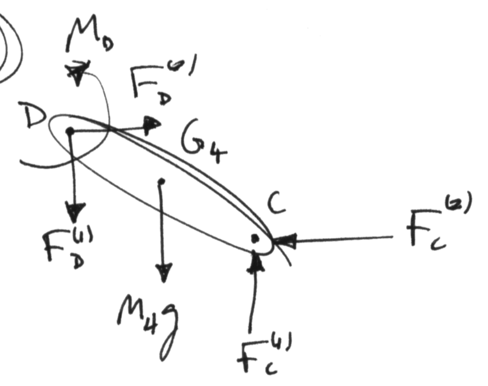
$$\sum \mathbf{F} = m_3 \mathbf{a}_{G3} = m_3 \mathbf{g} - F_B^{(1)}\mathbf{i} - F_B^{(2)}\mathbf{j} + F_C^{(1)}\mathbf{i} + F_C^{(2)}\mathbf{j}$$

$$\sum \mathbf{M}_{/B} = I_3 \boldsymbol{\alpha}_3 + m_3 \mathbf{r}_{G3/B} \times \mathbf{a}_{G3} = \mathbf{r}_{G3/B} \times (m_2 \mathbf{g}) + \dots + r_{BC}\, \mathbf{n}_{C/B} \times \mathbf{F}_C$$

$$\mathbf{n}_{C/B} \times \mathbf{F}_C = (\cos\theta_3\, F_C^{(2)} - \sin\theta_3\, F_C^{(1)})\,\mathbf{k}$$

$$\therefore\ I_3 \alpha_3 + m_3 (\mathbf{r}_{G3/B} \times \mathbf{a}_{G3})\cdot\mathbf{k} = r_{G2A}\, m_2 (\mathbf{n}_{C/B} \times \mathbf{g})\cdot\mathbf{k} + \dots + r_{BC}(\cos\theta_3\, F_C^{(2)} - \sin\theta_3\, F_C^{(1)})$$

4



$$\sum \underset{\sim}{F} = m_4 \underset{\sim}{a}_{G4} = m_4 \underset{\sim}{g} - F_C^{(1)} \underset{\sim}{i} - F_C^{(2)} \underset{\sim}{j} + \dots$$
$$+ F_D^{(1)} \underset{\sim}{i} + F_D^{(2)} \underset{\sim}{j}$$

$$\sum \underset{\sim}{M}_{/C} = I_4 \underset{\sim}{\alpha}_4 + m_4 \underset{\sim}{r}_{G4/C} \times \underset{\sim}{a}_{G4} = \dots$$
$$= M_D \underset{\sim}{k} + \underset{\sim}{r}_{D/C} \times \underset{\sim}{F}_D + \underset{\sim}{r}_{G4/C} \times (m_4 \underset{\sim}{g})$$
$$= M_D \underset{\sim}{k} + r_{CD} (\underset{\sim}{n}_{D/C} \times \underset{\sim}{F}_D) + m_4 r_{G4C} (\underset{\sim}{n}_{D/C} \times \underset{\sim}{g})$$

$$\underset{\sim}{n}_{D/C} \times \underset{\sim}{F}_D = (-\cos\theta_4 F_D^{(2)} + \sin\theta_4 F_D^{(1)}) \underset{\sim}{k}$$

$$\therefore m_4 r_{G4C} (\underset{\sim}{n}_{D/C} \times \underset{\sim}{a}_G) \cdot \underset{\sim}{k} + I_4 \alpha_4 = M_D + r_{CD} (-\cos\theta_4 F_D^{(2)} + \sin\theta_4 F_D^{(1)}) + m_4 r_{G4C} (\underset{\sim}{n}_{D/C} \times \underset{\sim}{g}) \cdot \underset{\sim}{k}$$

All together:

- 9 equations (3 from A.M.B., 6 from L.M.B.)
- 9 unknowns: $F_A^{(1)}, F_A^{(2)}, F_B^{(1)}, F_B^{(2)}, F_C^{(1)}, F_C^{(2)}, F_D^{(1)}, F_D^{(2)}, M_D$

Solve system in matlab for each position of the components at which we want to know M_D!