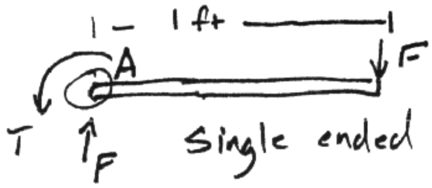
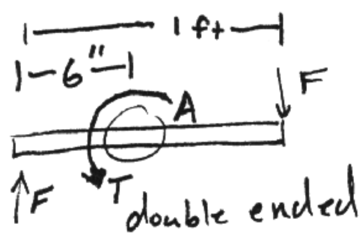
MAE 325 HW 11 Solutions

Due 11/17/99

① 6-21

How many cycles of tightening befor fatigue failure for (a) a single ended wrench and (b) a double ended wrench





$T_{max} = 100$ ft/lbs $\quad d = 0.625''$

$T_{min} = 0$ ft lbs $\quad S_{ut} = 60$ ksi

Max bending stress occurs near the point A in both cases.

Case (a)

$M = FL = T = 100$ ft lbs

$$I = \frac{\pi d^4}{64} = 0.00749 \text{ in}^4$$

Alternating: $M_a = \dfrac{T_{max} - T_{min}}{2} = 600$ in lbs

$M_m = \dfrac{T_{max} + T_{min}}{2} = 600$ in lbs

$$\sigma_{xa} = \frac{M_a c}{I} = 25.033 \text{ Ksi} \qquad \sigma_{xm} = \frac{M_m c}{I} = 25.033 \text{ Ksi}$$

No other stresses in the arm so

$$\sigma_I = \sigma_x \qquad \sigma_{II} = 0 \qquad \sigma_{III} = 0$$

Also, von Mises effective stress is just:

$$\sigma_a' = \sigma_{xa} \qquad \sigma_m' = \sigma_{xm}$$

Assume Case 3 loading and solve for the fatigue strength at which the wrench will fail (safety factor = 1)

$$N_f = 1 = \frac{S_f S_{ut}}{\sigma_a' S_{ut} + \sigma_m' S_f} \Rightarrow S_f = \frac{\sigma_a' S_{ut}}{S_{ut} - \sigma_m'}$$

Solving: $S_f = 42.954$ ksi

Calculate the unmodified endurance limit:

$$S_e' = 0.5\, S_{ut} \rightarrow S_e' = 30 \text{ ksi}$$

Modifying factors:

$C_{load} = 1$

$C_{surf} = A\left(\frac{S_{ut}}{ksi}\right)^b = 0.679$

$\boxed{A = 39.9,\ b = -0.995 \text{ as forged}}$

$C_{temp} = 1$

$C_{reliab} = 1$ (50 %)

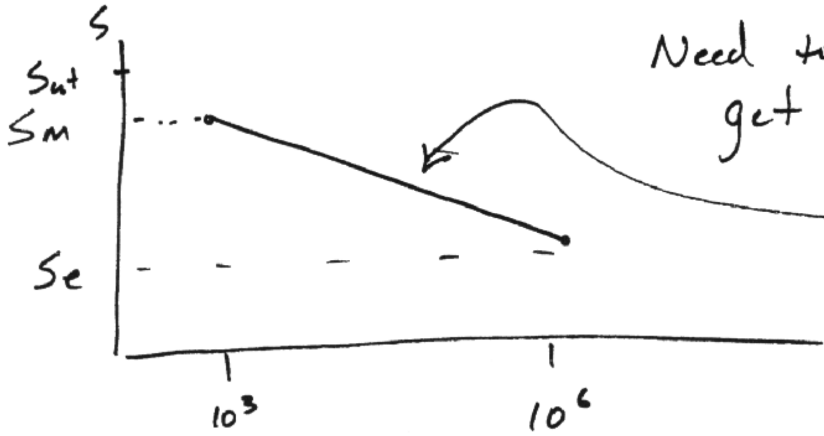
$C_{size} = 0.869\left(\frac{d_{equiv}}{in}\right)^{-0.097} = 1.002 \simeq 1$

$$S_e = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_e'$$
$$= (1)(1)(0.679)(1)(1)(30) = 20.398 \text{ ksi}$$

$S_m = 0.9 S_{ut} = 54 \text{ ksi}$ $(N = 10^3 \text{ cycles})$

$S_f = a N^b$ $N = 10^6 \Rightarrow z = -3.000$

S-N diagram



Need to find a, b to get eq. of line

after you have the eq, you can calc. S @ any amount of cycles.

$b = \frac{1}{z} \log \frac{S_m}{S_e} \Rightarrow b = -0.1409$

$a = \frac{S_m}{(10^3)^b} \Rightarrow a = 142.955 \text{ ksi}$

Now use: $N_a = \left(\frac{S_f}{a}\right)^{\frac{1}{b}}$ to find # of cycles to failur

$= \left(\frac{42.954}{142.955}\right)^{\frac{1}{-0.1409}} = \underline{\underline{5.1 \times 10^3 \text{ cycles}}}$

Case (b)

$M = T/2 \Rightarrow \quad \sigma_x = \frac{Mc}{I} = 12.5\,\text{ksi}$

as before, $\quad \sigma_a' = 12.5 = \sigma_m'$

$$S_f = \frac{\sigma_a' S_{ut}}{S_{ut} - \sigma_m'} \quad \Rightarrow \quad S_f = 15.9\,\text{ksi}$$

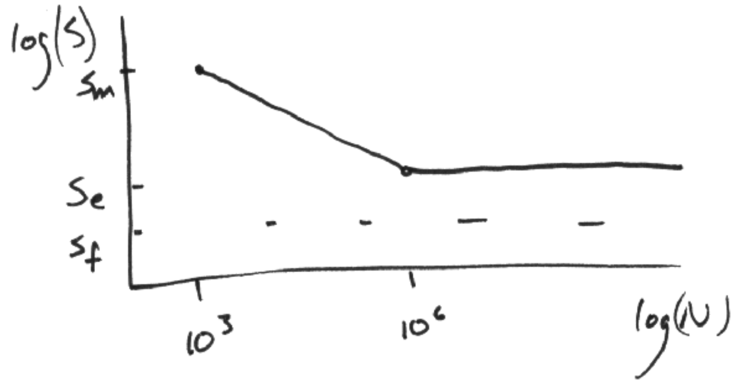
using a, b as found before

$$N_b = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{15.9}{142.955}\right)^{-\frac{1}{0.1409}}$$

$N_b = 6 \times 10^6$ cycles

Note: This answer is nonsense!!

Once you see that $S_f < S_e$ you know that the $\boxed{\text{life is infinite.}}$



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PROBLEM 6-27

Statement: **A storage rack is to be designed to hold the paper roll of Problem 6-8 as shown in Figure P6-12. Determine a suitable value for dimension *a* in the figure for an infinite-life fatigue safety factor of 2. Assume dimension *b* = 100 mm and that the mandrel is solid and inserts halfway into the paper roll.**

(a) The beam is a ductile material with $S_{ut} = 600$ MPa.

(b) The beam is a cast-brittle material with $S_{ut} = 300$ MPa.

Units: $N := newton$ $\quad kN := 10^3 \cdot N$ $\quad MPa := 10^6 \cdot Pa$ $\quad GPa := 10^9 \cdot Pa$

Given:

Paper roll dimensions	$OD := 1.50 \cdot m$	Roll density	$\rho := 984 \cdot kg \cdot m^{-3}$
	$ID := 0.22 \cdot m$		
	$L_{roll} := 3.23 \cdot m$	Design safety factor	$N_{fd} := 2$
Ductile tensile strength	$S_{uta} := 600 \cdot MPa$	Brittle tensile strength	$S_{utb} := 300 \cdot MPa$

Assumptions: The paper roll's weight creates a concentrated load acting at the tip of the mandrel. The mandrel's root fits tightly in the stanchion so it can be modeled as a cantilever beam. The mandrel is machined, reliability is 90%, and it operates at room temperature.

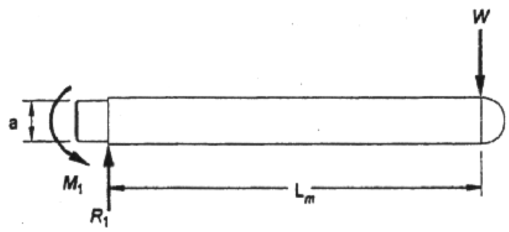

FIGURE 6-27
Free Body Diagram used in Problem 6-27

Solution: See Figure 6-27 and Mathcad file P0627.

1. Determine the weight of the roll and the length of the mandrel.

Weight $\quad W := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right) \cdot L_{roll} \cdot \rho \cdot g \qquad W = 53.9 \, kN$

Length $\quad L_m := 0.5 \cdot L_{roll} \qquad L_m = 1.615 \, m$

2. The maximum moment occurs at a section where the mandrel root leaves the stanchion and is

$$M_{max} := W \cdot L_m \qquad M_{max} = 87.04 \, kN \cdot m$$

4. The dynamic loading is repeated from 0 to M_{max} on each stress cycle, thus $\quad M_{min} := 0 \cdot kN \cdot m$

5. **Part (a)** - Calculate the alternating and mean components of the bending moment.

$$M_a := \frac{M_{max} - M_{min}}{2} \qquad M_a = 43520 \, N \cdot m$$

$$M_m := \frac{M_{max} + M_{min}}{2} \qquad M_m = 43520 \, N \cdot m$$

6. Determine the unmodified endurance limit. $\quad S'_e := 0.5 \cdot S_{uta} \qquad S'_e = 300 \, MPa$

7. Calculate the endurance limit modification factors for a nonrotating rectangular beam.

Load $\quad C_{load} := 1$

Size $\quad A_{95}(a) := 0.010462 \cdot a^2 \qquad d_{equiv}(a) := \sqrt{\frac{A_{95}(a)}{0.0766}}$

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$$C_{size}(a) := 1.189 \cdot \left(\frac{d_{equiv}(a)}{mm}\right)^{-0.097}$$

Surface $A := 4.51$ $b := -0.265$ (machined)

$$C_{surf} := A \cdot \left(\frac{S_{uta}}{MPa}\right)^{b} \qquad C_{surf} = 0.828$$

Temperature $C_{temp} := 1$

Reliability $C_{reliab} := 0.897$ $(R = 90\%)$

8. Calculate the modified endurance limit.

$$S_e(a) := C_{load} \cdot C_{size}(a) \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e$$

9. We can now determine the minimum required diameter, a. Using the distortion-energy failure theory with the modified Goodman diagram, the bending stress will also be the only nonzero principal stress, which will also be the von Mises stress. Assuming a Case 3 load line, use equation (6.18e) to determine the factor of safety. Guess $a := 100 \cdot mm$.

Bending stress $\sigma = \frac{M \cdot c}{I} = M \cdot \frac{a}{2} \cdot \frac{64}{\pi \cdot a^4} = \frac{32 \cdot M}{\pi \cdot a^3}$

Given

$$N_{fd} = \frac{\pi \cdot a^3}{32} \cdot \frac{S_e(a) \cdot S_{uta}}{M_a \cdot S_{uta} + M_m \cdot S_e(a)}$$

$a := find(a)$ $a = 186.864 \cdot mm$

Round this up to the next higher even value $a := 190 \cdot mm$

Using this value of a, the values of the functions of a are:

$C_{size}(a) = 0.787$ $S_e(a) = 175.371 \cdot MPa$

The realized safety factor is

$$N_{fa} := \frac{\pi \cdot a^3}{32} \cdot \frac{S_e(a) \cdot S_{uta}}{M_a \cdot S_{uta} + M_m \cdot S_e(a)} \qquad N_{fa} = 2.1$$

10. **Part (b)** - Determine the unmodified endurance limit. $S'_e := 0.4 \cdot S_{utb}$ $S'_e = 120 \cdot MPa$

11. Calculate the endurance limit size modification factor for a nonrotating rectangular beam.

Size $A_{95}(a) := 0.010462 \cdot a^2$ $d_{equiv}(a) := \sqrt{\frac{A_{95}(a)}{0.0766}}$

$$C_{size}(a) := 1.189 \cdot \left(\frac{d_{equiv}(a)}{mm}\right)^{-0.097}$$

12. Calculate the modified endurance limit.

$$S_e(a) := C_{load} \cdot C_{size}(a) \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e$$

13. We can now determine the minimum required diameter, a. Using the distortion-energy failure theory with the modified Goodman diagram, the bending stress will also be the only nonzero principal stress, which will also be the von Mises stress. Assuming a Case 3 load line, use equation (6.18e) to determine the factor of safety. Guess $a := 100 \cdot mm$.

Bending stress $$\sigma = \frac{M \cdot c}{I} = M \cdot \frac{a}{2} \cdot \frac{64}{\pi \cdot a^4} = \frac{32 \cdot M}{\pi \cdot a^3}$$

Given

$$N_{fd} = \frac{\pi \cdot a^3}{32} \cdot \frac{S_e(a) \cdot S_{utb}}{M_a \cdot S_{utb} + M_m \cdot S_e(a)}$$

$a := find(a)$ $\quad a = 251.687 \cdot mm$

Round this up to the next higher even value $\quad a := 252 \cdot mm$

Using this value of a, the values of the functions of a are:

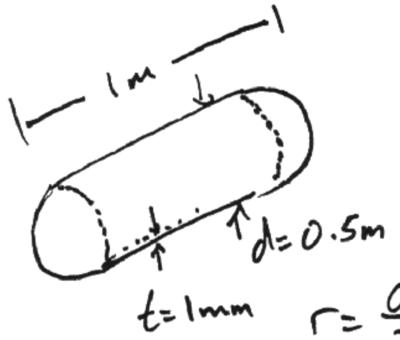
$C_{size}(a) = 0.766$ $\quad S_e(a) = 68.253 \cdot MPa$

The realized safety factor is

$$N_{fb} := \frac{\pi \cdot a^3}{32} \cdot \frac{S_e(a) \cdot S_{utb}}{M_a \cdot S_{utb} + M_m \cdot S_e(a)} \qquad N_{fb} = 2.0$$

(3) 6-42

A cylindrical tank w/ hemispherical ends is required to hold 150 psi of air at room temp. The pressure cycles from zero to maximum. Steel has $S_{ut} = 500$ MPa Find infinite life safety factor if $d = 0.5$ m and 1 mm wall thickness, $L = 1$ m



$r = \frac{d}{2}$

$p = 150$ psi $= 1034$ KPa

• Thin wall pressure vessel

$$\sigma_t = \frac{pr}{t} = 258.55 \text{ MPa}$$

$$\sigma_r = 0 \text{ MPa}$$

$$\sigma_a = \frac{pr}{2t} = 129.28 \text{ MPa}$$

These are alread principals (shear = 0)

so $\sigma_1 = \sigma_t$ $\quad \sigma_2 = \sigma_a$ $\quad \sigma_3 = \sigma_r$

Find von Mises effective stresses

$$\sigma'_{min} = 0 \text{ MPa} \qquad \sigma'_{max} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = 224.3 \text{ MPa}$$

So
$$\sigma'_a = \frac{\sigma'_{max} - \sigma'_{min}}{2} = 112.151 \text{ MPa}$$

$$\sigma'_m = \frac{\sigma'_{max} + \sigma'_{min}}{2} = 112.151 \text{ MPa}$$

Calculate the unmodified endurance limit:

$$S_e' = 0.5\, S_{ut} = 250\ \text{MPa}$$

modification factors:

$C_{load} = 0.7$ (axial)

$C_{size} = 1.0$ (axial)

$C_{temp} = 1$

$C_{reliab} = 0.659$ (99.999%)

$C_{surf} = A\left(\frac{S_{ut}}{MPa}\right)^b = 0.869$ $\boxed{\begin{array}{l} A = 4.51 \\ b = -0.265 \\ \text{machined} \end{array}}$

$$S_e = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_e'$$
$$= (0.7)(1.0)(0.869)(1)(0.659)(250) = 100.2\ \text{MPa}$$

$$N_f = \frac{S_e\, S_{ut}}{\sigma_a'\, S_{ut} + \sigma_m'\, S_e} \quad \text{(case 3)}$$

$N_f = 0.74$