MAE 325 Fall 1999
HW 12 Solutions Due 11/24

1. Norton 9-1f

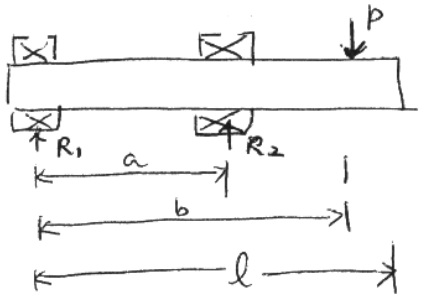
A constant magnitude transverse load p is applied as the shaft rotates ~~subject~~ subject to a time-varying torque that varies from T_{min} to T_{max}. Find the diameter of shaft required to obtain a safety factor of 2 in fatigue loading. Assume no stress concentrations are present.

; Given values ;

$a = 16$ in , $b = 18$ in

$p = 1000$ lbf , $S_{ut} = 108$ ksi

$T_{min} = 1000$ in lbf , $T_{max} = 2000$ in lbf , $N_f = 2$



$$q = R_1\langle x\rangle^{-1} + R_2\langle x-a\rangle^{-1} - P\langle x-b\rangle^{-1}$$

$$V = R_1\langle x\rangle^{0} + R_2\langle x-a\rangle^{0} - P\langle x-b\rangle^{0} + C_1$$

$$M(x) = R_1\langle x\rangle^{1} + R_2\langle x-a\rangle^{1} - P\langle x-b\rangle^{1} + C_1x + C_2$$

B.C

$V(0^-) = 0 \rightarrow C_1 = 0$

$M(0^-) = 0 \rightarrow C_2 = 0$

$V(l^+) = 0 \rightarrow R_1 + R_2 - P = 0 \rightarrow R_1 = P - R_2$

$M(l^+) = 0 \rightarrow R_1 l + R_2(l-a) - P(l-b) = 0$

$$(P - R_2)l + R_2(l-a) - P(l-b) = 0$$

$$R_2(-l + l - a) = -Pl + P(l-b) = -Pb$$

$$\therefore R_2 = P\left(\frac{b}{a}\right) \qquad R_1 = P - R_2 = P\left(\frac{a-b}{a}\right)$$

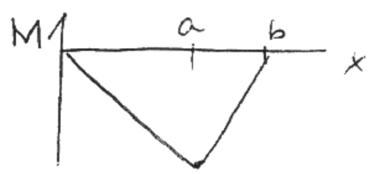
$$\therefore M(x) = P\left[\frac{a-b}{a}\langle x\rangle^1 + \frac{b}{a}\langle x-a\rangle^1 - \langle x-b\rangle^1\right]$$

$M(0) = 0$

$M(a) = P\left[\frac{a-b}{a}a + \frac{b}{a}\right] = P(a-b)$

$M(b) = P\left[\frac{a-b}{a}b + \frac{b}{a}(b-a)\right] = 0$

$\rightarrow$



M is maximum when $x = a$ (ie maximum moment in the shaft occurs at the right bearing)

$\therefore M_{max} = |P(a-b)| = 4500$ in lbf

Because the shaft rotates, a point on the surface sees a fluctuating normal stress, as though there were a fluctuating moment between $M_{min} = -M_{max}$ and M_{max}. However, the actual moment on the cross section is constant in time.

Alternate bending moment; $M_a = \frac{M_{max} - M_{min}}{2} = \frac{2M_{max}}{2} = M_{max} = 4500$ in lb.

mean bending moment; $M_m = \frac{M_{max} + M_{min}}{2} = 0$

Calculate the unmodified endurance limit

$$S_e' = 0.5\, S_{ut} = 54 \text{ kpsi}$$

Determine the endurance limit modification factors for a rotating round shaft.

$C_{load} = 1$

$C_{size}(d) = 0.869 \left(\frac{d}{in}\right)^{-0.097}$

$C_{surf} = A\left(\frac{S_{ut}}{ksi}\right)^b = 0.781$ ($A = 2.70$, $b = -0.265$ as machined)

$C_{temp} = 1$

$C_{reliab} = 0.814$ (99%)

Modified endurance limit

$$S_e(d) = C_{load}\, C_{size}(d)\, C_{surf}\, C_{temp}\, C_{reliab}\, S_e'$$

$$= 29.8325 \left(\frac{d}{in}\right)^{-0.097}$$

mean torque $T_m = \frac{T_{max} + T_{min}}{2} = 1{,}500$ in lbf

alternating torque $T_a = \frac{T_{max} - T_{min}}{2} = 500$ in lbf

Calculate shaft diameter (d) using equation (9.8) ← eq for fluctuating bending

$$d = \left\{ \frac{32 N_f}{\pi} \left[\frac{\sqrt{(K_f M_a)^2 + \frac{3}{4}(K_{fs} T_a)^2}}{S_f} + \frac{\sqrt{(K_{fm} M_m)^2 + \frac{3}{4}(K_{fsm} T_m)^2}}{S_{ut}} \right] \right\}^{1/3}$$

As no stress concentrations is assumed, all stress concentration factor in above equation is 1.

i.e. $K_f = K_{fs} = K_{fm} = K_{fsm} = 1$ & $M_m = 0$

$$\therefore d = \left\{ \frac{32 N_f}{\pi} \left[\frac{\sqrt{(M_a)^2 + \frac{3}{4}(T_a)^2}}{S_e} + \frac{\sqrt{\frac{3}{4}}\, T_m}{S_{ut}} \right] \right\}^{1/3}$$

Solve it to get d.

Note, – Se and Sut should be in psi unit
– Se is a function of d

→ use MATLAB 'fsolve' to solve for d

```
>> d = fsolve('d_ftn', 1)
```

↑ correspond to initial guess of d

where d_ftn.m is defined in next page.

Note,

→ answer ; $\boxed{d = 1.5123 \text{ in}}$

```
function out = d_ftn(d)

% define some variables
Nf = 2;
Ma = 4500; % in*lbf
Ta = 500;  % in*lbf
Tm = 1500; % in*lbf
Sut = 108 * 1.e3;   % in psi
Se = 29.8325*d^-0.097 *1.e3;  % in psi

% define right hand side of equation
d_rhs = (32*Nf/pi*(sqrt(Ma^2+3/4*Ta^2)/Se + ...
         sqrt(3/4)*Tm/Sut))^(1/3) ;

% define f(d) = 0
out = d_rhs - d;
```

2. Norton 10-5

; Find the Petroff no-load torque for the journal designed in Case study 9B (see p691)

Given values;

oil viscosity $\eta = 1.5\ \mu\text{reyn} = 1.5 \times 10^{-6}\ \text{lbf-sec-in}^{-2}$

diametral clearance in the bearing ; $C_d = 0.002\ \text{in}$

Shaft diameter ; $d = 2\ \text{in}$

Shaft speed ; $n' = 180\ \text{rpm} = 180\ \frac{\text{rev}}{\text{min}} = 3\ \frac{\text{rev}}{\text{sec}}$

Bearing length ; $l = 2\ \text{in}$

Petroff's eq. for no-load torque (eq. 10.2c in p695)

$$T_o = \eta \frac{\pi^2 d^3 l n'}{C_d}$$

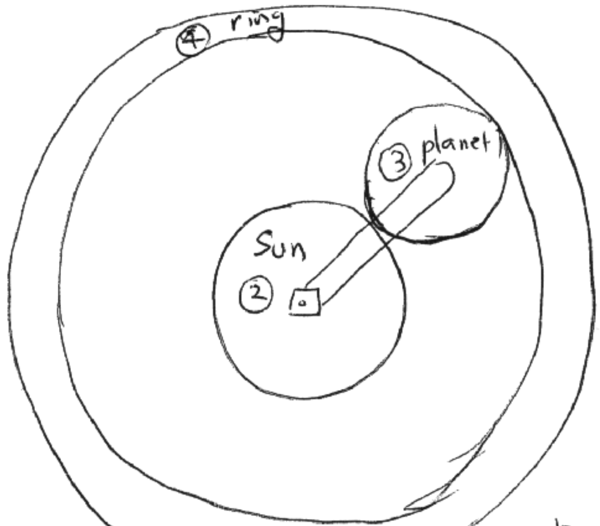
$$= \frac{1.5\times10^{-6}\ (\text{lbf}\cdot\text{sec in}^{-2})\ \pi^2\ (2\ \text{in})^3\ (2\ \text{in})\ (3/\text{sec})}{0.002\ \text{in}}$$

$$= 0.3353\ \frac{\text{lbf}\cdot\text{sec in}^{-2}\ \text{in}^3\ \text{in}/\text{sec}}{\text{in}}$$

$$\therefore \boxed{T_o = 0.3553\ \text{in}\cdot\text{lbf}}$$

3. Norton 11-11

; An epicycle spur gear train has a sun gear of 33 tooth and a planet gear of 21 tooth. Find the required # of teeth in the ring gear & determine the ratio between the arm and the sun if the ring gear is stationary Consider the arm to rotate at 1 rpm.



Given values;

$N_s = 33$; # of sun tooth

$N_p = 21$; # of planet tooth

$\omega_a = 1\ rpm$; arm speed

$\omega_r = 0\ rpm$; ring speed

From figure,

$$\underbrace{d_r}_{\text{diameter of gear}} = \underbrace{d_s}_{\text{diameter of sun gear}} + 2\underbrace{(d_p)}_{\text{diameter of planet gear}}$$

As sun, planets, and rings gear must have the same diametral pitch $(P = \frac{N}{d})$

$$d_r = d_s + 2d_p$$

$$P_r N_r = P_s N_s + 2 P_p N_p \quad \xrightarrow{P_r = P_s = P_p} \quad N_r = N_s + 2N_p = 75$$

$$\therefore \boxed{N_r = 75}$$

From the equation for the overall train ratio m_v

$$m_v = \frac{\omega_L - \omega_a}{\omega_F - \omega_a} \qquad \text{(eq 11.11c)}$$

Choose the sun gear as the first gear and the ring gear as the last, then $\omega_L = \omega_r$, $\omega_F = \omega_s$

$$\therefore \quad \frac{\omega_r - \omega_a}{\omega_s - \omega_a} = m_v = \underset{\text{External set}}{\left(-\frac{N_2}{N_3}\right)} \underset{\text{internal set}}{\left(+\frac{N_3}{N_4}\right)} = -\frac{N_2}{N_4} = -\frac{N_s}{N_r}$$

arrange to get ω_s

$$(\omega_s - \omega_a)\left(-\frac{N_s}{N_r}\right) = \omega_r - \omega_a$$

$$\omega_s = \frac{\omega_r - \omega_a}{\left(-\frac{N_s}{N_r}\right)} + \omega_a = 3.2727 \text{ rpm}$$

$\therefore$ ratio between between the sun and arm gear

$$\boxed{\frac{\omega_s}{\omega_a} = 3.2727}$$