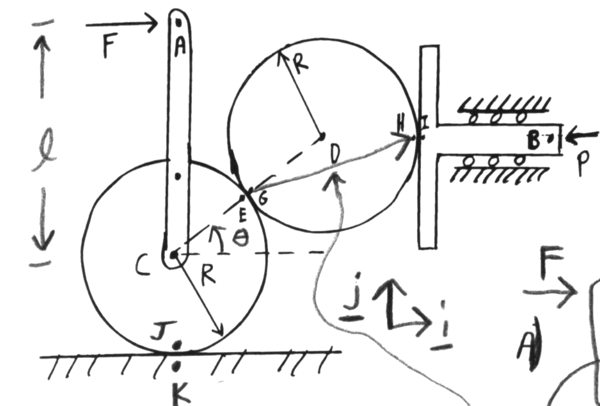
"Solutions" MAE 325 Prelim 1, Fall 1999, Oct 7, 1999

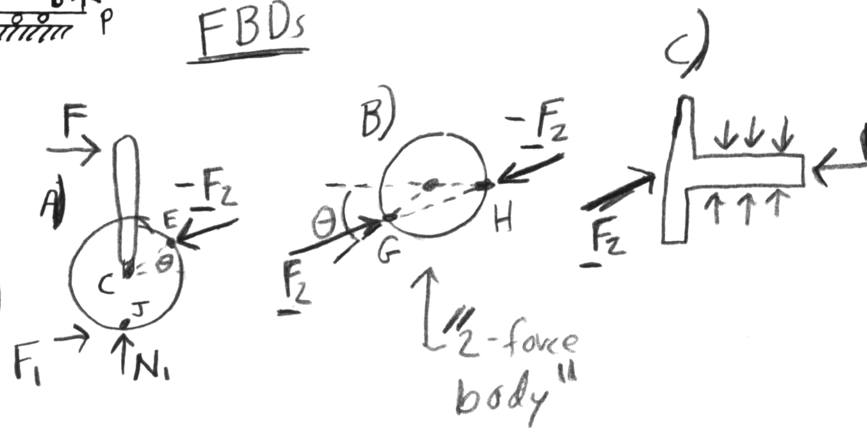
1) (35 pt) Do *either* part (a) *or* part (b). Five point bonus if you do them both fully correctly.). You are given F, ℓ, R, θ and that the point A is moving to the right at speed v_A. Assume statics, neglect gravity and asume roll without slip at all contacts.

⇐ *Please put scrap work for problem 1 on the page to the left* ⇐.
⇓ *Put neat, clear work to be graded for problem 1 below.* ⇓
(If you need the space, clearly mark work to be graded on the scrap page.)



a) Find P.

b) Find the velocity of point B.



$\underline{r}_{GH} = R[(1+\cos\theta)\underline{i} + \sin\theta \underline{j}]$

$$\underline{F}_2 = F_2 \underline{\lambda}_{GH} = F_2 \frac{\underline{r}_{GH}}{|\underline{r}_{GH}|}$$

$$\text{Body C: } \{\Sigma \underline{F} = \underline{0}\}\cdot\underline{i} \Rightarrow F_2 \underline{\lambda}_{GH}\cdot\underline{i} = P \Rightarrow \frac{F_2(1+\cos\theta)}{\sqrt{(1+\cos\theta)^2 + (\sin\theta)^2}} = P \quad (1)$$

$$\text{Body A: } \{\Sigma \underline{M}_{/J} = \underline{0}\}\cdot\underline{k} \Rightarrow (\underline{r}_{JE} \times -\underline{F}_2)\cdot\underline{k} - F(\ell+R) = 0$$

$$\underline{k}\cdot R[\cos\theta\underline{i} + (1+\sin\theta)\underline{j}] \times \frac{-F_2[(1+\cos\theta)\underline{i} + \sin\theta\underline{j}]}{\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}} = F(\ell+R)$$

$$\Rightarrow \frac{-RF_2[\cos\theta\sin\theta - (1+\sin\theta)(1+\cos\theta)]}{\sqrt{(1+\cos\theta)^2+(\sin\theta)^2}} = F(\ell+R) \quad (2)$$

$$(1)\,\&\,(2) \Rightarrow \frac{RP}{(1+\cos\theta)}[1+\cos\theta+\sin\theta] = F(\ell+R)$$

$$\boxed{P = F\frac{\ell+R}{R}\,\frac{1+\cos\theta}{1+\cos\theta+\sin\theta}} \quad (a)$$

Special cases
$\theta = \pi/2 \Rightarrow P = F\frac{\ell+R}{2R}$
$\theta = 0 \Rightarrow P = F\frac{\ell+R}{R}$

Take $\underline{v}_B = v_B \underline{i}$

Power Balance $\Rightarrow F v_A - P v_B = 0$

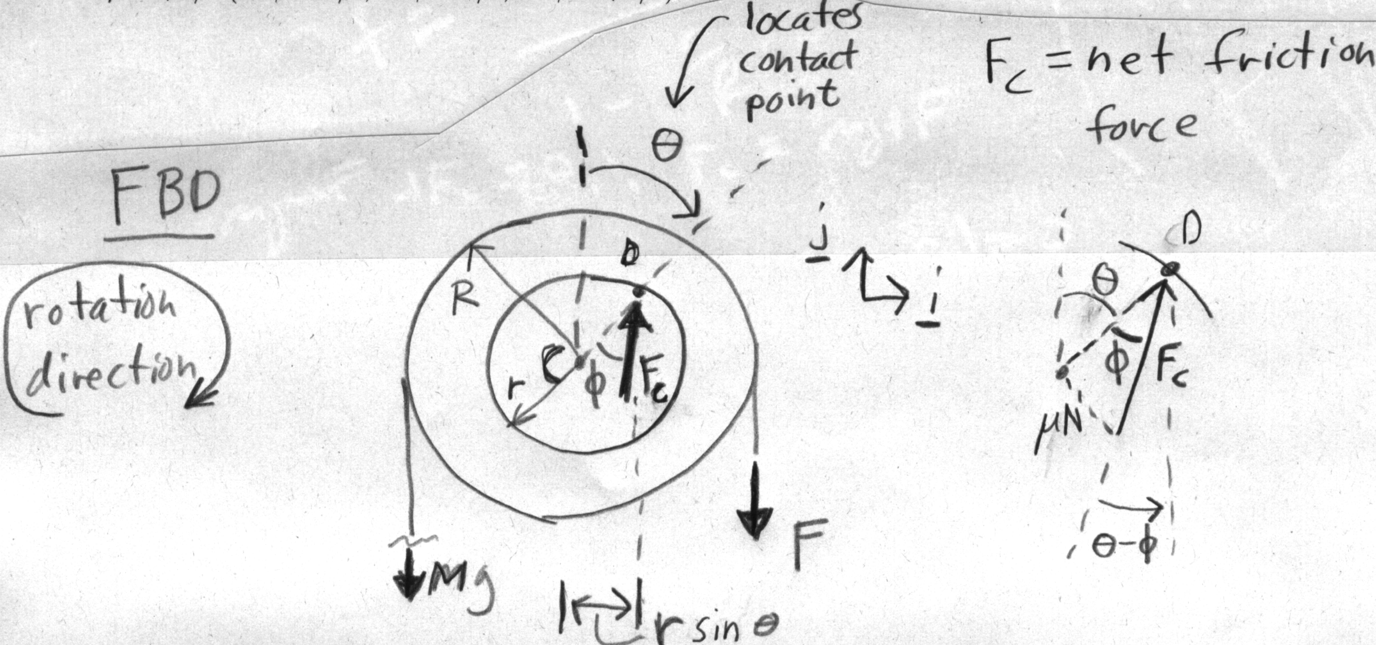
$$\Rightarrow v_B = \frac{F}{P} v_A \Rightarrow \boxed{v_B = v_A \frac{R}{\ell+R}\,\frac{1+\cos\theta+\sin\theta}{1+\cos\theta}} \quad (b)$$

2) (35 pt) A weight M is steadily raised by application of a force F on a rope going over a pulley on an unlubricated journal bearing (no ball bearings). The friction coefficient between the bearing and its axle is $\mu = \tan\phi$. (25 pt if either part below is answered correctly, so you can jump to part (b) if you strongly prefer numbers.)

⇐ Please put scrap work for problem 2 on the page to the left ⇐.
⇓ Put neat, clear work to be graded for problem 2 below. ⇓
(If you need the space, clearly mark work to be graded on the scrap page.)

a) Find F in terms of M, g, R, r and μ (or ϕ or $\sin\phi$ or $\cos\phi$ — whichever is most convenient. For example $\cos(\tan^{-1}(\mu))$ is just $\cos\phi$).

b) Evaluate F in the special case that $M = 100\,\mathrm{kg}, g = 10\mathrm{m/s^2}, r = 1\,\mathrm{cm}, R = 2\,\mathrm{cm}$, and $\mu = \sqrt{3}/3$ (so $\phi = \pi/6, \sin\phi = 1/2, \cos\phi = \sqrt{3}/2$).



$$\sum \underline{F} = \underline{0} \Rightarrow (-mg - F)\underline{j} + (\cos(\theta-\phi)\underline{j} + \sin(\theta-\phi)\underline{i})F_c = \underline{0}$$

$$\{\sum \underline{F} = \underline{0}\}\cdot\underline{i} = 0 \Rightarrow \sin(\theta-\phi) = 0 \Rightarrow \theta - \phi = 0 \Rightarrow \boxed{\theta = \phi}$$ Now we know where D is.

$$\{\sum \underline{M}_{/D} = \underline{0}\}\cdot\underline{k} = 0 \Rightarrow Mg(R + r\sin\theta) - F(R - r\sin\theta) = 0$$

$$\Rightarrow F = \frac{R + r\sin\theta}{R - r\sin\theta} Mg$$

$$\theta = \phi \Rightarrow \boxed{F = \frac{1 + \frac{r}{R}\sin\phi}{1 - \frac{r}{R}\sin\phi} Mg} \quad (a)$$

$$= \frac{1 + (\frac{1}{2})(\frac{1}{2})}{1 - (\frac{1}{2})(\frac{1}{2})}(100\,\mathrm{kg})\cdot 10\,\mathrm{N/kg}$$

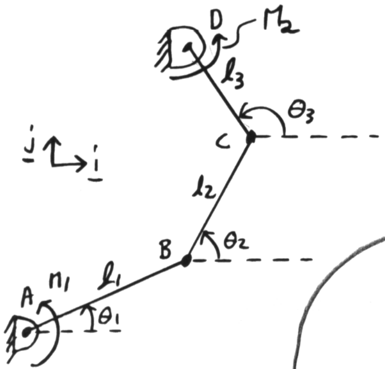
$$= \frac{5}{3}1000\,\mathrm{N} \Rightarrow \boxed{F = 1667\,\mathrm{N}} \quad (b)$$

3) (30 pt) Do *either* part (a) *or* part (b). Five point bonus if you do them both correctly. Consider the mechanism below with a motor at A driving a load at D. Shown at A and D are the torques acting on the mechanism from the motor and load. Assume statics. Neglect gravity. Assume $\theta_1, \ell_1, \theta_2, \ell_2, \theta_3, \ell_3, \dot{\theta}_1$, and M_1 are known.

⇐ *Please put scrap work for problem 3 on the page to the left* ⇐.

⇓ *Put neat, clear work to be graded for problem 3 below.* ⇓

(If you need the space, clearly mark work to be graded on the scrap page.)



a) Find M_2, *or*

b) Find $\dot{\theta}_3$.

Use soln. to part (b) to solve (a)

Power Balance $\Rightarrow M_1\omega_1 + M_2\omega_3 = 0$

$$\Rightarrow M_2 = -M_1 \frac{\omega_1}{\omega_3}$$

$$\boxed{M_2 = -M_1 \frac{\ell_3 \sin(\theta_2-\theta_1)}{\ell_1 \sin(\theta_3-\theta_2)}} \quad (a)$$

$$\underline{v}_D = \underline{v}_D$$

$$\underline{0} = \underline{v}_B + \underline{v}_{C/B} + \underline{v}_{D/C}$$

$$= \omega_1 \underline{k} \times \underline{r}_{AB} + \omega_2 \underline{k} \times \underline{r}_{BC} + \omega_3 \underline{k} \times \underline{r}_{CD}$$

$$\Rightarrow \underline{0} = \underline{k} \times \Big[\underbrace{\omega_1 \underline{r}_{AB} + \omega_2 \underline{r}_{BC} + \omega_3 \underline{r}_{CD}}_{\text{in x-y plane}}\Big]$$

$$\Rightarrow \quad \omega_1 \underline{r}_{AB} + \omega_2 \underline{r}_{BC} + \omega_3 \underline{r}_{CD} = \underline{0}$$

$$\Rightarrow \quad \{\omega_2 \underline{r}_{BC} + \omega_3 \underline{r}_{CD} = -\omega_1 \underline{r}_{AB}\}$$

$$\underline{r}_{BC} \times \{\,\} \Rightarrow \left[\omega_3\, \underline{r}_{BC} \times \underline{r}_{CD} = -\omega_1 \underline{r}_{BC} \times \underline{r}_{AB}\right]$$

$$[\;]\cdot\underline{k} \quad \Rightarrow \quad \boxed{\omega_3 = -\omega_1 \frac{(\underline{r}_{BC} \times \underline{r}_{AB})\cdot\underline{k}}{(\underline{r}_{BC} \times \underline{r}_{CD})\cdot\underline{k}}}$$

$$= \omega_1 \frac{-\ell_2 \ell_1 (\cos\theta_2 \underline{i} + \sin\theta_2 \underline{j}) \times (\cos\theta_1 \underline{i} + \sin\theta_1 \underline{j}) \cdot \underline{k}}{\ell_2 \ell_3 ((\cos\theta_2 \underline{i} + \sin\theta_2 \underline{j}) \times (\cos\theta_3 \underline{i} + \sin\theta_3 \underline{j})) \cdot \underline{k}}$$

$$\boxed{\dot{\theta}_3 = \omega_3 = -\underset{\substack{\uparrow \\ \dot{\theta}_1}}{\omega_1} \frac{\ell_1}{\ell_3} \frac{\cos\theta_2 \sin\theta_1 - \sin\theta_2 \cos\theta_1}{\cos\theta_2 \sin\theta_3 - \sin\theta_2 \cos\theta_3} = \frac{-\omega_1 \ell_1 \sin(\theta_2-\theta_1)}{\ell_3 \sin(\theta_3-\theta_2)}} \quad (b)$$

Note: answers do not depend on ℓ_2. Would you expect this?

Alternate Solution to part B

$r_{AB} = l_1(-\cos\theta_1 \underline{i} - \sin\theta_1 \underline{j})$

$r_{BC} = l_2(-\cos\theta_2 \underline{i} - \sin\theta_2 \underline{j})$

$r_{CD} = l_3(-\cos\theta_3 \underline{i} - \sin\theta_3 \underline{j})$

$V_D \equiv V_D$

$$\underline{0} = \omega_3 \hat{k} \times \underline{r}_{CD} + \omega_2 \hat{k} \times r_{BC} + \omega_1 \hat{k} \times \underline{r}_{AB}$$

$$\left\{ \underline{0} = (-\omega_3 l_3 \cos\theta_3 \underline{j} + l_1 \omega_3 \sin\theta_3 \underline{i}) + (-\omega_2 l_2 \cos\theta_2 \underline{j} + l_3 \omega_2 \sin\theta_2 \underline{i}) + (-\omega_1 l_1 \cos\theta_1 \underline{j} + l_1 \omega_1 \sin\theta_2 \underline{i}) \right\}$$

$$\{ \ \} \cdot \underline{i} \Rightarrow \quad 0 = l_3 \omega_3 \sin\theta_3 + l_2 \omega_2 \sin\theta_2 + l_1 \omega_1 \sin\theta_1 \qquad (1)$$

$$\{ \ \} \cdot \underline{j} \Rightarrow \quad 0 = l_3 \omega_3 \cos\theta_3 + l_2 \omega_2 \cos\theta_2 + l_1 \omega_1 \cos\theta_1 \qquad (2)$$

using (1)

$$\omega_2 = \frac{-l_3 \omega_3 \sin\theta_3 - l_1 \omega_1 \sin\theta_1}{l_2 \sin\theta_2} \qquad (3)$$

plug (3) into (2)

$$0 = l_3 \omega_3 \cos\theta_3 + \cancel{l_2}\left(\frac{-l_3 \omega_3 \sin\theta_3 - l_1 \omega_1 \sin\theta_1}{\cancel{l_2}\sin\theta_2}\right)\cos\theta_2 + l_1 \omega_1 \cos\theta_1$$

$$0 = l_3 \omega_3 \left(\cos\theta_3 - \frac{\sin\theta_3 \cos\theta_2}{\sin\theta_2}\right) + l_1 \omega_1 \left(\cos\theta_1 - \frac{\sin\theta_1 \cos\theta_2}{\sin\theta_2}\right)$$

$$\boxed{\omega_3 = -\omega_1 \frac{l_1}{l_3} \frac{\cos\theta_1 \sin\theta_2 - \sin\theta_1 \cos\theta_2}{\cos\theta_3 \sin\theta_2 - \sin\theta_3 \cos\theta_2}}$$