Prelim 1, solution to problem 1, part b, by kinematics

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This is a sample solution to problem 1 part b on the prelim, using a kinematics based approach. The solution that has already been posted is complete and correct, but does not show you this way of doing the problem. For the sake of brevity (that is to save myself some work) the problem is not restated here.

Define

- body 1, with angular velocity $\overline{\omega}_1$, contains point C
- body 2, with angular velocity $\overline{\omega}_2$, contains point D
- body 3, which cannot spin, contains point B
- point P, not depicted in the original problem description, is the point of contact between body 1 and the ground

To begin, note that given the velocity of point A, and no slipping between the ground and body 1, the motion of body 1 is completely defined. Also note that the velocity of point B, $\overline{v}_B$, is along the $\hat{\imath}$ direction and completely describes the motion of body 3. Finally, notice that body 2 remains in contact with body 3, so if we know $\overline{v}_B$ and how quickly body 2 is rolling along the plunger (body 3) then we know everything about the kinematics of the problem. Our unknowns will then be v_B (the magnitude of $\overline{v}_B$) and ω_2 (the magnitude of $\overline{\omega}_2$).

One way to get the necessary equations is to write $\overline{v}_D$ in two ways. For the first, use point P as the reference point of zero velocity.

$$\begin{aligned}
\overline{v}_D &= \overline{v}_{D/G} + \overline{v}_G \\
&= \overline{v}_{D/G} + \overline{v}_{E/P} \\
&= \overline{\omega}_2 \wedge \overline{r}_{D/G} + \overline{\omega}_1 \wedge \overline{r}_{E/P} && (1) \\
&= R\omega_2 \hat{k} \wedge (\cos\theta\hat{\imath} + \sin\theta\hat{\jmath}) + (\omega_1 \hat{k}) \wedge (R\hat{\jmath} + R\cos\theta\hat{\imath} + R\sin\theta\hat{\jmath}) && (2)
\end{aligned}$$

For the second way of writing $\overline{v}_D$, trace the velocity through to the plunger.

$$\begin{aligned}
\overline{v}_D &= \overline{v}_{D/H} + \overline{v}_H \\
&= \overline{\omega}_2 \wedge \overline{r}_{D/H} + \overline{v}_B \\
&= -R\omega_2 \hat{j} + v_B \hat{i}
\end{aligned} \tag{3}$$

Equating 2 and 3 above, and taking the $\hat{j}$ component, we see that

$$\omega_2 = -\omega_1 \frac{\cos\theta}{\cos\theta + 1}$$

Substituting this result into that for the $\hat{i}$ component, we get an expression for v_B:

$$\begin{aligned}
v_B &= R\omega_1 \left(\frac{\sin\theta\cos\theta}{\cos\theta + 1} - (1 + \sin\theta) \right) \\
&= R\omega_1 \left(\frac{-1 - \cos\theta - \sin\theta}{1 + \cos\theta} \right)
\end{aligned} \tag{4}$$

To get the final answer we need to know ω_1. From $\overline{v}_A = \overline{\omega}_1 \wedge \overline{r}_{A/P}$ we get

$$\omega_1 = -\frac{v_A}{l + R}$$

if we define $\overline{v}_A \equiv -v_A \hat{i}$. Substituting into Equation 4, we arrive at the same result as that obtained by the equilibrium approach:

$$v_B = v_A \frac{R}{l + R} \frac{1 + \cos\theta + \sin\theta}{1 + \cos\theta}$$