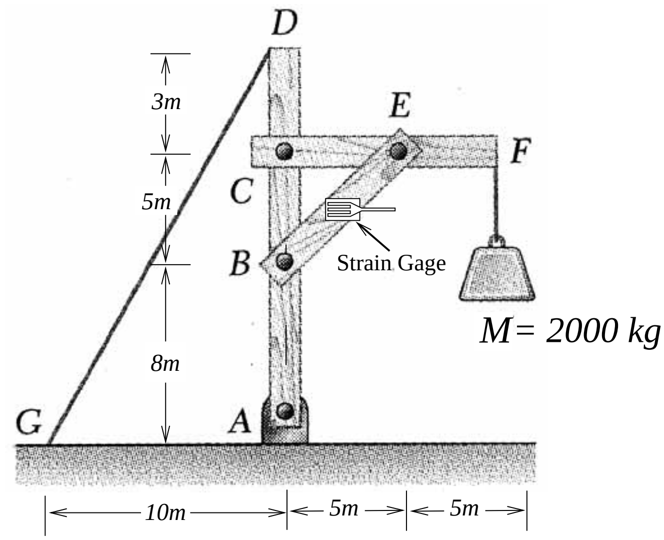
ENGRD 2020 – MECHANICS OF SOLIDS

Final Examination **Maximum 110 Points** **Monday, December 16, 2013**

Academic integrity is expected of all students at all times whether in the presence or absence of members of the teaching staff. If you understand this and agree that you will neither give, nor use, nor receive unauthorized aid in this examination, please sign the Academic Integrity space on the examination booklet. Thank you.
• *Please turn off your cell phone. No calls, texts or tweets during the exam.* •

20 Points
(a) - 2
(b) - 3
(c) - 3
(d) - 6
(e) - 6

1. **Frames. Stresses, Strains.** The figure at right shows a crane in the form of the frame $\overline{AB \ldots F}$ made of near weightless elements which are supporting a mass of 2000 [kg]. The frame is anchored by cable D-G.



A frame used as a crane.

   (a) Draw the *free body diagram* for the crane.

   (b) Dismember the frame and show the *free-body diagram* of each member.

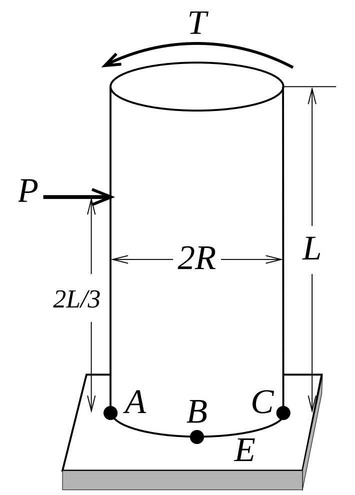
   (c) Determine the force in cable DG.

   (d) Determine the magnitude and the direction (or components) of every force acting on member $\overline{CEF}$.

   (e) If member BE has cross-section: width 200 mm and thickness 50 mm - and is fabricated out of Aluminum (*Young's Modulus*, 70 GPa) what will be the expected strain value measured by a strain gage attached at $45°$ to the line BE?

20 Points
(a) - 3
(b) - 5
(c) - 2
(d) - 5
(e) - 5

2. **Torsion; Bending; Stresses.** Consider the vertical rod of length L and diameter $2R$ shown in the figure. The rod is rigidly attached to a base at the bottom and it is subjected to the torque $\boldsymbol{T}$ and lateral force $\boldsymbol{P}$ at $2L/3$ from the bottom as shown.
The rod is fabricated out of 6061-T6 Aluminum which has a yield stress σ_{yield} of 35 ksi and a yield shear stress τ_{yield} of 20 ksi.



A rod subjected to torsion and bending.

   (a) Letter E denotes the end of the rod which is attached to the base at the bottom. Draw the *free body diagram* of the rod when loaded as shown.

   (b) Find the reaction(s).

   (c) The three locations A, B and C are identified on the outer surface of the rod at end E.
      i. At which location(s) will the shear stresses τ due to torsion be maximal?

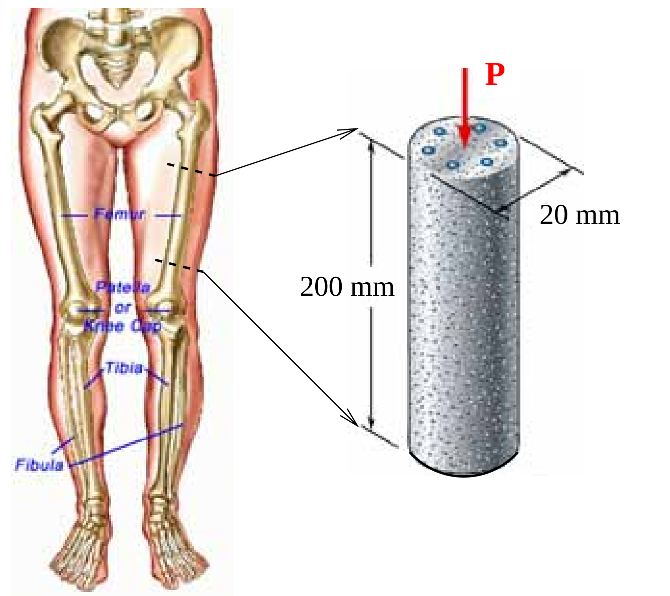
ii. At which location(s) will the tensile/compressive bending stresses σ be maximal?

(d) What will be the maximum permissable applied load $\boldsymbol{P}$ (with $\boldsymbol{T}=0$)?
(Express the result in terms of the given parameters.)

(e) What will be the maximum permissable applied torque $\boldsymbol{T}$ (with $\boldsymbol{T}=0$)?
(Express the result in terms of the given parameters.)

20 Points
(a) - 3
(b) - 3
(c) - 10
(d) - 4

3. **Axial Deformations; Statically Indeterminate Problem.** It's proposed to fabricate a prosthetic *femur* to replace one that was shattered in an accident. Specifically, it's hoped to replace the central 8-inch (200 mm) portion as shown in the figure using a *polyether-ether-ketone (PEEK)* polymer.



Sketch of prosthetic femur.

	σ_f [MPa]	*Young's Modulus* [GPa]
PEEK	90 [MPA]	3.6 [GPa]
Stainless Steel	520 [MPa]	190 [GPa]

(a) For a 225 lb ($\approx$1 kN) man what will be the force $\boldsymbol{P}$ that the femur in the man's leg needs to support? *For all calculations in this problem you may assume that the femur in the leg is nearly vertical.*

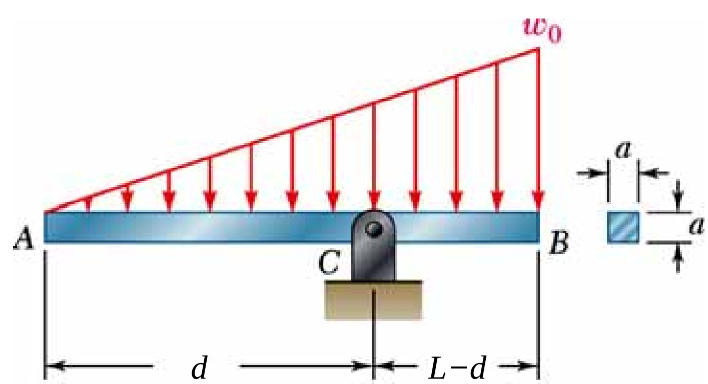
(b) It's known that the *failure stress* σ_f of PEEK is about 90 MPa, what must be the minimal cross-sectional area of the prosthetic femur fabricated of PEEK if a *factor of safety FS* of 3.0 is to be used.

(c) A bright engineer proposes to increase the strength of the prosthetic by imbedding six (6) stainless steel rods in a PEEK cylinder as shown schematically in the figure. Not shown (for clarity) is that the load $\boldsymbol{P}$ is carried by **both** the 20 mm diameter PEEK cylinder and the six steel rods. What diameter rod should she specify, so that the load the prosthetic carries is equally distributed between the *PEEK* cylinder and the six stainless steel rods?

(d) What will be the *factor of safety (FS)* of the composite prosthetic?

25 Points
(a) - 2
(b) - 2
(c) - 2
(d) - 5
(e) - 5
(f) - 4
(g) - 5

4. **Static Equilibrium; Shear and Moment Diagrams; Beam Stresses.** A "balanced beam" of length L has a linearly increasing load applied over its length whose maximum value reaches w_0 at B. The beam is near weightless and is supported by a pin-joint at C.



A "balanced beam".

(a) Draw the *free body diagram* of this beam.

(b) Find the value of d so that the beam remains horizontal.

If you cannot find the value of $\mathbf{d}$ *leave it as a constant in the subsequent calculations. It will result in a slightly messier result.*

(c) Find the reaction(s) at C when the beam is horizontal.

(d) Draw the shear force diagram for this beam when it is in the equilibrium position shown. Be sure to label all important values and clearly show where they occur.

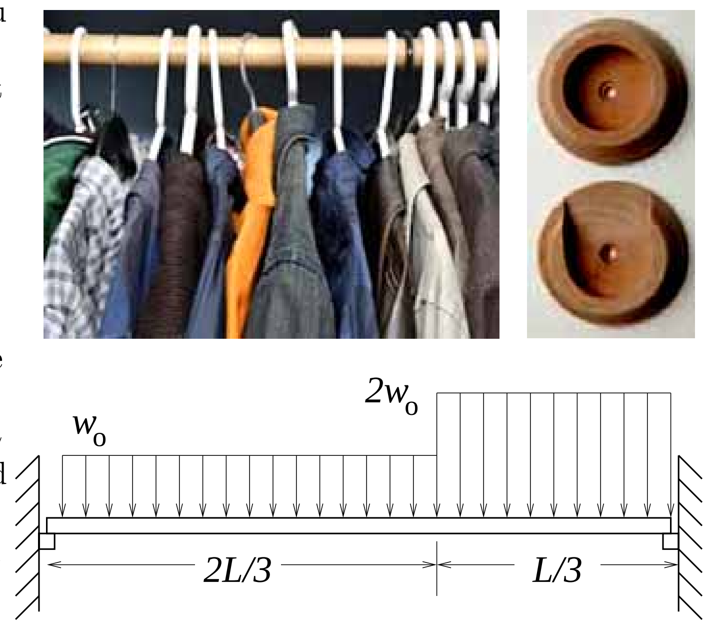
(e) Draw the bending moment diagram for this beam when it is in the equilibrium position shown. Be sure to label all important values and clearly show where they occur.

(f) The beam has cross-section given by $a \times a$. Determine the maximum bending stress for this beam when it is balanced. Where along the beam axis and where on the beam cross-section does it occur?

(g) Determine the maximum shear stress for this beam when it is balanced. Where along the beam axis and where on the beam cross-section (i. e. $a \times a$) does it occur?

25 Points
(a) - 2
(b) - 3
(c) - 8
(d) - 5
(e) - 4
(f) - 3

5. **Design of Beams for Strength and Stiffness.** As a special gift for a friend you decide to design a clothes hanger bar made of white oak for a walk-in closet. The closet is 12 ft wide and the bar will be a rod of circular cross-section which is supported at the ends by two hangers which permit approximating the bar to be simply supported. Your friend loves clothes whose weight is uniformly distributed along the length of the bar. The Summer clothes take up $2/3\,L$ and weigh 20 lbs/ft while the Winter clothes take up the remaining $1/3\,L$ and weigh 40 lb/ft. You need to select a rod that will support all those clothes but also have a deflection δ_{allow} that will not exceed $\leq L/300$ when the closet is full of clothes. You may neglect the weight of the rod.



A wooden rod used as a coat hanger in a closet.

(a) Draw the free body diagram of the loaded rod.

(b) Determine the reactions of the beam for distributed load $\boldsymbol{w}_o$.

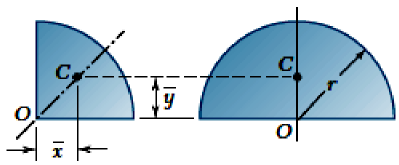
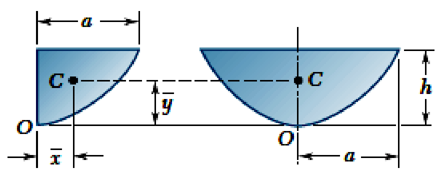
(c) Draw the shear and bending moment diagrams for the beam and label the critical values.

(d) Select the diameter of rod to use if the white oak has a failure bending stress of 4.7 ksi, a failure shear stress of 1.9 ksi and you want to have a *factor of safety, (FS)* of 2.0.

(e) Determine the equation of the elastic curve of the beam in terms of $\boldsymbol{w}_o, L, E$ and I.

(f) Using $\boldsymbol{w}_o = 20$ lb/ft and $L = 12$ ft, calculate the maximum deflection. State whether the value is within the allowable value. Use $E = 1.8 \times 10^6$ psi. If it's not, what would you propose to do?

Centroids of Common Shapes of Areas and Lines

Shape		$\bar{x}$	$\bar{y}$	Area
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$

$$\sigma = \frac{F}{A} \qquad \epsilon = \frac{\Delta L}{L_o} \qquad \tau = \frac{V}{A} \qquad \sigma(\theta) = \frac{F}{A_0}\cos^2\theta \qquad \tau(\theta) = \frac{F}{A_0}\sin\theta\,\cos\theta \qquad \epsilon_{th} = \alpha\,\Delta T \qquad \Delta L \equiv \delta = \frac{PL}{AE}$$

$$\sigma = E\,\epsilon \qquad \sigma_{all} = \frac{\sigma_{ult}}{SF} \quad \text{or} \quad \sigma_{all} = \frac{\sigma_{yield}}{SF} \qquad SF \equiv \text{Safety Factor} \qquad \nu = -\frac{\epsilon_y}{\epsilon_x} \qquad \tau = \frac{Tc}{J} \qquad \tau = G\,\gamma \qquad \phi = \frac{TL}{JG}$$

$$\epsilon_x = \frac{1}{E}\{\sigma_x - \nu\,(\sigma_y + \sigma_z)\} \quad \text{similarly for } \epsilon_y \text{ and } \epsilon_z\text{ ; also} \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \quad \text{similarly for } \gamma_{yz} \text{ and } \gamma_{zx}$$

$$I_P \equiv J_{\bullet} = \frac{\pi c^4}{2} \qquad J_{\odot} = \frac{\pi}{2}(c_2^4 - c_1^4) \qquad I_{\square} = \frac{bh^3}{12} \qquad I_{\odot} = \frac{\pi R^4}{4} \qquad I = \bar{I}_{\mathrm{CM}} + A\,d^2 \qquad Q \equiv \int_{y=y_1}^{y=c} y\,dA = A_1\,\bar{y}_1$$

$$\sigma(y) = \frac{-M\,y}{I} \qquad \tau(y) = \frac{V\,Q(y)}{I\,t} \qquad EI\frac{d^2y}{dx^2} = M(x) \qquad \frac{dM}{dx} = V(x) \qquad \frac{dV}{dx} = -\,w(x)$$

Please remember to draw a clear free-body diagram when solving each problem.
Also, where appropriate, please indicate magnitudes, directions and units.