Your TA, Section # and Section time:

SOLUTIONS

Your name:

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Cornell TAM/ENGRD 2030

Prelim 2

March 20, 2014

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- : Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- $\vec{\bullet}$: Use correct **vector notation**.
- A+ : Be (I) neat, (II) clear and (III) well organized.
- □ : TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> : Make appropriate `Matlab code` clear and correct. You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`". Small syntax errors will have small penalties.
- : Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| : **Justify** your results so a grader can distinguish an informed answer from a guess.
- : If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ : Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** The last page is blank for your use. Ask for more extra paper if you need it. Put your name on each extra sheet, fold it in, and refer to back pages or extra sheets on the page of the relevant problem.

Problem 4: ______ /25

Problem 5: ______ /25

Problem 6: ______ /25

4) Consider ideal massless pulleys and massless inextensible strings. Each pulley system as one and only one mass m and one and only one force F. Points A, B, C and D can be any point on any string or at the center of a pulley that you like. Show enough reasoning so it is clear that you could justify your result in detail if needed. You can count the accelerations as being positive in any direction you like.

a) Read the cover page. Initial this statement. "I have read the cover page and I understand it." ALR

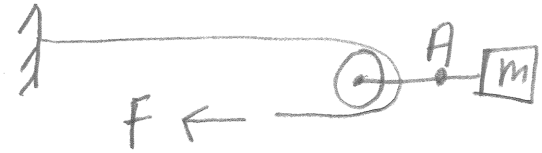
b) Design a pulley system so that $a_A = 2F/m$.

c) Design a pulley system so that $a_B = F/(4m)$.

d) Design a pulley system using 2 or fewer pulleys so that $a_C = 9F/m$.

e) Design a pulley system using 6 or fewer pulleys so that $a_D = 1024F/m$.

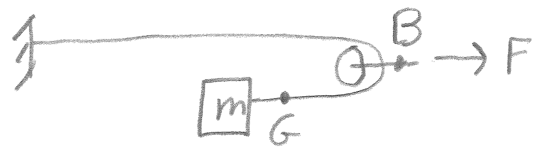
a)



tension at A = 2F

$\Rightarrow \boxed{a_A = 2F/m}$ (a)

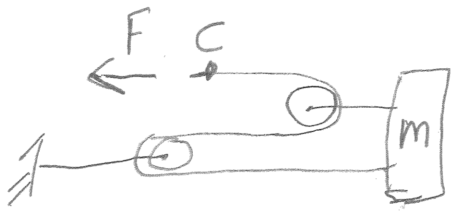
b)



tension at G = F/2

$a_B = a_G/2 \Rightarrow \boxed{a_B = F/(4m)}$ (b)

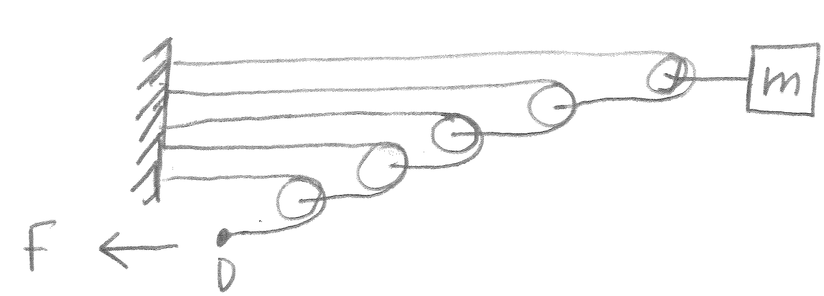
c)



Force on C = 3F

$a_C = 3a_m \Rightarrow \boxed{a_C = 9F/m}$ (c)

d)



Force on mass = $2^5 \cdot F$

$a_D = 2^5 \cdot a_m$

$a_D = 2^{10} F/m$

$\boxed{a_D = 1024F/m}$ (d)

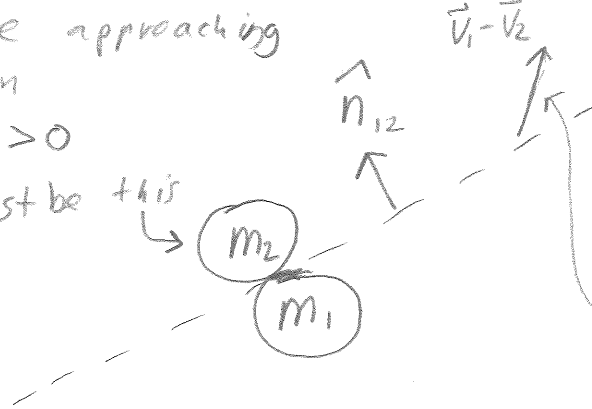
5) 2D. Two disks m_1 and m_2 collide with negligible friction. Their velocities before the collision are known, as is the coefficient of restitution is e. The line of common tangency at the instant of collision makes a 30° angle with the $+x$ axis (that is, the x axis rotated 30° counter clockwise is the common tangent line). Assume consistent units.

Complete the Matlab code at right to find the system kinetic energy after the collision.

For balls to be approaching before collision

$(\vec{V}_1^b - \vec{V}_2^b)\cdot\hat{n}_{12} > 0$

$\Rightarrow$ ball 2 must be this one

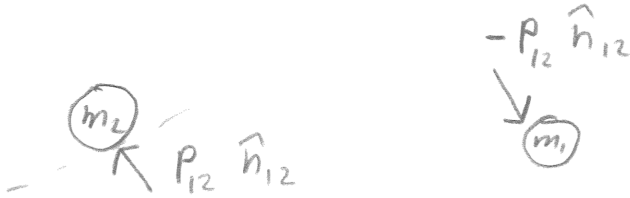


$\vec{V}_1^b = 2\hat{\imath} + 9\hat{\jmath}$

$\vec{V}_2^b = 1\hat{\imath} - 3\hat{\jmath}$

$\vec{V}_1^b - \vec{V}_2^b = \hat{\imath} + 12\hat{\jmath}$

FBDs



$\hat{n}_{12} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

LMB$_1$: $\quad m_1\vec{V}_1^a = m_1\vec{V}_1^b - P_{12}\hat{n}_{12} \qquad (1)$

LMB$_2$: $\quad m_2\vec{V}_2^a = m_2\vec{V}_2^b + P_{12}\hat{n}_{12} \qquad (2)$

Restitution: $\quad (\vec{V}_2^a - \vec{V}_1^a)\cdot\hat{n}_{12} = -e(\vec{V}_2^b - \vec{V}_1^b)\cdot\hat{n}_{12} \qquad (3)$

$1,2,3 \Rightarrow$

$$\begin{array}{c} (1) \\ \\ (2) \\ \\ (3) \end{array}
\underbrace{\begin{bmatrix} m_1 & 0 & 0 & 0 & +n_{12x} \\ 0 & m_1 & 0 & 0 & +n_{12y} \\ 0 & 0 & m_2 & 0 & -n_{12x} \\ 0 & 0 & 0 & m_2 & -n_{12y} \\ -n_{12x} & -n_{12y} & n_{12x} & n_{12y} & 0 \end{bmatrix}}_{\text{Known} \rightarrow A}
\underbrace{\begin{bmatrix} V_{1x}^a \\ V_{1y}^a \\ V_{2x}^a \\ V_{2y}^a \\ P_{12} \end{bmatrix}}_{z \text{ find}}
=
\underbrace{\begin{bmatrix} m_1 V_{1x} \\ m_1 V_{1y} \\ m_2 V_{2x} \\ m_2 V_{2y} \\ -e(\vec{V}_2^b - \vec{V}_1^b)\cdot\hat{n}_{12} \end{bmatrix}}_{b \text{ known}}$$

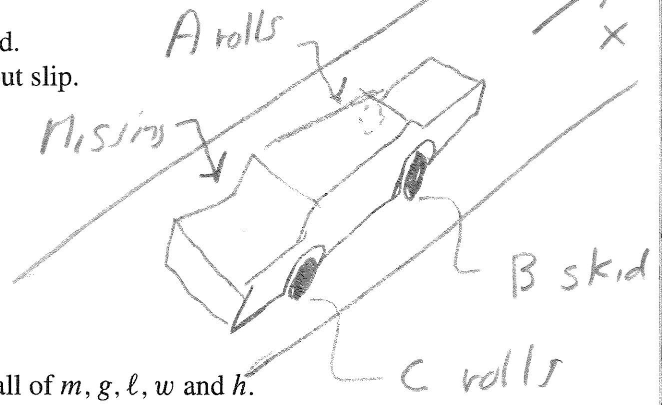
$$\boxed{E_k = \frac{1}{2} m_1 \vec{V}_1^a\cdot\vec{V}_1^a + \frac{1}{2} m_2 \vec{V}_2^a\cdot\vec{V}_2^a}$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function prel2()
m1 = 1  ;    v1b =   [2   9];
m2 = 2  ;    v2b =   [1  -3];
e  = 0.5;
θ = π/6;
n     =  [-sin(θ)   cosθ];      % = n12

b  =  [ m1*V1b      m2*V2b      -e*(V2b - V1b)*n12' ]';
          1x2         1x2                1x1
                      5x1

A = [  m1     0      0      0      n(1);
       0      m1     0      0      n(2);
       0      0      m2     0     -n(1);
       0      0      0      m2    -n(2);
       [-n           n      0]          ];

z =  A\b; % The great backslash!
V1a =  z(1:2);
V2a =  z(3:4);

Ek = 1/2 (m1 * V1a * V1a'  +  m2 V2a * V2a')

end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

6) A car with mass m travels without tipping or turning on a straight path on a level road in the $+x$ direction ($+y$ is to the left and $+z$ is up).

* The back left wheel is missing so only three wheels touch the ground.
* The front left wheel A and back right wheel C both roll freely without slip.
* The front right wheel B is jammed and slides with friction $\mu = 1$.

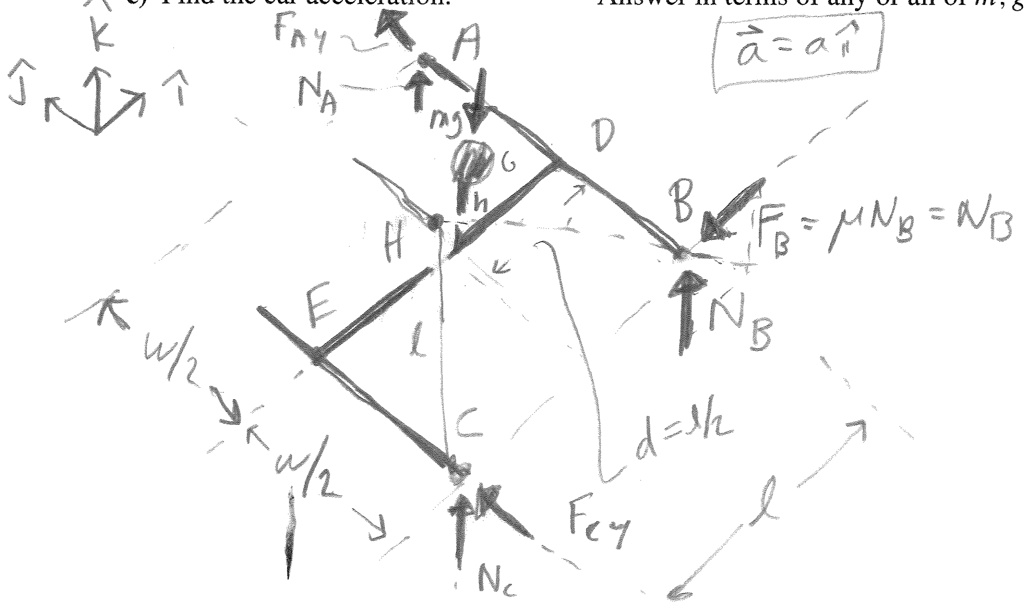
$\ell =$ the distance between the front and rear wheels,
$w =$ the width of the car,
$h =$ the height of G, the car's center of mass (assume $h < \ell/2$),
$d = \ell/2$ the distance of G back from the front axle.



a) Draw a clear sketch. Draw a clear free body diagram.

b) Find any unknown force component. Answer in terms of any or all of m, g, ℓ, w and h.

c) Find the car acceleration. Answer in terms of any or all of m, g, ℓ, w and h.



$\boxed{\vec{a} = a\hat{\imath}}$

LMB

$\{\sum \vec{F} = m\vec{a}\}$

$\{\}\cdot\hat{k} \Rightarrow$

$N_A + N_B + N_c - mg = 0$ (1)

AMB$_{ED}$:

$\{\sum \vec{M}_E = \dot{\vec{H}}_{/E}\}\cdot\hat{\imath}$

$-(N_c + N_B)\frac{w}{2} + N_A\frac{w}{2} = \underbrace{(\vec{r}_{G/E} \times ma\hat{\imath})\cdot\hat{\imath}}_{0}$ (2)

$\Rightarrow \quad N_A = N_B + N_c$

(2) & (1) $\Rightarrow \quad \boxed{N_A = mg/2}$ (a)

Use pt. H: * directly above C, a height ℓ
* on line of action of $\vec{F}_B = N_B\hat{k} - N_B\hat{\imath}$

Consider axis Hy: through H, parallel to $\hat{j}$.

N_C intersects axis $\Rightarrow$ no moment

F_{Cy} $\parallel$ to axis $\Rightarrow$ " "

$\vec{F}_B$ intersects axis $\Rightarrow$ " " $[\vec{F}_B = N_B \hat{k} - N_B \hat{i}]$

F_{Ay} $\parallel$ to axis $\Rightarrow$ " "

$$\sum M_{Hy} = \dot{H}_{Hy}$$

$[\vec{r}_{G/H} \times (ma\hat{i})] \cdot \hat{j}$

$$mg\underbrace{(l-d)}_{=l/2} - \underbrace{N_A}_{=mg/2} l = -(l-h)ma$$

$$0 = -(l-h)\cancel{m}a$$

$$\boxed{a = 0} \quad (b)$$

Why? w/ $d = l/2$ car is supported by wheels A & C

$\Rightarrow$ No normal force at B

Note: If $h > l/2$ then there could be another solution where pt. B locks (like frictional self-locking in statics) & car would flip. But this is beyond where we are in this class, hence the assumption $h < l/2$