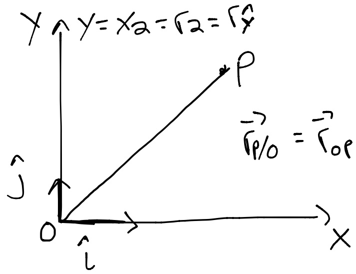
Vectors: (see ruina/pratap)

Position Vectors

Fixed or Newtonian coordinate system

$y = x_2 = r_2 = r\hat{y}$



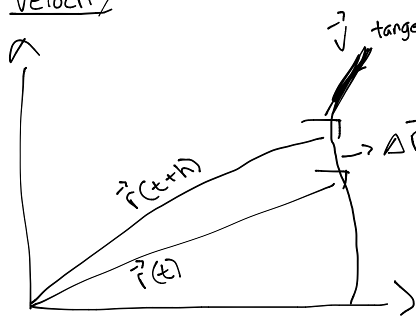
$\vec{r}_{P/O} = \vec{r}_{OP} = \vec{r}_P = \vec{r}$

$x = x_1 = r_1 = r\hat{x}$

$\hat{i} = \hat{e}_x = \hat{e}_1 = \hat{x}$

$\hat{j} = \hat{e}_y = \hat{e}_2 = \hat{y}$

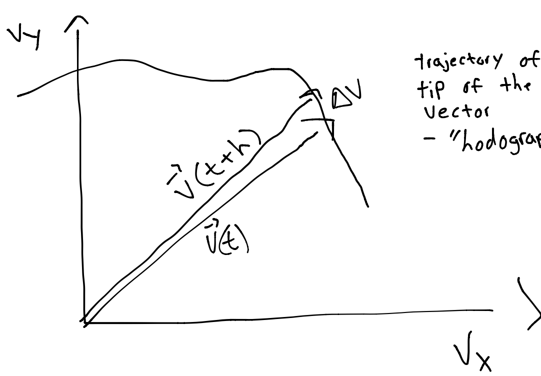
Velocity



$\Delta\vec{r} = \vec{r}(t+h) - \vec{r}(t)$

$$\vec{V}_{h \to 0} = \frac{\vec{r}(t+h) - r(t)}{h} \qquad V = \dot{x}\hat{i} + \dot{y}\hat{j} = \dot{x}\hat{x} + \dot{y}\hat{y}$$

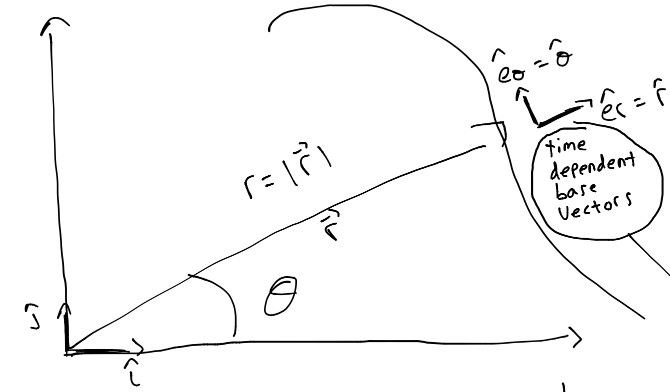
Acceleration



trajectory of the tip of the velocity vector
- "hodograph"

$$\lim_{\Delta t = 0} \vec{a} = \frac{\Delta v}{\Delta t} = \dot{v}_x\hat{i} + \dot{v}_y\hat{j} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

Polar Coordinates



you have to take derivatives of base vectors !

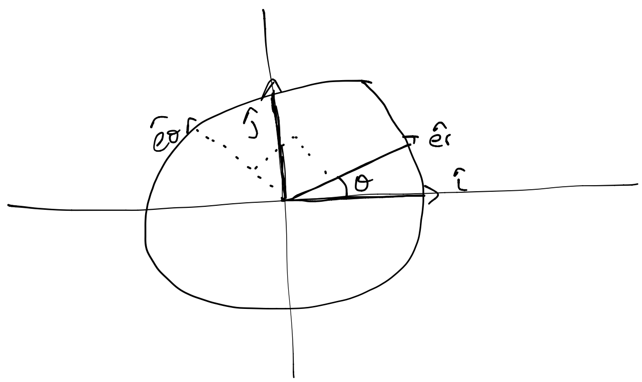
Position Vector

$\hat{r} = r\hat{e}_r, \quad \hat{e}_r = \frac{\vec{r}}{|\vec{r}|}$

Velocity Vector

product rule

$$\vec{V} = \frac{d}{dt}\vec{r} = \frac{d}{dt} r\hat{e}_r = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r$$



$$\boxed{\begin{array}{ll} \hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} & \dot{\hat{e}}_r = \dot{\theta}\,\hat{e}_\theta \\ \hat{e}_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j} & \dot{\hat{e}}_\theta = -\dot{\theta}\,\hat{e}_r \end{array}}$$

Back to Velocity

$\vec{V} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r, \quad \underline{\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta}$

$$\boxed{\vec{V} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta}$$

$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\hat{e}}_r + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta + r\dot{\theta}\dot{\hat{e}}_\theta, \quad \underline{\dot{\hat{e}}_r = \dot{\theta}\hat{e}_\theta, \dot{\hat{e}}_\theta = -\dot{\theta}\hat{e}_r}$

$\vec{a} = \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$

$$\boxed{\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta}$$

Note:

$$\vec{r} = r$$
$$x\hat{i} + y\hat{j} = r\hat{e_r}$$

$$\vec{v} = v$$
$$\dot{x}\hat{i} + \dot{y}\hat{j} = \dot{r}\hat{e_r} + r\dot{\theta}\hat{e_\theta}$$

$$\vec{a} = a$$
$$\ddot{x}\hat{i} + \ddot{y}\hat{j} = (\ddot{r} - r\dot{\theta}^2)\hat{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e_\theta}$$