Your year in school (semester, degree sought):

"SOLUTIONS"

Your name:

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Cornell MAE 4735/5735

Prelim 1

Oct 10, 2012

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes extra)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.

A+ Be (I) neat, (II) clear and (III) well organized.

□ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).

>> Make appropriate `Matlab code` clear and correct.
You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
Small syntax errors will have small penalties.

Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\boldsymbol{\lambda}}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.

→| **Justify** your results so a grader can distinguish an informed answer from a guess.

If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).

≈ Work for **partial credit** (from 60–100%, depending on the problem)

- Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
- Reduce the problem to a clearly defined set of equations to solve.
- Provide Matlab code which would generate the desired answer (and explain the nature of the output).

Extra sheets. Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 1: ____ /25

Problem 2: ____ /25

Problem 3: ____ /25

0) Read the directions on the front cover.

1) 2D in side view. A car, moving to the right in the figure below, screeches to a stop, skidding the rear wheels (coefficient of friction $=\mu$, friction angle $= \phi$, with $\tan\phi = \mu$). The brakes are not applied to the light front wheels which roll easily.

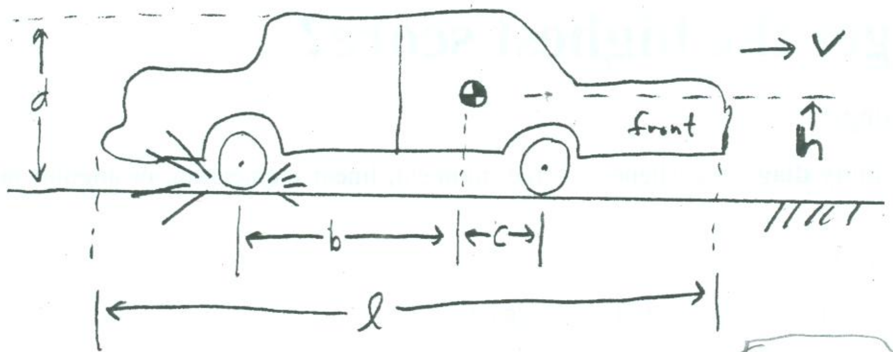
What is the vertical force from the ground on the rear wheels ("the" force is the sum from both wheels)?

Answer in terms of some or all of the variables on this page. Extra credit for showing that your answer agrees with one or more special cases that you can evaluate more simply.

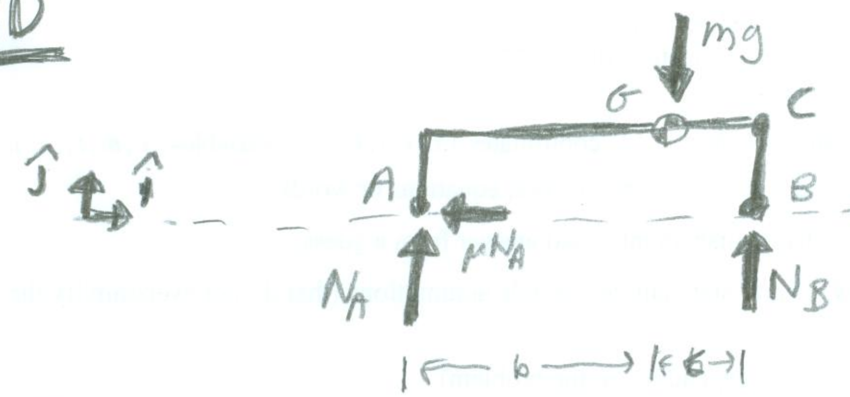
Could use up to 3 LMB & AMB eqs. Look for one that doesn't have a_G or N_B in it (saves algebra).

$\downarrow g$

m = mass
I^G = mom. of inertia about C.O.M



FBD



Assume: $\vec a_G = a_G \hat\imath$ (no rotation)

AMB$_{/C}$

$$\sum \vec M_{/C} = \dot{\vec H}_{/C}$$

both ∥ to $\hat\imath$.

$$\left\{ (-\mu N_A h + (b+c) N_A + mgc)\hat k = \underbrace{\vec r_{G/C} \times m\vec a_G}_{\vec 0} + I\cancel{\ddot\theta}\hat k \right\}$$

$$\{\}\cdot\hat k \Rightarrow \boxed{N_A = \frac{mgc}{b+c+\mu h}}$$

Checks: 1) if $h=0$ or $\mu = 0 \Rightarrow N_A = mg\frac{c}{b+c}$ (agrees w/ lever rule) ✓

2) if $c=0 \Rightarrow N_A = 0$ (all weight on front wheel) ✓

3) if $g = 0 \Rightarrow N_A = 0$ (also, $N_A \propto m$ and g) ✓

2) A motor, not shown, is mounted a distance d off center (eccentrically) in the disk that is in a rectangular hole. Therefore the base moves up and down a distance

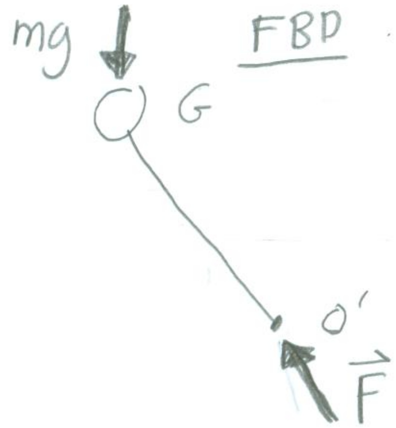
$$\left\{ x_{0'} = d \sin \omega_0 t. \right\}$$

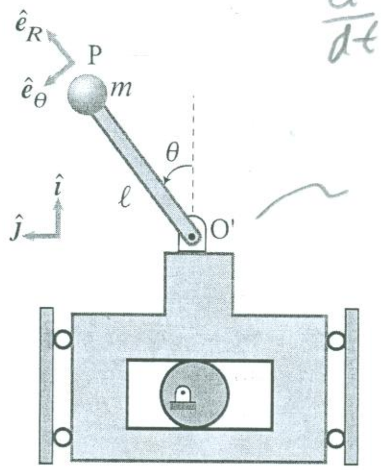
The rod has negligible mass. There is gravity pointing down. At $t = 0$ the rod is at θ_0 with angular speed $\dot{\theta}_0$. You are given m, g, ℓ, d, ω_0 and t_{final}.

(1) Find the equations of motion.

(2) Write Matlab code that would give θ at t_{final}. Assume any non-zero values that please you for all constants that you need. The code need *not* have detailed comments. And, if you need to comment, you can use normal writing and arrows and such (you do not have to use `% standard Matlab comment style`).

$\hat{e}_R \times \hat{\imath} = -\sin\theta \hat{k}$





$$\frac{d^2}{dt^2}\{\ \} \Rightarrow \vec{a}_{0'} = -d\omega_0^2 \sin\omega_0 t\, \hat{\imath}$$

$\underline{AMB_{/0'}}$

$$\sum \vec{M}_{/0'} = \dot{\vec{H}}_{/0'}$$

$$mg\ell \sin\theta\, \hat{k} = \underbrace{\vec{r}_{G/0'}}_{=\ell\hat{e}_R} \times m\vec{a}_G + \cancel{I^G}\ddot{\theta}\hat{k}$$

pt. mass ⇒ 0

$$\vec{a}_G = \vec{a}_{0'} + \ddot{\theta}\ell\hat{e}_\theta - \ell\dot{\theta}^2\hat{e}_R$$

$$\Rightarrow \left\{ mg\ell\sin\theta\hat{k} = (\ell\hat{e}_R)\times\left[-d\omega_0^2\sin(\omega_0 t)\hat{\imath} + \ddot{\theta}\ell\hat{e}_\theta - \ell\dot{\theta}^2\hat{e}_R\right]m \right.$$

$$\left. = \left[\ell d\omega_0^2 \sin(\omega_0 t) + \ddot{\theta}\ell^2 + 0\right] m\hat{k} \right\}$$

$$\{\ \}\cdot\hat{k} \Rightarrow \quad \ddot{\theta} = \frac{\cancel{m}\ell\left[g - d\omega_0^2\sin(\omega_0 t)\right]}{\cancel{m}\ell^2}\sin\theta$$

$$\boxed{\ddot{\theta} = \frac{(g + \underbrace{a_{0'}}_{-d\omega_0^2\sin(\omega_0 t)})}{\ell}\sin\theta}$$

{check: accel. base is like altered gravity}

Informal Matlab (e.g. $\dot\theta$ means thetadot)

```
function exam2()

θ0 = .5;  θ̇0 = 0;
z0 = [θ0; θ̇0];
tf = 7;  tspan = [0 tf];

[t zarray] = ode23(@rhs, tspan, z0);

θf = zarray(end,1);
disp(['The final theta is:  ' num2str(θf)])
end
```

```
function zdot = rhs(t,z)
θ = z(1); ω = z(2); % put params here even though its bad form. Quick&dirty
m=1; g=10; l=1; d=.1; ω0=70; %

θ̇ = ω;
a0' = -d.*ω0^2 .* sin(ω0 t);
θ̈ = ((g+a0')/l) .* sin(θ); % The ODE

zdot = [θ̇ ; θ̈];

end
```

3) Assume you want to find the motion of the 4-bar linkage shown.

(1) List all constant parameters needs (e.g., g, etc.)

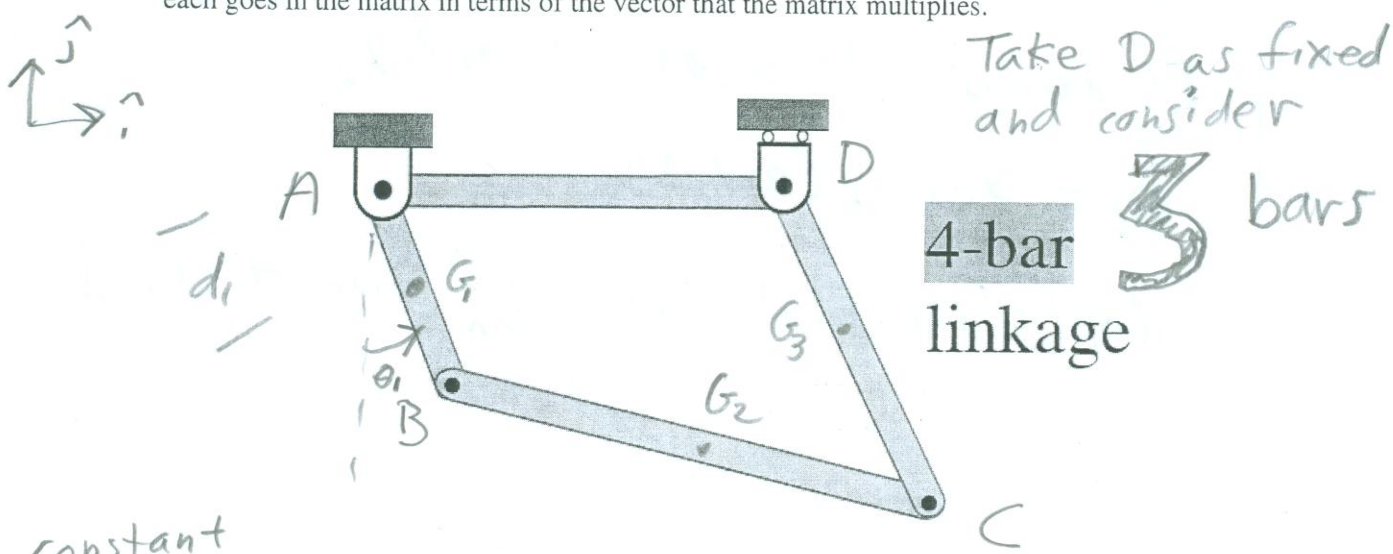
(2) How many degrees of freedom does the system have?

(3) Give an example of a good set of minimal coordinates?

(4) The big matrix for the DAEs will have how many rows and how many columns? Please be clear about the meanings of the rows and columns.

(5) Give an example of each type of equation used in a row or column (hint: there are 3 distinct types). It should be explicit (a readable equation) but need not be in matrix form (need not have all the zeros needed in the matrix). But it should be separated into appropriate left and right hand sides. Explain clearly where each goes in the matrix in terms of the vector that the matrix multiplies.

Take D as fixed and consider



① constant parameters

g

masses: m_1, m_2, m_3

lengths: l_1, l_2, l_3, l_4 ← needed to locate D, Inertias: $I^{G_1}, I^{G_2}, I^{G_3}$

com positions: d_1, d_2, d_3 (or, assume each is at center, e.g. $d_1 = l_1/2$)

②, ③ $\boxed{\text{1 DOF, eg. } \theta_1}$ ← only one minimal coord. needed.

How to count to 1?

Each bar floating has 3 DOFs ⇒ 9 DoFs

2 constraints on position at A, B, C, D ⇒ 8 Constraints

$\boxed{9-8=1}$

(*)

$$\underbrace{\begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix}}_{17 = \#\text{ unknowns}} \begin{bmatrix} \ddot{x}_1 \; (a_{1x}) \\ \ddot{x}_2 \\ \ddot{\theta}_1 \\ \vdots \\ A_x \\ A_y \\ B_x \\ B_y \\ \vdots \end{bmatrix} = \begin{bmatrix} \text{Vector w/} \\ \text{forces \&} \\ V^2 \text{ terms} \\ \\ \\ \end{bmatrix}$$

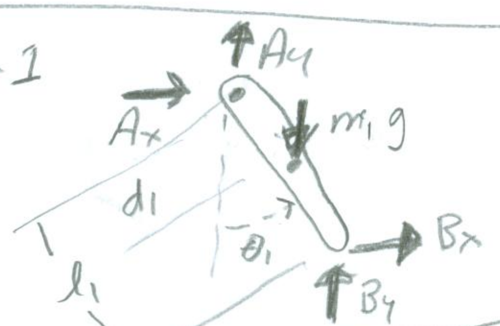
17 = # eqs

9 accels & 8 constraint forces = unknowns

M is 17×17: 17 unknowns = # of colums

17 equations
- 6 Lin Mom
- 3 Ang. Mom
- 8 constraints

FBD of bar 1



Examples of eqns in (*):

Lin. comb. of unknowns = "knowns" (given the state)

entries of M

$\{LMB\}\cdot\hat{j} \Rightarrow$ (6 of these)	$m_1 a_{1y} - \cancel{+}\, 1\cdot A_y - \cancel{+}\, 1\cdot B_y = -m_1 g$	2nd row
$\{AMB\}\cdot\hat{k} \Rightarrow$ (3 of these)	$I^{G_1}\ddot{\theta}_1 + [d_1\cos\theta_1]A_x + [d_1\sin\theta_1]A_y + [(l_1-d_1)\cos\theta_1]B_x + [(l_1-d_1)\sin\theta_1]B_y = 0$	3rd row
$\{\text{acel. of } A = \vec{0}\}\cdot\hat{i} \Rightarrow$ (8 of these)	$1\cdot a_{1x} - (d_1\cos\theta_1)\ddot{\theta}_1 = -(\sin\theta_1)\, d_1\, \dot{\theta}_1^2$	10th row