Solve the 3 mass 4 spring equations analytically w/ example

25. From 23: $M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}$ $K = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix}$ ✓

$m = 1, k = 1$

$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ ✓

given X_0, V_0

$[U] = \text{eig}(M^{-1}K)$

$\vec{X} = \sum A_i \vec{U}_i \cos(\omega_i t) + \sum B_i \vec{U}_i \sin(\omega_i t)$

at $t=0 \rightarrow [U]\vec{A}_i = \vec{X}_0$ w $[U]\vec{B}_i = \vec{V}_0$ ✓

$\vec{A}_i = [U]^{-1}\vec{X}_0$ $\vec{B}_i = \frac{1}{\omega}([U]^{-1}\vec{V}_0)$ ✓

$\vec{\omega} = \begin{bmatrix} 1.8478 \\ 1.4142 \\ .7654 \end{bmatrix}$

$\vec{U} = \begin{bmatrix} -1/2 & -\sqrt{2}/2 & 1/2 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ -1/2 & \sqrt{2}/2 & -1/2 \end{bmatrix}$

X only

$\vec{X}_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\vec{V}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \vec{A}_i = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $\vec{B}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\vec{X} = \begin{bmatrix} 1/2 \\ \sqrt{2}/2 \\ 1/2 \end{bmatrix} \cos(1.8478t) + \begin{bmatrix} 1/2 \\ -\sqrt{2}/2 \\ 1/2 \end{bmatrix} \cos(.7654t)$

Correct, as seen in Plots #1 and #2

V only

$\vec{X}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\vec{V}_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \rightarrow \vec{A}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\vec{B}_i = \begin{bmatrix} -1.3066 \\ 0 \\ .5412 \end{bmatrix}$

$\vec{X} = 1.3066 \begin{bmatrix} 1/2 \\ \sqrt{2}/2 \\ 1/2 \end{bmatrix} \sin(1.8478t) + .5412 \begin{bmatrix} 1/2 \\ -\sqrt{2}/2 \\ 1/2 \end{bmatrix} \sin(.7654t)$

Correct, as seen in plots #3 and #4

Normal Mode

$$\vec{x}_0 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \vec{v}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow A_1 = \begin{bmatrix} 0 \\ -\sqrt{2} \\ 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\vec{x} = -\sqrt{2} \begin{bmatrix} -\sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix} \cos(1.4142t)$$

seen on plots ✓
#5 and #6

Random

$$\vec{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{v}_0 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \rightarrow A_1 = \begin{bmatrix} -3.414 \\ 1.4142 \\ .5858 \end{bmatrix} \quad B_1 = \begin{bmatrix} -11.152 \\ 1 \\ -.7926 \end{bmatrix}.$$

numerical agrees on plot
#7 and #8