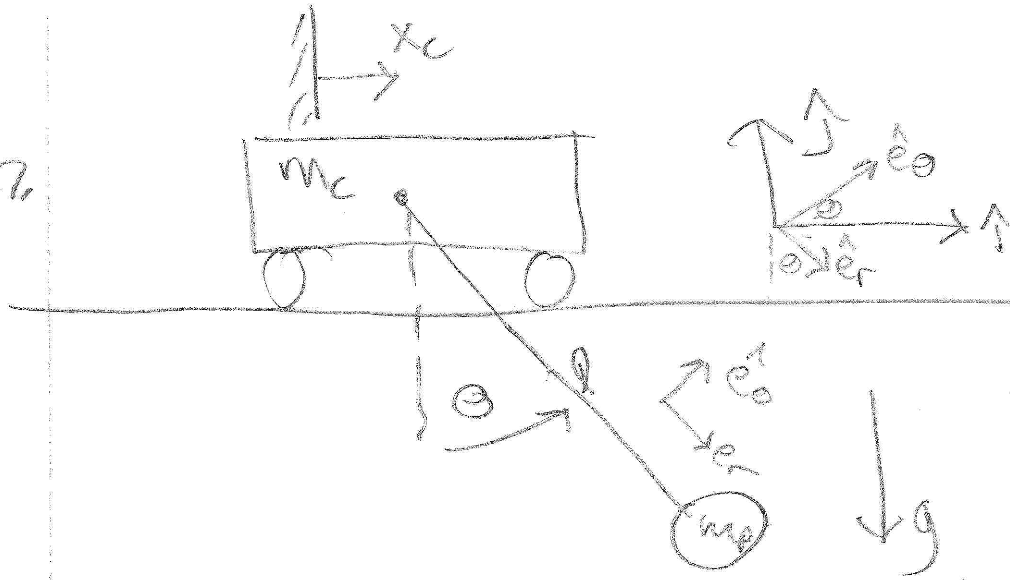
27.



FBD
good

$\hat{e}_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$
$\hat{e}_r = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$

Determine the full nonlinear equations two ways.

Lagrange

$PE = -m_p g l \cos\theta$

$KE = \Sigma \tfrac{1}{2} M V^2 = \tfrac{1}{2} m_c \dot{x}_c^2 + \tfrac{1}{2} m_p V_p^2$

$\vec{V}_p = \dot{x}_c \hat{i} + \omega \hat{k} \times l \hat{e}_r \rightarrow V_p = \dot{x}_c \hat{i} + \dot{\theta} l \hat{e}_\theta$

$\vec{V}_p = (\dot{x}_c + \dot{\theta} l \cos\theta)\hat{i} + (\dot{\theta} l \sin\theta)\hat{j}$

$V_p^2 = \vec{V}_p \cdot \vec{V}_p = \dot{x}_c^2 + 2\dot{x}_c \dot{\theta} l \cos\theta + \dot{\theta}^2 l^2 \cos^2\theta + \dot{\theta}^2 l^2 \sin^2\theta$

$\boxed{V_p^2 = \dot{x}_c^2 + 2\dot{x}_c \dot{\theta} l \cos\theta + \dot{\theta}^2 l^2}$ ✓

$\mathcal{L} = KE - PE = \tfrac{1}{2} m_c \dot{x}_c^2 + \tfrac{1}{2} m_p(\dot{x}_c^2 + 2\dot{x}_c \dot{\theta} l \cos\theta + \dot{\theta}^2 l^2) + m_p g l \cos\theta$

$\frac{\partial \mathcal{L}}{\partial \theta} = -m_p l \sin\theta (g + \dot{\theta} \dot{x}_c)$

$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \; m_p l (l\dot{\theta} + \dot{x}_c \cos\theta) \rightarrow \frac{d}{dt} = m_p l (l\ddot{\theta} + \ddot{x}_c \cos\theta - \dot{x}_c \dot{\theta} \sin\theta)$

$\frac{\partial \mathcal{L}}{\partial \theta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \boxed{0 = -m_p l (l\ddot{\theta} + g\sin\theta + \ddot{x}_c \cos\theta)}$ ✓

$$\frac{\partial \mathcal{L}}{\partial x_c} = 0 \qquad \frac{\partial \mathcal{L}}{\partial \dot{x}_c} = m_p(\dot{x}_c + l\dot{\theta}\cos\theta) + m_c\dot{x}_c$$

$$\frac{d}{dt} = m_p(\ddot{x}_c + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) + m_c\ddot{x}_c$$

$$\frac{\partial \mathcal{L}}{\partial x_c} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}_c} = \boxed{0 = -m_c\ddot{x}_c - m_p(\ddot{x}_c - l\dot{\theta}^2\sin\theta + l\ddot{\theta}\cos\theta)}$$

$$\ddot{\theta} = -\frac{(g\sin\theta + \ddot{x}_c\cos\theta)}{l}$$

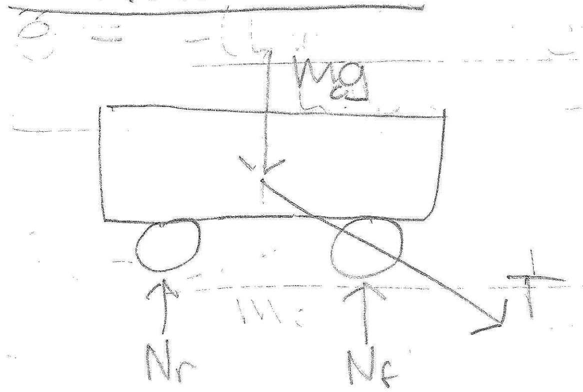
$$\ddot{x}_c = \frac{l m_p(\dot{\theta}^2\sin\theta - \ddot{\theta}\cos\theta)}{m_c + m_p}$$

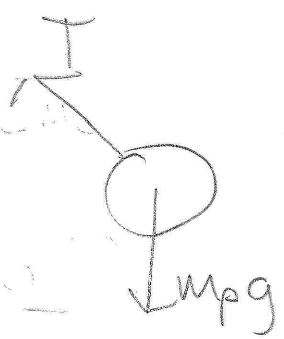
$$\boxed{\ddot{\theta} = -\frac{(l m_p \cos\theta\sin\theta\,\dot{\theta}^2 + g(m_p + m_c)\sin\theta}{(l m_c + l m_p\sin^2\theta)}}$$

$$\boxed{\ddot{x}_c = \frac{l m_p\dot{\theta}^2\sin\theta + g m_p\sin\theta\cos\theta}{(m_c + m_p\sin^2\theta)}}$$

Matlab Symbolic

Newton's Laws





$\vec{a}_c = \ddot{x}_c \hat{\imath}$ $\qquad \vec{a}_p = \vec{a}_c + \vec{a}_{p/c} = \ddot{x}_c \hat{\imath} + l\ddot{\theta}\hat{e}_\theta - l\dot{\theta}^2 \hat{e}_r$

LMB cart

$\vec{a}_p = (\ddot{x}_c + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\hat{\imath} + (l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta)\hat{\jmath}$

$\sum \vec{F} = m\vec{a}_c \rightarrow T\sin\theta = m_c \ddot{x}_c$

woo

$$\boxed{\ddot{x}_c = \frac{T}{m_c}\sin\theta}$$

LMB pend

$-T\hat{e}_r - m_p g\hat{\jmath} = m_p \vec{a}_p$

$-T\sin\theta\hat{\imath} + T\cos\theta\hat{\jmath} - m_p g\hat{\jmath} = m_p(\ddot{x}_c + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\hat{\imath} + m_p(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta)\hat{\jmath}$

$-T\sin\theta = -m_p(\ddot{x}_c + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)$

$m_c\ddot{x}_c = -m_p(\ddot{x}_c + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)$

$$\boxed{\ddot{x}_c = -\frac{m_p(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)}{m_p + m_c}}$$

$$T\cos\theta - m_p g = m_p(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta)$$

$$T\cos\theta = m_p(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta + g)$$

$$T\sin\theta = \frac{m_p(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta + g)\sin\theta}{\cos\theta}$$

$$T\sin\theta = \frac{m_p(l\ddot{\theta}\sin^2\theta + l\dot{\theta}^2\sin\theta\cos\theta + g\sin\theta)}{\cos\theta}$$

$$\boxed{\ddot{x}_c = \frac{m_p(l\ddot{\theta}\sin^2\theta + l\dot{\theta}^2\sin\theta\cos\theta + g\sin\theta)}{m_c\cos\theta}}$$

$$\boxed{\ddot{\theta} = -\frac{(l m_p\cos\theta\sin\theta\,\dot{\theta}^2 + g(m_p+m_c)\sin\theta)}{l m_c + l m_p\sin^2\theta}}$$

$$\boxed{\ddot{X}_c = \frac{l m_p\dot{\theta}^2\sin\theta + g m_p\sin\theta\cos\theta}{m_c + m_p\sin^2\theta}}$$

Matlab symbolic Toolbox

<u>These equations match the lagrange solution</u>

b) $$\boxed{\ddot{\theta} = -\frac{g(m_p+m_c)\theta}{l\,m_c}}$$

$\theta \approx \sin\theta$
$1 \approx \cos\theta$
$\dot{\theta} \approx 0$
$\dot{\theta}^2 \approx 0$

$$\boxed{\ddot{x}_c = \frac{g m_p \theta}{m_c}}$$

✓

c) $$\begin{bmatrix} l m_c & 0 \\ 0 & m_c \end{bmatrix}\begin{bmatrix} \ddot{\theta} \\ \ddot{x}_c \end{bmatrix} + \begin{bmatrix} g(m_p+m_c) & 0 \\ -g m_p & 0 \end{bmatrix}\begin{bmatrix} \theta \\ x_c \end{bmatrix} = 0$$ ✓

Nice