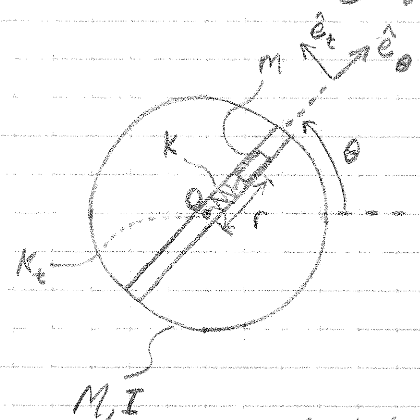
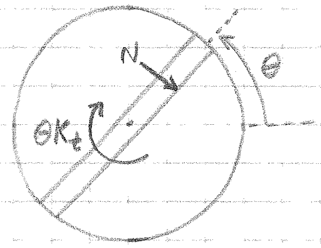
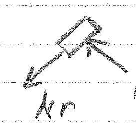
30. A turntable with mass 'M' and moment of inertia I is held in place at its center with a bearing and a torsional spring of constant k_t. Along a diameter of the disk is a slot in which mass 'm' slides with no friction. A zero-rest-length spring holds it at the center with spring constant k. Find the equations of motion two different ways and find the normal modes and frequencies.



Mass 1:



Mass 2:



Note: torsional spring and linear spring are at rest when $\theta=0$ and $r=0$

Constraint:

$$\vec{r}_1 = r\hat{e}_\theta$$

AMB for mass 2 about O:

$$\sum \vec{r}_{/O} \times \vec{F} = \dot{\vec{H}}_{m2/O} \implies (r\hat{e}_\theta)\times(N\hat{e}_t) = (r\hat{e}_\theta)\times m\left(\ddot{r}\hat{e}_\theta + \vec{\omega}\times(\vec{\omega}\times r\hat{e}_\theta) + \dot{\vec{\omega}}\times r\hat{e}_\theta + 2\vec{\omega}\times\dot{r}\hat{e}_\theta\right)$$

$$rN\hat{k} = (r\hat{e}_\theta)\times m\left(\ddot{r}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_\theta + r\ddot{\theta}\hat{e}_t + 2\dot{\theta}\dot{r}\hat{e}_t\right)$$

$$= m(r^2\ddot{\theta} + 2r\dot{\theta}\dot{r})\hat{k}$$

$$N = m(r\ddot{\theta} + 2\dot{\theta}\dot{r})$$

30. (continued)

AMB for mass 1 about O:

$$\Sigma \vec{r}_{/O} \times \vec{F} = \dot{H}_{m1/O} \Rightarrow (r\hat{e}_\theta)\times(-N\hat{e}_t) - \theta k_t \hat{k} = I\ddot{\theta}\hat{k}$$

$$-rN\hat{k} - \theta k_t \hat{k} = I\ddot{\theta}\hat{k}$$

$$-rm(r\ddot{\theta}+2\dot{\theta}\dot{r}) - \theta k_t = I\ddot{\theta}$$

$$\boxed{(I+r^2m)\ddot{\theta} + 2mr\dot{\theta}\dot{r} + \theta k_t = 0}$$

LMB for mass 2:

$$\Sigma \vec{F} = m\vec{a}_{m2} \Rightarrow -kr\hat{e}_\theta + N\hat{e}_t = m(\ddot{r}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_\theta + r\ddot{\theta}\hat{e}_t + 2\dot{\theta}\dot{r}\hat{e}_t)$$

dot both sides with $\hat{e}_\theta$

$$-kr = m(\ddot{r} - r\dot{\theta}^2)$$

$$\boxed{\ddot{r} + \left(\frac{k}{m} - \dot{\theta}^2\right) r = 0}$$ ✓

Normal modes:

1. Constant rotation at $\dot{\theta} = \sqrt{k/m}$; frequency = 0 ✗

just rotation $\Rightarrow \boxed{\omega_n = \sqrt{\frac{k_T}{I}}}$

2. Oscillation of small mass with no rotation ; frequency $= \sqrt{k/m}$ ✓

3. Linear and rotational oscillation in phase ;

$$\theta(t) = Te^{i\omega t} ; \quad r = Re^{i\omega t} ;$$

$$-\omega^2 Re^{i\omega t} + \left(\frac{k}{m} - (i\omega T e^{i\omega t})^2\right) Re^{i\omega t} = 0$$

$$-\omega^2 + k/m + \omega^2 T^2 e^{2i\omega t} = 0$$ ✗