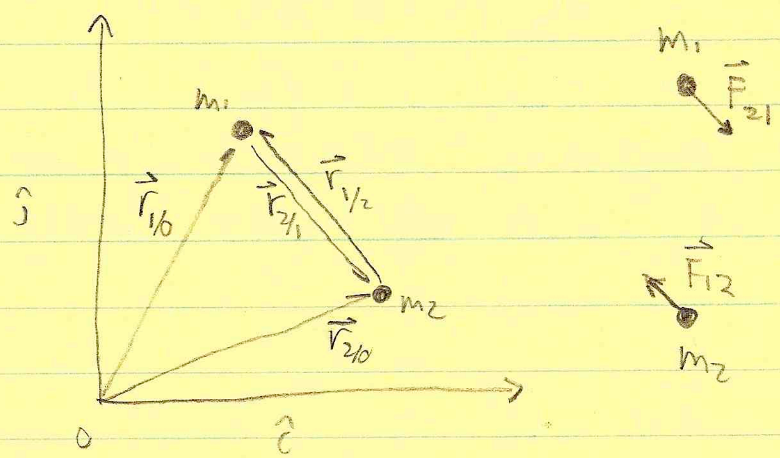
worked w/ Daniel C
and Spyros M.

MAE 5735
HW3, Due 9/12/12

8 hours

6 a) From two particles w/ mass m_1 and m_2, what is the period of circular motion if the distance between them is d and the only force between them $F = \frac{G m_1 m_2}{r^2}$?

First, we make a free-body diagram



where $\vec{F}_{21} = \frac{-G m_1 m_2}{|\vec{r}_{1/2}|^2} \frac{\vec{r}_{1/2}}{|\vec{r}_{1/2}|}$

$$\vec{F}_{12} = \frac{-G m_1 m_2}{|\vec{r}_{2/1}|^2} \frac{\vec{r}_{2/1}}{|\vec{r}_{2/1}|}$$

And $\vec{r}_{1/2} = \vec{r}_{1/0} - \vec{r}_{2/0}$, $\vec{r}_{2/1} = \vec{r}_{2/0} - \vec{r}_{1/0}$

We can define the center of mass as $\vec{r}_G$, where $m_{tot}\vec{r}_G = \sum_i m_i \vec{r}_i = m_1\vec{r}_1 + m_2\vec{r}_2$

Since the system is not being acted upon by external forces, we know

$$\sum \vec{F}_{ext} = 0 = \sum m_i \vec{a}_i$$

since $M_{TOT} \vec{a}_G = \sum m_i \vec{a}_i$

$$\Rightarrow \vec{a}_G = 0$$

$$\Rightarrow \vec{V}_G = \text{constant}$$

which means we can use the COM as the origin of a Newtonian reference frame. Re-defining the locations of these particles wrt that reference frame,

* This step inspired by p 594 of Rvina & Pratap

$$\vec{r}\,'_1 = \vec{r}_{1/0} - \vec{r}_G$$

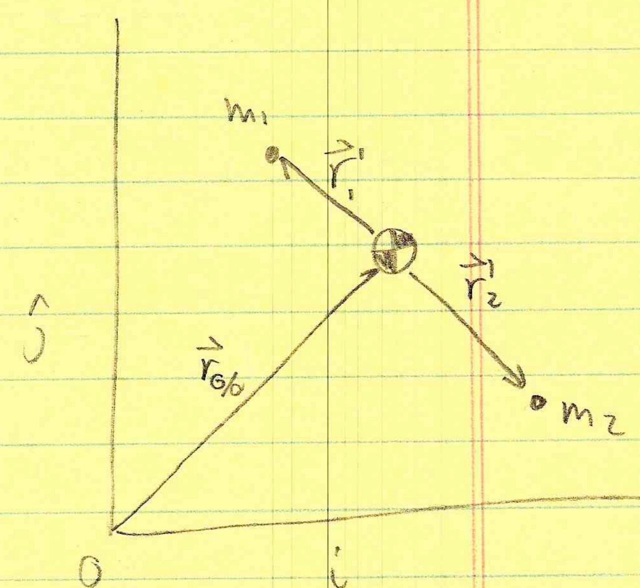
$$\vec{r}\,'_2 = \vec{r}_{2/0} - \vec{r}_G$$

$$\vec{V}\,'_1 = \vec{V}_{1/0} - \vec{V}_G$$

$$\vec{V}\,'_2 = \vec{V}_{2/0} - \vec{V}_G$$

$$\vec{a}\,'_1 = \vec{a}_1$$

$$\vec{a}\,'_2 = \vec{a}_2$$



So, we can redefine the unit vector $\frac{r_{1/2}}{|\vec{r}_{1/2}|}$ as $\frac{\vec{r}_1'}{|\vec{r}_1'|}$

and

$$\frac{r_{2/1}}{|\vec{r}_{2/1}|} \text{ as } \frac{\vec{r}_2'}{|\vec{r}_2'|}$$

And therefore redefine the forces as

$$\vec{F}_1 = \frac{-Gm_1M_2}{|\vec{r}_{1/2}|^2} \frac{\vec{r}_1'}{|\vec{r}_1'|}$$

$$\vec{F}_2 = \frac{-Gm_1M_2}{|\vec{r}_{2/1}|^2} \frac{\vec{r}_2'}{|\vec{r}_2'|}$$

Looking at Fig 1 and Fig 2, we can see:

$$\vec{r}_{1/2} = \vec{r}_1' - \vec{r}_2'$$

$$\vec{r}_{2/1} = \vec{r}_2' - \vec{r}_1'$$

And, going back to the definition of center of mass with respect to G:

$$M_{TOT}\vec{r}_{G/G} = \sum m_{i/G}\vec{a}_{i/G}$$

$$\Rightarrow M_{TOT}(0) = m_1 r_1' + m_2 r_2'$$

$$\Rightarrow \vec{r}_2{}' = -\left(\frac{m_1}{m_2}\right)\vec{r}_1{}'$$

$$\vec{r}_1{}' = -\left(\frac{m_2}{m_1}\right)\vec{r}_2{}'$$

$$\Rightarrow \vec{F}_1 = \frac{-Gm_1m_2}{\left|\vec{r}_1{}' + \frac{m_1}{m_2}\vec{r}_1{}'\right|^2}\frac{\vec{r}_1{}'}{|\vec{r}_1{}'|} = \frac{-Gm_1m_2}{\left(1+\frac{m_1}{m_2}\right)^2|\vec{r}_1{}'|^2}\cdot\frac{\vec{r}_1{}'}{|\vec{r}_1{}'|^2}$$

$$\vec{F}_2 = \frac{-Gm_1m_2}{\left|\vec{r}_2{}' + \frac{m_2}{m_1}\vec{r}_2{}'\right|^2}\frac{\vec{r}_2{}'}{|\vec{r}_2{}'|} = \frac{-Gm_1m_2}{\left(1+\frac{m_2}{m_1}\right)^2|\vec{r}_2{}'|^2}\cdot\frac{\vec{r}_2{}'}{|\vec{r}_2{}'|}$$

Let's look at just $\vec{F}_1$. We've effectively reduced this to a one-dimensional problem: 2?

1 particle, 2DOF

$$\vec{F}_1 = \frac{-Gm_1M}{|\vec{r}_1{}'|^2}\cdot\frac{\vec{r}_1{}'}{|\vec{r}_1{}'|}, \quad M = \frac{m_2}{\left(1+\frac{m_1}{m_2}\right)^2}$$

$$\Rightarrow \quad m_1 \ddot{\vec{r}}_1' = \frac{-G m_1 M}{|\vec{r}_1'|^3} \vec{r}_1'$$

let's assume periodic circular motion of the x component of this

$$r_{1x}' = \cos(\omega t + \phi)$$

if $\ddot{r}_{1x}' = \frac{-GM}{|\vec{r}_1'|^3} \vec{r}_1'$

$$\Rightarrow -\omega^2 = \frac{-GM}{|\vec{r}_1'^3|}$$

we know that $d = \vec{r}_1' - \vec{r}_2'$

And also that $\vec{r}_2' = -\left(\frac{m_1}{m_2}\right) \vec{r}_1'$

$$\Rightarrow d = \vec{r}_1' + \frac{m_1}{m_2} \vec{r}_1'$$

$$\Rightarrow \vec{r}_1' = \frac{d}{1 + \frac{m_1}{m_2}}$$

subbing in,

$$\Rightarrow w^2 = \frac{GM}{d^3}\left(1+\frac{m_1}{m_2}\right)^3$$

but $M = \frac{m_2}{\left(1+\frac{m_1}{m_2}\right)^2}$

$$\Rightarrow w^2 = \frac{Gm_2\left(1+\frac{m_1}{m_2}\right)}{d^3}$$

$$\Rightarrow w = \sqrt{\frac{G(m_1+m_2)}{d^3}}$$

since $w = \frac{2\pi}{T}$ $\Rightarrow$ $T = \frac{2\pi}{\sqrt{\frac{G(m_1+m_2)}{d^3}}}$

SOLUTION

$$\boxed{\Rightarrow T = \sqrt{\frac{4\pi^2 d^3}{G(m_1+m_2)}}}$$

✓

And since the orbit is circular, the period would be the same for the y componant, and if you'd derived the whole thing from F_2 instead of F_1. ✓

6

b) Pick numbers for G, m, m_2, and r, and, using appropriate ICs, test your analytical result

Again, going back to the very beginning,

$$\vec{F}_{21} = \frac{-Gm_1m_2}{|\vec{r}_{1/2}|^2}\frac{\vec{r}_{1/2}}{|\vec{r}_{1/2}|}$$

$$\vec{F}_{12} = \frac{-Gm_1m_2}{|\vec{r}_{2/1}|^2}\frac{\vec{r}_{2/1}}{|\vec{r}_{2/1}|}$$

Using Eulers,

$$\vec{r}_i[n+1] = \vec{r}_i[n] + \vec{v}_i[n]\Delta t$$

$$\vec{v}_i[n+1] = \vec{v}_i[n] + \vec{a}_i[n]\Delta t$$

$\vec{a}_i[n] = \vec{F}_i/m_i$ where $n = t/\Delta t$, and Δt is some timestep

For parameters, I chose $G=1$, $m_1=1$, $m_2=2$ and $d=1$. This means that for position ICs, $\vec{r}_{1_0} = \begin{bmatrix}0\\0\end{bmatrix}$, $\vec{r}_{2_0} = \begin{bmatrix}1\\0\end{bmatrix}$ satisfy d. →

7

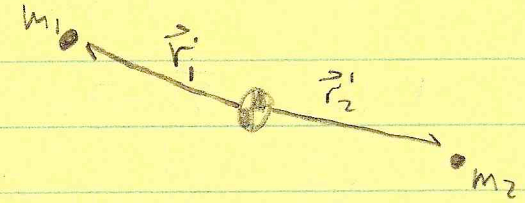
For V_0, the magnitude of velocity tangent to the circle, we know that the mass will complete one circumference of the circle in one period. so,

$$|V_{1_0}| \frac{m}{s} = \frac{2\pi |\vec{r}_1'|}{T} \quad , \quad |V_{2_0}| = \frac{2\pi |\vec{r}_2'|}{T}$$

we know from before that $\vec{r}_1' = -\frac{m_2}{m_1}\vec{r}_2'$

$$\vec{r}_2' = -\frac{m_1}{m_2}\vec{r}_1'$$

and that $d = |\vec{r}_1' - \vec{r}_2'|$, minus because they point in different directions:



$$\Rightarrow \quad d = \left|\vec{r}_1' + \frac{m_1}{m_2}\vec{r}_1'\right| = \left(1 + \frac{m_1}{m_2}\right)|\vec{r}_1'|$$

$$\Rightarrow \quad |\vec{r}_1'| = \frac{d}{\left(1 + \frac{m_1}{m_2}\right)} \quad , \quad |\vec{r}_2'| = \frac{d}{\left(1 + \frac{m_2}{m_1}\right)}$$ (by symmetry)

and if T is given by $T = \sqrt{\dfrac{4\pi^2 d^3}{G(m_1+m_2)}}$ $\longrightarrow$

Kind of cheating, should use $T = m m_2 G / r^2$

$$\Rightarrow |V_{1_0}| = \frac{2\pi \, d / \left(1 + \frac{m_1}{m_2}\right)}{\sqrt{\frac{4\pi^2 d^3}{G(m_1+m_2)}}} , \quad |V_{2_0}| = \frac{2\pi \, d / \left(1 + \frac{m_2}{m_1}\right)}{\sqrt{\frac{4\pi^2 d^3}{G(m_1+m_2)}}}$$

since they are both going in opposite directions, we can choose

$$\vec{V}_{1_0} = \begin{bmatrix} 0 \\ |V_{10}| \end{bmatrix} , \quad \vec{V}_{2_0} = \begin{bmatrix} 0 \\ -|V_{20}| \end{bmatrix}$$

(A more robust solution is

$$\vec{V}_{1_0} = \begin{bmatrix} \cos(\pi/2 + \theta) \\ \sin(\pi/2 + \theta) \end{bmatrix} |V_{10}| , \quad \vec{V}_{2_0} = -\begin{bmatrix} \cos(\pi/2 + \theta) \\ \sin(\pi/2 + \theta) \end{bmatrix} |V_{20}|$$

where θ is the angle between the two masses, and the $+\frac{\pi}{2}$ is because we want our initial velocity to be $\perp$ to that angle)

And, if $G = 1$, $m_1 = 1$, $m_2 = 2$, $d = 1$, we should expect

$$\boxed{T = \sqrt{\frac{4\pi^2}{3}}} \approx 3.63 \text{ seconds}$$

we see the attached trajectory, plotted parametrically, for four seconds with $\Delta t = 10^{-5}$

In order to test our hypothesis for the period, the two bodies should have the same x coordinate they started with after $T = 3.63$ seconds

SOLUTION:

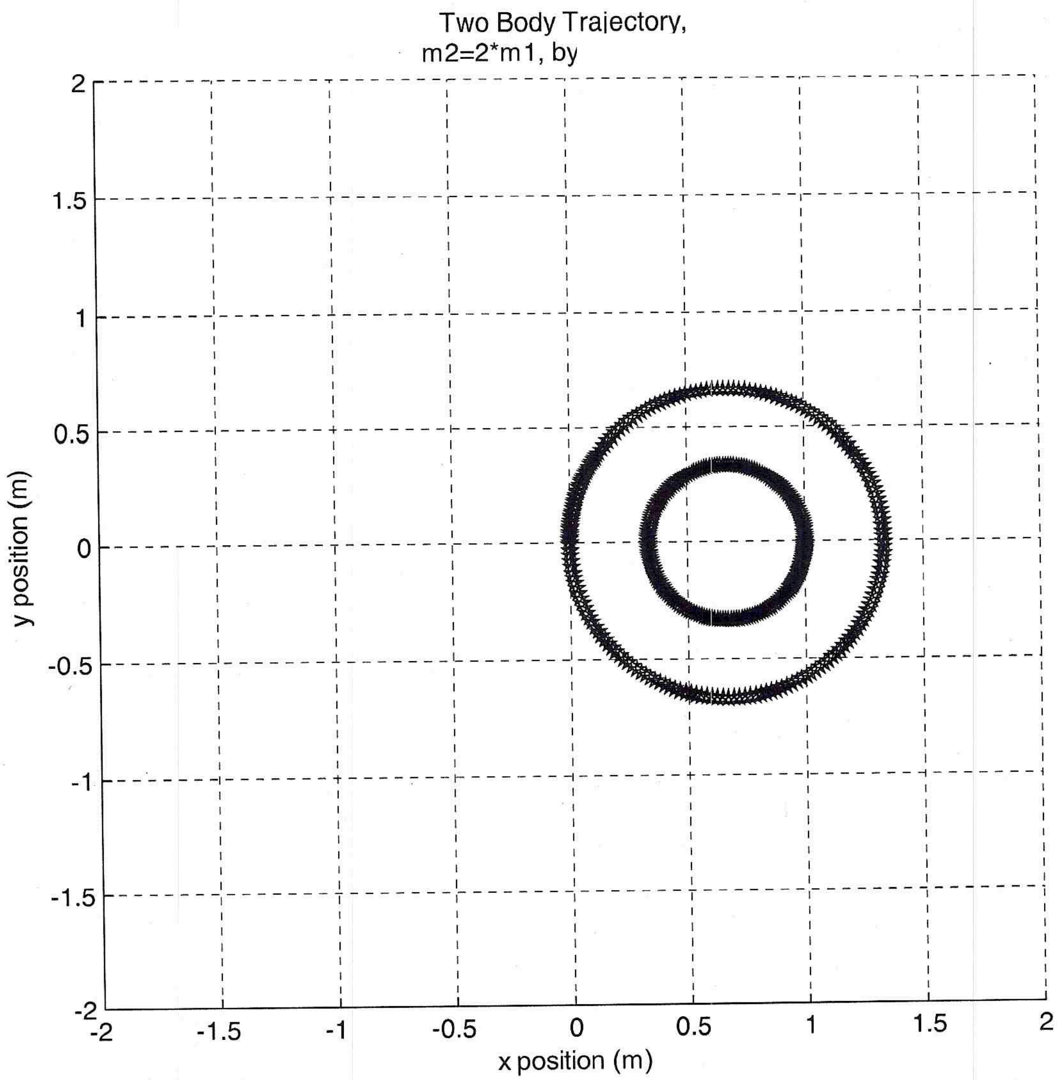
The attached figure "X coordinate of the two masses" shows a numerical justification of our analytical solution, as each mass has returned to its initial position in close to the predicted period.

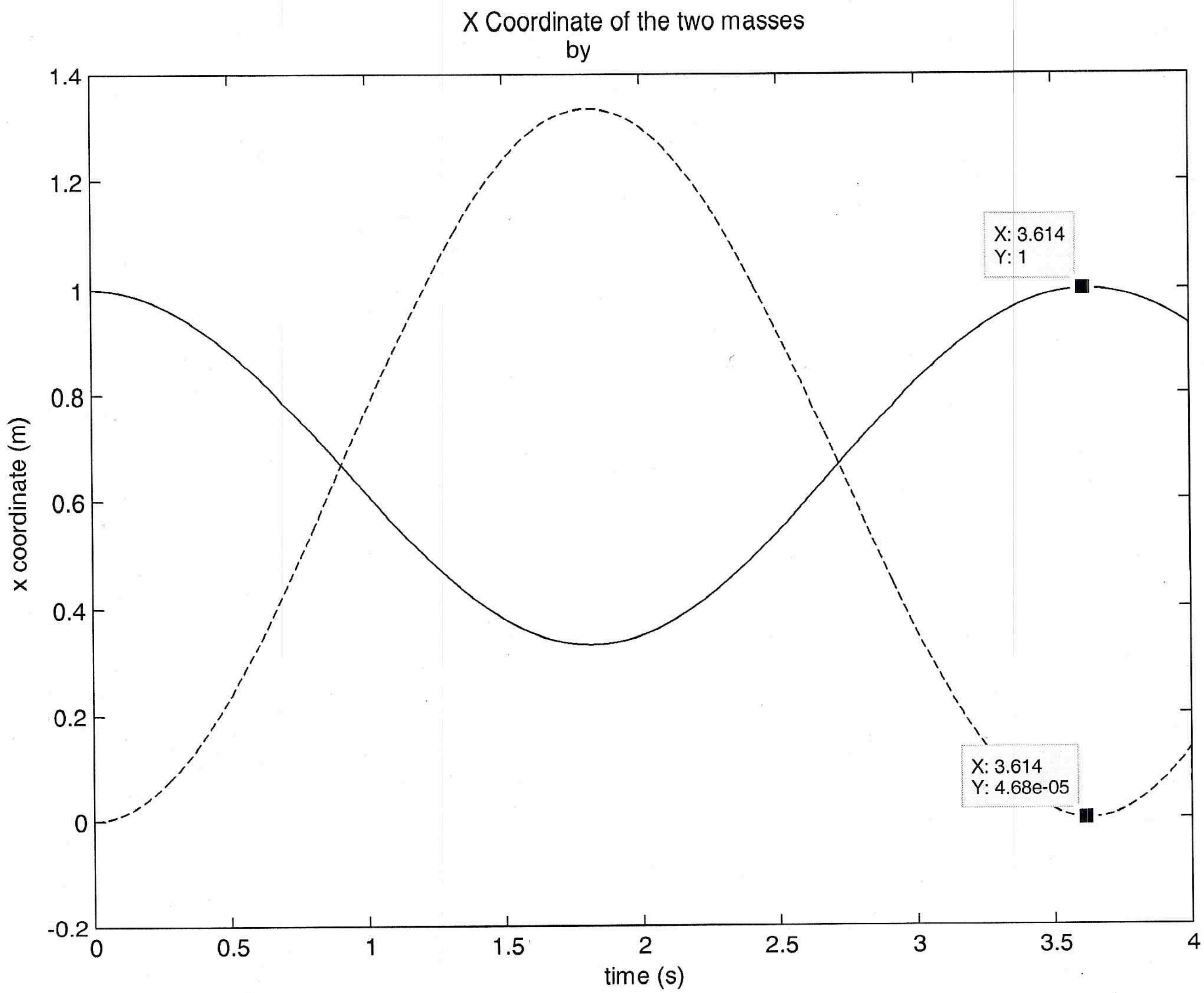
```
function [time,t1,t2]=orbit2(h,totalTime) %by

%this function outputs the orbit of the two particles subjected to the
%following initial conditions, from time 0 to 'totalTime'
%at a timestep of 10^-h. It's a little more complicated than it has to be
%to allow for any initial conditions, not just those along the x axis
deltaT=10^-h;
numSteps=totalTime/deltaT;  %set up time step
time=0:deltaT:totalTime;
t1=zeros(2,length(time));
t2=zeros(2,length(time));

G=1; %define parameters
m1=1;
m2=2;

p1x0=0; %define initial position on particle 1
p1y0=0;
p2x0=1; %define initial position on particle 2
p2y0=0;

r1=[p1x0,p1y0]; %put the position ICs in vector form
r2=[p2x0,p2y0];

rCOM=(m1*r1+m2*r2)/(m1+m2);

scatter(rCOM(1),rCOM(2),'g*')

Ang=atan2((r2(2)-r1(2)),(r2(1)-r1(1)))+pi/2

d=norm(r1-r2);
r1=r1-rCOM; %put the position ICs in vector form
r2=r2-rCOM;


v1x0=cos(Ang)*2*pi*d/(1+m1/m2)/sqrt(4*pi^2*d^3/(G*(m1+m2))); %initial velocity on
particle 1
v1y0=sin(Ang)*2*pi*d/(1+m1/m2)/sqrt(4*pi^2*d^3/(G*(m1+m2)));

v2x0=-cos(Ang)*2*pi*d/(1+m2/m1)/sqrt(4*pi^2*d^3/(G*(m1+m2))); %initial velocity on
particle 2
v2y0=-sin(Ang)*2*pi*d/(1+m2/m1)/sqrt(4*pi^2*d^3/(G*(m1+m2)));

v1=[v1x0,v1y0]; %put the velocity ICs in vector form
v2=[v2x0,v2y0];


for n=1:numSteps %perform Euler's method

    t1(1,n)=rCOM(1)+r1(1); %first save the trajectories
    t1(2,n)=rCOM(2)+r1(2);
```

11

```
    t2(1,n)=rCOM(1)+r2(1);
    t2(2,n)=rCOM(2)+r2(2);

    a1=-G*m2/norm(r1-r2)^3*(r1-r2); %then update them
    a2=-G*m1/norm(r2-r1)^3*(r2-r1);

    r1=r1+v1*deltaT;
    r2=r2+v2*deltaT;

    v1=v1+a1*deltaT;
    v2=v2+a2*deltaT;
end
```

good


Two Body Trajectory,
m2=2*m1, by
y position (m)
x position (m)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

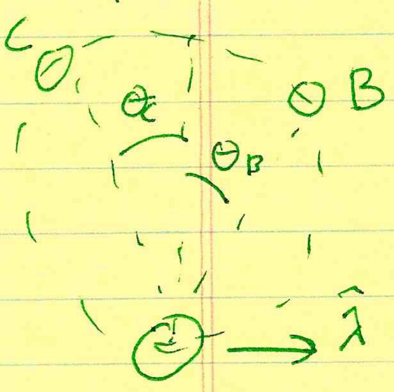

X Coordinate of the two masses
by
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
x coordinate (m)
X: 3.614
Y: 1
X: 3.614
Y: 4.68e-05
0
0.5
1
1.5
2
2.5
3
3.5
4
time (s)

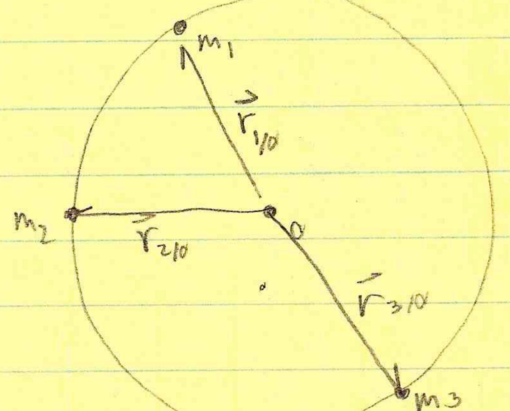
c) For three equal particles $m_1 = m_2 = m_3 = 1$, and $G = 1$, what is the angular speed for circular motion on a circle with diameter D.

Claim - This orbit is only possible if the three masses are equidistant and have the same velocities

Proof: Assume the three masses are on a circular orbit, around a circle of radius $d/2$.

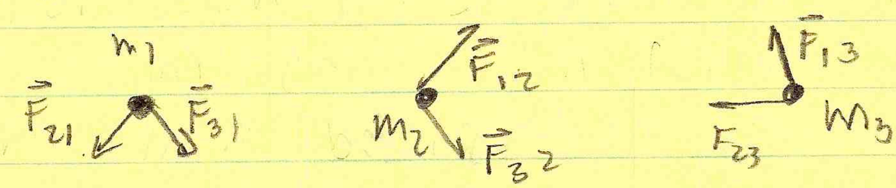
OR, LOOK at A





where $|\vec{r}_1| = |\vec{r}_2| = |\vec{r}_3| = d$

FBD:



if $\theta_B > \theta_C$
$\Rightarrow \hat{a}_A \cdot \hat{\lambda} > 0$
which can't be $\Rightarrow$
$\boxed{\theta_B = \theta_C}$
etc.

For each mass, we can construct a fictional centrigital force acting on it that is equivalent in magnitude to the gravitational forces exerted by the two other masses, wher $F_{cent_i} = \frac{d}{2}\hat{r}_i m_i w_i^2$

$$\Rightarrow \frac{d}{2} m_1 \hat{r}_1 w_1^2 = F_{21} \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|} + F_{31} \frac{\vec{r}_1 - \vec{r}_3}{|r_1 - r_2|}$$

$$\frac{d}{2} m_2 \hat{r}_2 w_2^2 = F_{12} \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} + F_{32} \frac{\vec{r}_2 - \vec{r}_3}{|\vec{r}_2 - \vec{r}_3|}$$

$$\frac{d}{2} m_3 \hat{r}_3 w_3^2 = F_{13} \frac{\vec{r}_3 - \vec{r}_1}{|r_3 - \vec{r}_1|} + F_{23} \frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|}$$

we know $d=1$, $m_i=1$. Furthurmore, let us introduce notation $\hat{e}_{ij} = \frac{\vec{r}_i - \vec{r}_j}{|r_i - r_j|} = -\hat{e}_{ji}$

$$\Rightarrow \frac{1}{2} \hat{r}_1 w_1^2 = F_{21} \hat{e}_{21} + F_{31} \hat{e}_{31} \quad (1)$$

$$\frac{1}{2} \hat{r}_2 w_2^2 = F_{12} \hat{e}_{12} + F_{32} \hat{e}_{32} \quad (2)$$

$$\frac{1}{2} \hat{r}_3 w_3^2 = F_{13} \hat{e}_{13} + F_{23} \hat{e}_{23} \quad (3)$$

From ①,

$$\frac{1}{2}\hat{r}_1 w_1^2 - F_{21}\hat{e}_{21} = F_{31}\hat{e}_{31}$$

From ②

$$\frac{1}{2}\hat{r}_2 w_2^2 - F_{12}\hat{e}_{12} = F_{32}\hat{e}_{32}$$

F_{ij} is the magnitude of gravitational force, or $F_{ij} = \frac{G m_1 m_2}{|\vec{r}_i - \vec{r}_j|^2} = F_{ji}$

if, as we said earlier, $\hat{e}_{ij} = -\hat{e}_{ji}$

$$\Rightarrow \frac{1}{2}\hat{r}_1 w_1^2 - F_{21}\hat{e}_{21} = -F_{13}\hat{e}_{13}$$

$$\frac{1}{2}\hat{r}_2 w_2^2 - F_{12}\hat{e}_{12} = -F_{23}\hat{e}_{23}$$

I got the Idea to do this → from seyros M.

subbing these into ③,

$$\Rightarrow \frac{1}{2}\hat{r}_3 w_3^2 = -\frac{1}{2}\hat{r}_1 w_1^2 + F_{21}\hat{e}_{21} - \frac{1}{2}\hat{r}_2 w_2^2 + F_{12}\hat{e}_{12}$$

$$\Rightarrow \frac{1}{2}\left(\hat{r}_3 w_3^2 + \hat{r}_2 w_2^2 + \hat{r}_1 w_1^2\right) = F_{21}\hat{e}_{21} + F_{12}\hat{e}_{12}$$

but, $F_{21}\hat{e}_{21} = -F_{12}\hat{e}_{12}$ ⟶

so,

$$\hat{r}_1 w_1^2 + \hat{r}_2 w_2^2 + \hat{r}_3 w_3^2 = 0$$

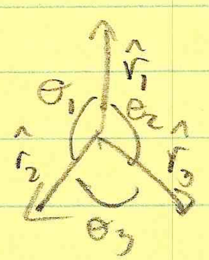
Assure $w_1 = w_2 = w_3$ (if they didn't, the paths would eventually crash, and we would have a two body problem)

$$\Rightarrow \hat{r}_1 + \hat{r}_2 + \hat{r}_3 = 0$$

Dot both sides with $\hat{r}_1$:

$$\Rightarrow 1 + \hat{r}_2 \cdot \hat{r}_1 + \hat{r}_3 \cdot \hat{r}_1 = 0$$

$$\Rightarrow |\hat{r}_2||\hat{r}_1| \cos(\theta_1) + |\hat{r}_3||\hat{r}_1| \cos(\theta_2) = -1$$

for 

, and by symmetry, dotting by $\hat{r}_2, \hat{r}_3$,

$$\cos(\theta_1) + \cos(\theta_2) = -1$$
$$\cos(\theta_2) + \cos(\theta_3) = -1$$
$$\cos(\theta_3) + \cos(\theta_1) = -1$$

$\Rightarrow \cos(\theta_1) = -1 - \cos(\theta_2)$

$\Rightarrow \cos(\theta_3) - \cos(\theta_2) = 0 \quad \Rightarrow \theta_3 = \theta_2$

$\Rightarrow \theta_3 = \theta_1$ (by symmetry)

$\Rightarrow \theta_3 = \theta_1 = \theta_2$

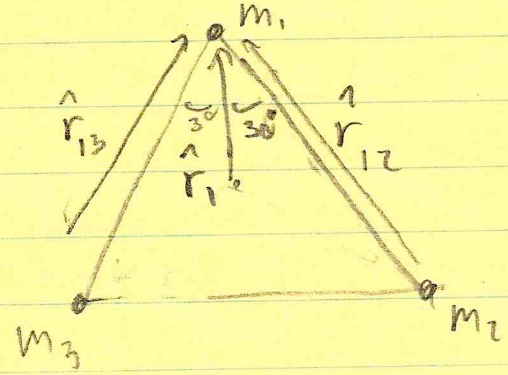
so, all angles must be equal, and that can only be satisfied for a $60°$-$60°$-$60°$ equilateral triangle.

To find the angular velocity, we can do another one of our force balances, equating centripital force with gravitational:

$$\frac{d}{2} m_1 \hat{r}_1 w^2 = +\frac{G m_1 m_2}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_{12} + \frac{G m_1 m_3}{|\vec{r}_1 - \vec{r}_3|^2} \hat{r}_{13}$$

$$\Rightarrow \frac{d}{2} \hat{r} w^2 = \frac{G}{|\vec{r}_1 - \vec{r}_2|^2} \hat{r}_{12} + \frac{G}{|\vec{r}_1 - \vec{r}_3|^2} \hat{r}_{13}$$

If we're on an equilateral triangle,



The forces contributed by m_2 and m_3 orthogonal to m_1 will cancel out. Therefore, we are only concerned with forces parallel to $\hat{r}_1$

$$\hat{r}_{12} = \hat{r}_1 \cos(30^\circ)$$

$$\hat{r}_{13} = \hat{r}_1 \cos(30^\circ)$$

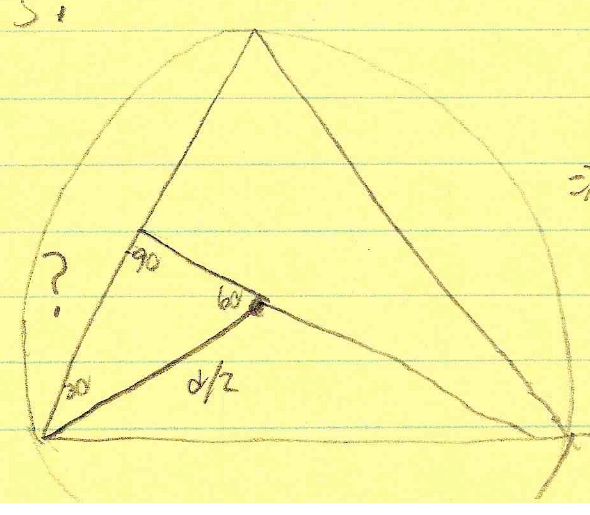
$$\Rightarrow \frac{dw^2}{2} = G\left(\frac{1}{|\vec{r}_1 - \vec{r}_2|^2} + \frac{1}{|\vec{r}_1 - \vec{r}_3|^2}\right)\cos(30)$$

since they are equidistant,

$$|\vec{r}_1 - \vec{r}_2| = |\vec{r}_1 - \vec{r}_3|$$

$$\Rightarrow w^2 = \frac{4G}{d|\vec{r}_1 - \vec{r}_2|^2}\cos(30)$$

so what is the distance between two planets:



$\Rightarrow ? = \frac{d}{4}\sqrt{3}$

$= \frac{1}{2}$ distance between them

so, the distance $|\vec{r}_1 - \vec{r}_2| = \frac{d}{2}\sqrt{3}$

$$\Rightarrow w^2 = \frac{4G}{d\left(\frac{d}{2}\sqrt{3}\right)^2} \cos(30)$$

$$= \frac{4G \cos(30)}{d^3 \frac{3}{4}} = \frac{16G\cos(30)}{d^3 \cdot 3}$$

since $\cos(30) = \frac{\sqrt{3}}{2}$, $G = 1$, $d = 1$

$$\Rightarrow w^2 = \frac{8\sqrt{3}}{3}$$

$$\Rightarrow \boxed{w = \sqrt{\frac{8\sqrt{3}}{3}}} \ \frac{\text{rad}}{\text{seconds}}$$

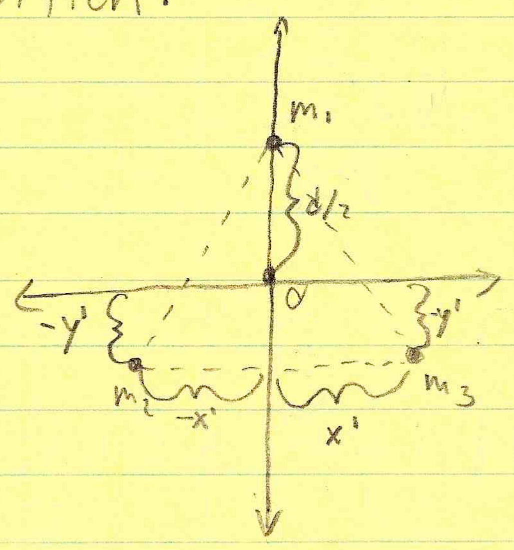
Checked with
Spyros M. →
and Daniel C. ✓

d) Check your results against numerical solution

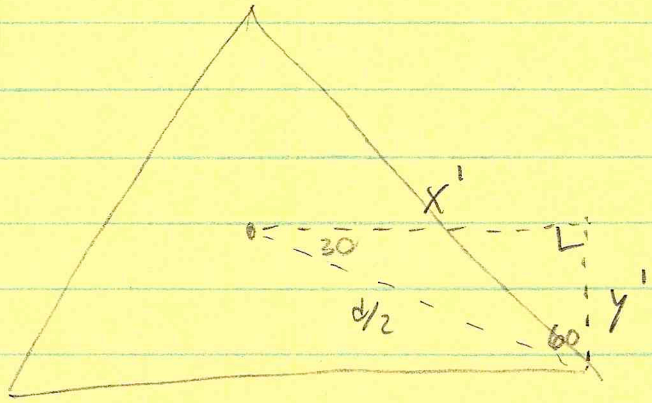
Using a modified version of code with the same principals as those used in b), the script "Threebody.m" was written to perform this computation.

As for what initial conditions to use, they must satisfy
- equilateral triangle
- velocity tangent to circle

For position:



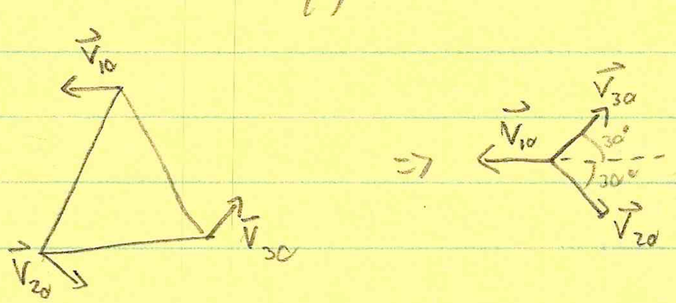
It is clear to see that $\vec{r}_{1_0} = \begin{bmatrix} 0 \\ d/2 \end{bmatrix}$. If we set $\vec{r}_{2_0} = \begin{bmatrix} -x' \\ -y' \end{bmatrix}$, $\vec{r}_{3_0} = \begin{bmatrix} x' \\ -y' \end{bmatrix}$ →



This is a 60-30-90, so $y' = \frac{d}{4}$, $x' = \frac{d}{4}\sqrt{3}$

so, $\vec{r}_{20} = \begin{bmatrix} -\frac{d\sqrt{3}}{4} \\ -d/4 \end{bmatrix}$, $\vec{r}_{30} = \begin{bmatrix} \frac{d\sqrt{3}}{4} \\ -d/4 \end{bmatrix}$

For velocity, if we define $\vec{V}_{10} = \begin{bmatrix} -V_0 \\ 0 \end{bmatrix}$:



$\Rightarrow \vec{V}_{20} = \begin{bmatrix} V_0 \cos(30°) \\ -V_0 \sin(30°) \end{bmatrix}$, $\vec{V}_{30} = \begin{bmatrix} V_0 \cos(30°) \\ V_0 \sin(30) \end{bmatrix}$

But what is V_0? If we know from 6, that $\omega = \sqrt{\frac{8\sqrt{3}}{3}} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\sqrt{\frac{8\sqrt{3}}{3}}} \longrightarrow$

If $V_0 = \frac{2\pi r}{T}$, and $r = d/2$

$$\Rightarrow V_c = \left(\frac{\pi d}{2\pi}\right) \cdot \sqrt{\frac{8\sqrt{3}}{3}} = \left(\sqrt{\frac{8\sqrt{3}}{3}}\right) \cdot \frac{d}{2}$$

timestep 10^{-4} → We can see the trajectory over 4 seconds plotted in "Three body trajectory"

SOLUTION

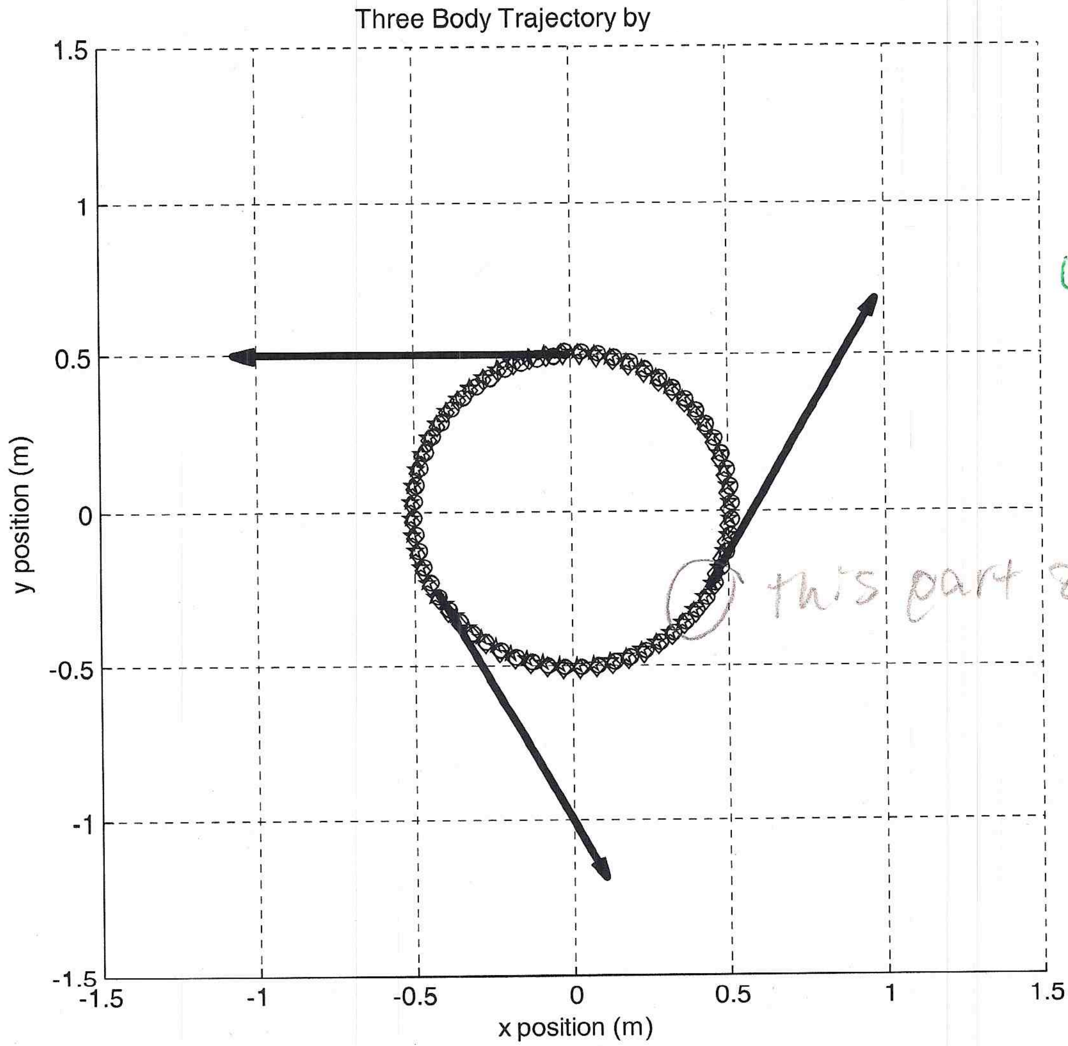
By plotting the angle $\left(\text{atan2}(r_{ix}, r_{iy})\right)$ Yes of each mass over time, the angle should come back to its starting point after T seconds. If

$$T = \frac{2\pi}{\sqrt{\frac{8\sqrt{3}}{3}}} = 2.92 \text{ seconds}$$

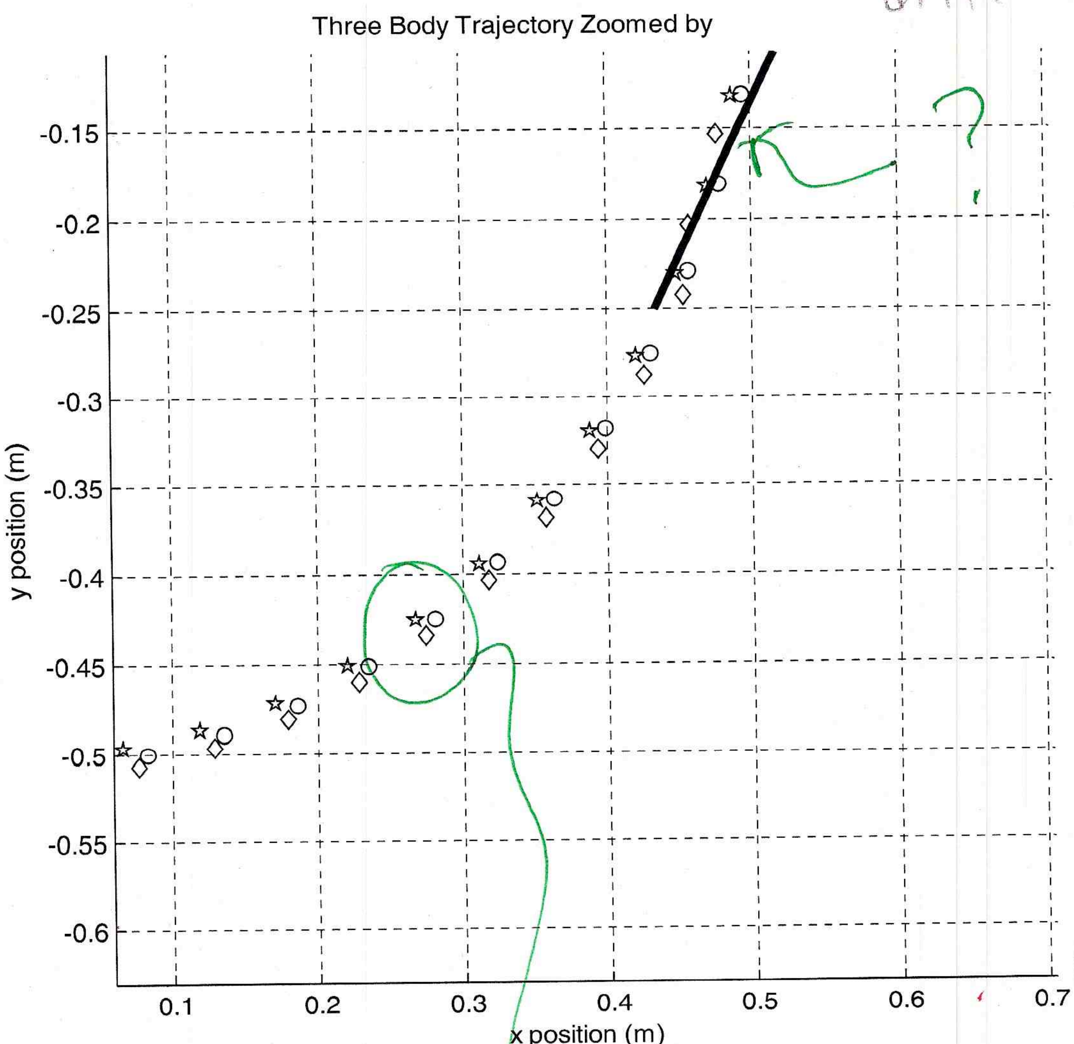
we can see from the plot "Angle of masses in three body problem" a numerical justification of our analytical solution.

```
function [time,t1,t2,t3]=ThreeBody(h) %by

%this function outputs the orbit of three particles subjected to the
%specific initial conditions gven from time 0 to 'totalTime'
%at a timestep of 10^-h

deltaT=10^-h;
totalTime=4;

%setup
numSteps=totalTime/deltaT;
time=0:deltaT:totalTime;
t1=zeros(2,length(time));
t2=zeros(2,length(time));
t3=zeros(2,length(time));

G=1; %define parameters
m1=1;
m2=1;
m3=1;
d=1;


p1x0=0; %define initial position on particle 1
p1y0=d/2;
p2x0=-d*sqrt(3)/4; %define initial position on particle 2
p2y0=-d/4;
p3x0=d*sqrt(3)/4; %define initial position on particle 3
p3y0=-d/4;

r1=[p1x0,p1y0]; %put the position ICs in vector form
r2=[p2x0,p2y0];
r3=[p3x0,p3y0];

V=sqrt(8*sqrt(3)/3)*d/2;

v1x0=-V; %initial velocity on particle 3
v1y0=0;
v2x0=V*cos(pi/3);   %initial velocity on particle 1
v2y0=-V*sin(pi/3);
v3x0=V*cos(pi/3);   %initial velocity on particle 2
v3y0=V*sin(pi/3);


v1=[v1x0,v1y0]; %put the velocity ICs in vector form
v2=[v2x0,v2y0];
v3=[v3x0,v3y0];


for n=1:numSteps
    t1(1,n)=r1(1); %first save the trajectories
```

```
    t1(2,n)=r1(2);

    t2(1,n)=r2(1);
    t2(2,n)=r2(2);

    t3(1,n)=r3(1);
    t3(2,n)=r3(2);

                         %then update
    a1=-G*m2/norm(r1-r2)^3*(r1-r2)-G*m3/norm(r1-r3)^3*(r1-r3);
    a2=-G*m1/norm(r2-r1)^3*(r2-r1)-G*m3/norm(r2-r3)^3*(r2-r3);
    a3=-G*m2/norm(r3-r2)^3*(r3-r2)-G*m1/norm(r3-r1)^3*(r3-r1);

    r1=r1+v1*deltaT;
    r2=r2+v2*deltaT;
    r3=r3+v3*deltaT;

    v1=v1+a1*deltaT;
    v2=v2+a2*deltaT;
    v3=v3+a3*deltaT;
end
```


Three Body Trajectory by
1.5
1
0.5
0
-0.5
-1
-1.5
y position (m)
-1.5
-1
-0.5
0
0.5
1
1.5
x position (m)
this part zoomed

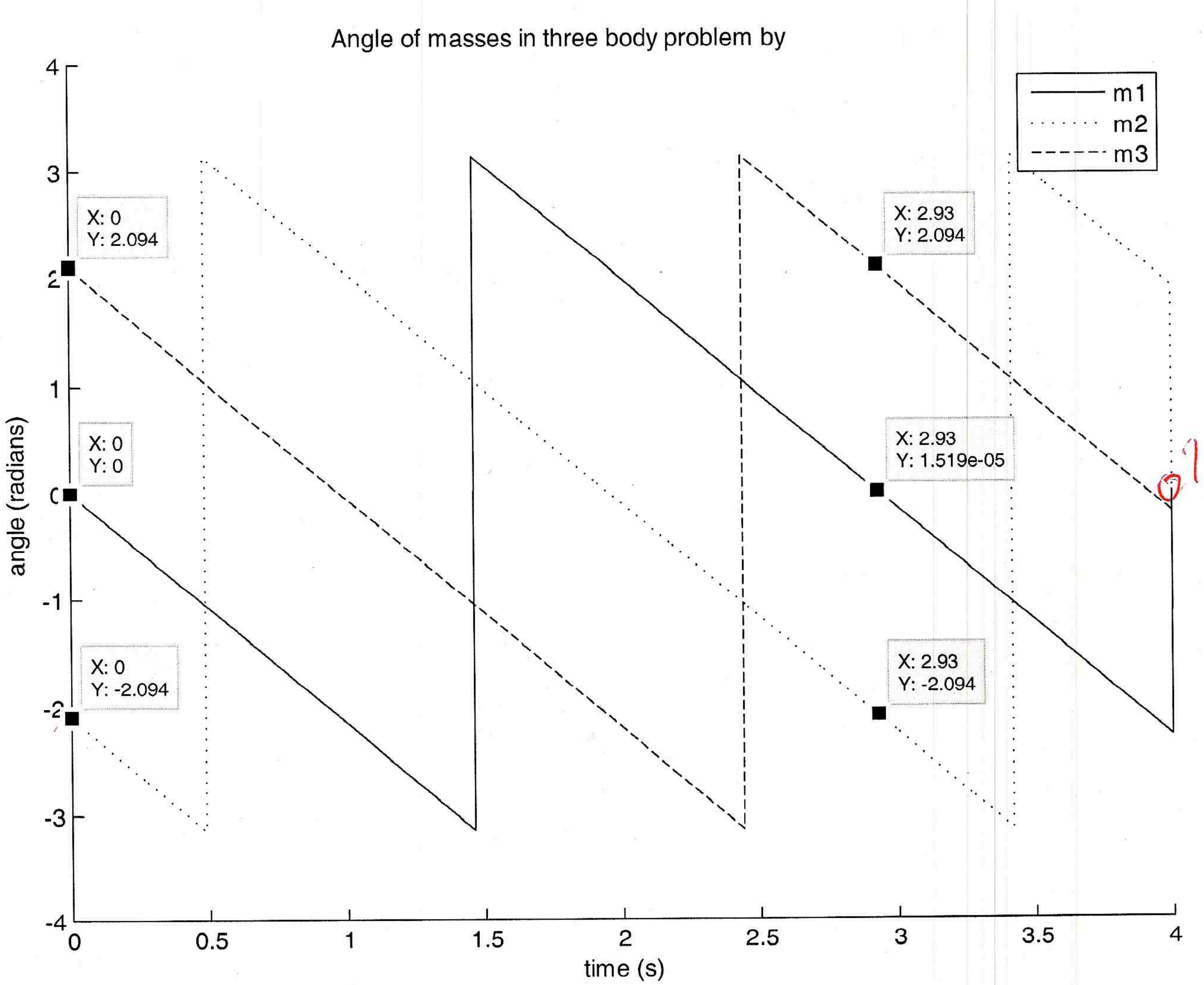
each mass is plotted as a different symbol



?

pretty big ~~error~~ error

use axis('equal') for trajectories


Angle of masses in three body problem by
m1
m2
m3
X: 0
Y: 2.094
X: 0
Y: 0
X: 0
Y: -2.094
X: 2.93
Y: 2.094
X: 2.93
Y: 1.519e-05
X: 2.93
Y: -2.094
angle (radians)
time (s)
4
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2
2.5
3
3.5
4
0?

O.K.