4 hours

8. Konig's Theorem

The total kinetic energy of a system of particles is

$$E_K = \frac{1}{2} \sum m_i v_i^2$$

a) Derive an expression of this form

$$E_K = \frac{1}{2} M_{tot} V_G^2 + [\ldots]$$

First, lets define the system of particles w/r/t a center of mass:

$$V_{i/0} = V_{G/0} + V_{i/G}$$

subbing this into the given statement,

$$\Rightarrow E_K = \frac{1}{2} \sum m_i \left(V_{G/0} + V_{i/G}\right)^2$$

$$= \frac{1}{2} \sum m_i \left(V_{G/0}^2 + V_{G/0} V_{i/G} + V_{i/G}^2\right)$$

$$\Rightarrow E_K = \frac{1}{2} M_{Tot} V_G^2 + \frac{1}{2} V_G \sum m_i V_{i/G} + \frac{1}{2} \sum m_i V_{i/G}^2$$

If $m_{TOT}\vec{V}_G = \sum m_i \vec{V}_i$

$$\Rightarrow \sum m_i \vec{V}_{i/G} = m_{Tot}\vec{V}_{G/G} = 0$$

because the relative velocity of G with respect to G must be zero.

→ reasoned through this w/ Daniel C.

⇒

SOLUTION:

$$E_K = \frac{1}{2} M_{Tot} V_G^2 + \frac{1}{2}\sum m_i V_{i/G}^2$$

✓

b) Is it always true that

$$(\Sigma \vec{F}^{ext}) \cdot \vec{V}_G = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{V}_G^2\right)$$

we know that $\Sigma \vec{F}_{ext} = \Sigma m_i \bar{a}_{i/o}$

$$= m_{TOT} \cdot \vec{a}_G = m_{TOT} \cdot \dot{\vec{V}}_G$$

$$\Rightarrow m_{TOT} \cdot \dot{\vec{V}}_G \cdot \vec{V}_G = \frac{d}{dt}\left(\frac{1}{2} m_{tot} \vec{V}_G \cdot \vec{V}_G\right)$$

Differentiating w/ chain rule,

$$\Rightarrow m_{TOT} \dot{\vec{V}}_G \cdot \vec{V}_G = \frac{1}{2}\left(m_{TOT}\left(\dot{\vec{V}}_G \cdot \vec{V}_G + \vec{V}_G \cdot \dot{\vec{V}}_G\right)\right)$$

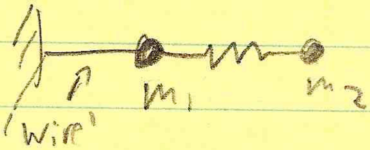
SOLUTION

$$m_{TOT} \dot{\vec{V}}_G \cdot \vec{V}_G = m_{TOT} \dot{\vec{V}}_G \cdot \vec{V}_G$$

so yes, it is always true

c) Is it always true that the power due to internal forces is equal to the rate of change of the quantity you filled in for part a)?

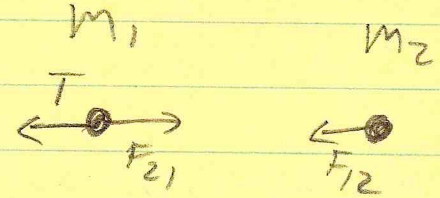
so, let's create a system where there are internal forces, but some of the particles are constrained, and see if its power is equal to $\sum m_i \vec{a}_{i/O} \cdot \vec{v}_{i/O}$:



I get the idea to do this from office hours

The spring is in tension, then released.

FBD:



For m_1, the system is constrained by the wire; $T = F_{21}$, and m_1 does not move.

We can write out the power due to internal forces as

$$P_{int} = \vec{F}_{21} \cdot \vec{V}_{1/O} + \vec{F}_{12} \cdot \vec{V}_{2/O}$$

We know $\vec{V}_{1/O} = 0$, so

$$P_{int} = \vec{F}_{12} \cdot V_{2O}$$

Now, looking at our claim, we ask:

$$P_{int} \stackrel{?}{=} \sum m_i \vec{a}_{i/O} \cdot \vec{V}_{i/O} \longrightarrow$$

expanding RHS:

$$= m_1 \vec{a}_{1/G} \cdot \vec{v}_{1/O} + m_2 \vec{a}_{2/G} \cdot \vec{v}_{2/O}$$

Again, $\vec{v}_{1/O} = 0$, so we have

$$\vec{F}_{12} \cdot \vec{v}_{2/O} \stackrel{?}{=} m_2 \vec{a}_{2/G} \cdot \vec{v}_{2/O}$$

we know $\vec{F}_{12} = m_2 \vec{a}_{2/O}$,

$$\Rightarrow m_2 \vec{a}_{2/O} \cdot \vec{v}_{2O} \stackrel{?}{=} m_2 \vec{a}_{2/G} \cdot v_{2/O}$$

If $\vec{a}_{2/O} = \vec{a}_{2/G} + a_{G/O}$

$$\Rightarrow \vec{a}_{2/G} \cdot \vec{v}_{2/O} + \vec{a}_{G/O} \cdot \vec{v}_{2/O} \stackrel{?}{=} m_2 \vec{a}_{2/G} \cdot \vec{v}_{2/O}$$

SOLUTION:

This is not true, and only could be true if the second mass were not moving (which it is) or the center of gravity is not accelerating