1.6 Matrix Operations

MATH 294 SPRING 1982 PRELIM 1 # 1 294SP82P1Q1.tex

1.6.1 **a)** Write the system of equations

$$\begin{array}{ccccccc} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ 2x_1 & + & 3x_2 & + & 4x_3 & = & -2 \\ 3x_1 & + & 4x_2 & + & 6x_3 & = & 0 \end{array}$$

in the form $A\vec{x} = \vec{b}$.

b) Find the det A for A in part (a) above.

c) Does A^{-1} exist?

d) Solve the above system of equations for $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

e) Let $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}$. Find $A \cdot B$ (i.e. calculate the product AB).

MATH 294 SPRING 1983 FINAL # 10 294SP83FQ10.tex

1.6.2 **a)** Find a basis for the vector space of all 2x2 matrices.

b) $\underline{\underline{A}}$ is the matrix given below, $\underline{v}$ is an eigenvector of $\underline{\underline{A}}$. Find any eigenvalue of $\underline{\underline{A}}$.

$$\underline{\underline{A}} = \begin{bmatrix} 3 & 0 & 4 & 2 \\ 8 & 5 & 1 & 3 \\ 4 & 0 & 9 & 8 \\ 2 & 0 & 1 & 6 \end{bmatrix} \text{ with } \underline{v} = [\text{an eigenvector of } \underline{\underline{A}}] = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

c) Find one solution to each system of equations below, if possible. If not possible, explain why not.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} \cdot \underline{x} = \underline{b},\ \underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

d) Read carefully. Solve for $\underline{\mathbf{x}}$ in the equation $\underline{\underline{A}} \cdot \underline{b} = \underline{\mathbf{x}}$ with:

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } \underline{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

e) Find the inverse of the matrix

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

MATH 293 SPRING 1990 PRELIM 1 # 5 293SP90P1Q5.tex

1.6.3 **a**) Find the 2 x 2 matrix A such that if $\vec{x} = (x_1,\ x_2)$ in V_2, $A\vec{x} = (3x_1,\ 2x_2)$.
If B is a matrix and $\vec{x}$ is a vector, compute $B\vec{x}$ whenever possible.

b) $B = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$, $\vec{x} = (c, -b, a)$

c) $B = (1\ 3\ 2\ -1)$, $\vec{x} = (3, -1, -1, 2)$

d) $B = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & 2 \end{bmatrix}$, $\vec{x} = (1, 3)$

MATH 293 FALL 1991 PRELIM 3 # 3 293FA91P3Q3.tex

1.6.4 Write the matrix

$$\begin{pmatrix} -11 & -3 \\ 4 & 1 \end{pmatrix}$$

as a product of elementary matrices.

MATH 293 FALL 1991 PRELIM 3 # 6 293FA91P3Q6.tex

1.6.5 Given a matrix A is mxn with $m \neq n$ and $AX = I$ and $YA = I$ (where I is the corresponding identity matrix), it follow that $X = Y$. True or false.

MATH 293 SPRING 1992 PRELIM 2 # 1 293SP92P2Q1.tex

1.6.6 **a**) Calculate $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$.

b) Calculate $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 9 & 11 \end{bmatrix}$.

MATH 293 SUMMER 1992 PRELIM 2 # 1 293SU92P2Q1.tex

1.6.7 Given three matrices A, B and C,

$$A = \begin{pmatrix} -1 & 2 & 3 \\ 4 & 1 & -5 \end{pmatrix},\ B = \begin{pmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 1 \end{pmatrix},\ C = \begin{pmatrix} -1 & 1 & 0 & -3 & 2 \\ 0 & 2 & 1 & -1 & 1 \end{pmatrix}$$

Find the following products whenever possible

a) AB

b) BA

c) $A(BC)$

d) CB

MATH 293 SPRING 1993 PRELIM 2 # 2 293SP93P2Q2.tex

1.6.8 Given the matrices

$$B = \begin{pmatrix} -1 & 2 \\ 4 & 2 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & 2 \\ 2 & 3 \end{pmatrix}.$$

a) Compute BC.
b) Compute CB.
c) Compute $(BC - CB)^2$.

MATH 293 SPRING 1993 PRELIM 2 # 4 293SP93P2Q4.tex

1.6.9 Factor the given matrix, A, a the product LU, where L is a lower triangular matrix, and U is an upper triangular matrix. Label clearly which is L and which is U:

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 5 & -8 \\ 2 & 2 & 5 \end{pmatrix}$$

MATH 293 SPRING 1994 PRELIM 2 # 2 293SP94P2Q2.tex

1.6.10 (True/false) The following properties hold for the matrix

$$A = \begin{pmatrix} 2 & -3 & 7 \\ -1 & 4 & 0 \end{pmatrix}:$$

a) If $AM = AN$ then $M = N$, where M and N are 3 x 2 matrices.
b) A has an inverse.
c) A is in reduced row echelon form.
d) A is equal to the matrix $B = \begin{pmatrix} -1 & 4 & 0 \\ 2 & -3 & 7 \end{pmatrix}$.
e) A and B are row equivalent.
f) A and B have the same row reduced form.
g) $(A^T)^T = A$.
h) $B^TA = BA^T$.

MATH 293 FALL 1994 FINAL # 9 293FA94FQ9.tex

1.6.11 Let A and B be n by n matrices. Then
a) $AB = BA$,
b) $(AB)^T = A^TB^T$,
c) $A^{-1}B^{-1} = (AB)^{-1}$,
d) $(A^TB^T)^T = BA$,
e) $BAB = AB^2$

MATH 293 SPRING 1996 PRELIM 2 # 4 293SP96P2Q4.tex
1.6.12 Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -1 & 2 \\ 1 & 0 & 0 & -1 \end{pmatrix}.$$

Let $\mathbf{x} = (0, \frac{1}{2}, 1, 0)$. We know that $A\mathbf{x} = 0$. True or false:
1. $\mathbf{x}$ is a *trivial* solution to $A\mathbf{x} = 0$.

MATH 293 SPRING 1996 PRELIM 2 # 8 293SP96P2Q8.tex
1.6.13 Let M be the matrix

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

Find the entry in the 4th row, 4th column of the matrix M^3. Hint: this matrix is the adjacency matrix of a certain graph. Draw the graph and use this to interpret the entries of M^3.

MATH 293 SPRING 1996 FINAL # 34 293SP96FQ34.tex
1.6.14 If A and B are matrices, then $(A+B)(A+B) = A^2 + 2AB + B^2$. True or false.

MATH 293 SPRING 1996 FINAL # 35 293SP96FQ35.tex
1.6.15 If A and B are nxn matrices, then $(A^T B^T)^T = BA$. True or false.

MATH 294 FALL 1997 PRELIM 1 # 1 294FA97P1Q1.tex
1.6.16 **a**) Consider the problem $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & 2 \\ -1 & 2 & 1 & -3 \end{pmatrix}, \text{and } \vec{b} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$

Determine the general solution to this problem, in vector form.
b) Find a 2 by 2 matrix B, which is not the zero matrix, with $B^2 = 0$.

MATH 294 FALL 1997 FINAL # 4 294FA97FQ4.tex
1.6.17 Let

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}.$$

a) A is the adjacency matrix of a graph G. Find this graph.
b) Using many pencils, we computed the following powers of A:

$$A^9 = \begin{pmatrix} 656 & 533 & 452 & 533 & 737 & 452 \\ 533 & 368 & 328 & 369 & 452 & 328 \\ 452 & 328 & 368 & 328 & 533 & 369 \\ 533 & 369 & 328 & 368 & 452 & 328 \\ 737 & 452 & 533 & 452 & 656 & 533 \\ 452 & 328 & 369 & 328 & 533 & 368 \end{pmatrix},$$

$$A^{10} = \begin{pmatrix} 1803 & 1189 & 1189 & 1189 & 1560 & 1189 \\ 1189 & 902 & 780 & 901 & 1189 & 780 \\ 1189 & 780 & 902 & 780 & 1189 & 901 \\ 1189 & 901 & 780 & 902 & 1189 & 780 \\ 1560 & 1189 & 1189 & 1189 & 1803 & 1189 \\ 1189 & 780 & 901 & 780 & 1189 & 902 \end{pmatrix},$$

$$A^{11} = \begin{pmatrix} 3938 & 2992 & 2749 & 2992 & 4181 & 2749 \\ 2992 & 2090 & 1969 & 2091 & 2749 & 1969 \\ 2749 & 1969 & 2090 & 1969 & 2992 & 2091 \\ 2992 & 2091 & 1969 & 2090 & 2749 & 1969 \\ 4181 & 2749 & 2992 & 2749 & 3938 & 2992 \\ 2749 & 1969 & 2091 & 1969 & 2992 & 2090 \end{pmatrix}.$$

How many paths of length 10 (i.e. 10 edges) from vertex 2 to vertex 3 are there in the graph G?

MATH 294 SPRING 1998 PRELIM 2 # 3 294SP98P2Q3.tex

1.6.18 **a)** All matrices A, B, C, X, Y, Z, and I are nxn and I is the identity matrix. The inverse of

$$\begin{bmatrix} I & 0 & 0 \\ C & I & 0 \\ A & B & I \end{bmatrix} \text{ is } \begin{bmatrix} I & 0 & 0 \\ Z & I & 0 \\ X & Y & I \end{bmatrix}.$$

Find X.

b) Complete the L, U factorization of

$$\begin{bmatrix} 1 & 3 & -5 & -3 \\ -1 & -5 & 8 & 4 \\ 4 & 2 & -5 & -7 \\ -2 & -4 & 7 & 5 \end{bmatrix}.$$

MATH 293 SPRING xx xx # 4 293SPxxQ4.tex

1.6.19 **a)** Find a basis for the row space of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 3 & 6 & 1 & 12 \\ 9 & 18 & 1 & 36 \end{bmatrix}.$$

b) If A is an mxn matrix show that $B = A^t A$ and $C = AA^t$ are both square. What are their sizes? Show that $B = B^t$, $C = C^t$.

MATH 293 SPRING xx xx # 5 293SPxxQ5.tex

1.6.20 **a)** If U is an r x m matrix and D and F are r x 1 column vectors, express the relation

$$f_k = \sum_{i=1}^{m} \sum_{j=1}^{r} u_{ji} d_j u_{ki}, \; k = 1, ..., r$$

in matrix form.

b) If A is an n x n matrix show that $\det(cA) = c^n \det A$.