4.1 General 2-D Integrals

MATH 293 SPRING ?? FINAL # 7 294SPXXFQ7.tex

4.1.1 Show that the transformation $x = au\cos 2\pi v,\ y = bu\sin 2\pi v$, where a and b are positive constants, takes the unit square $0 \le u \le 1,\ 0 \le v \le 1$ in the (u, v) plane onto the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the (x, y) plane. (Hint: draw a picture and show where each edge of the unit square in the (u, v) plane is taken by the transformation.) Then compute the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, by a suitable integral over the unit square in the (u, v) plane.

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4.1.2 Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-y}} e^{(3x-x^3)}\,dxdy.$$

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4.1.3 Recall that the moment of the inertia of a planar region R about the origin is defiend by

$$I_0 = \int\int_R \delta(x, y)(x^2 + y^2)dA,$$

where δ denotes the mass density (per unit area) For $\delta(x, y) = \cos\left[(x^2 + y^2)^2\right]$ and R defined by $1 \le x^2 + y^2 \le \frac{\pi^2}{4}$, compute I_0.

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4.1.4 Set up (but do not evaluate) the integrals necessary to find the area of the region bounded by the curves $x = y^2,\ y = 2x - 6$, and the x axis,

a) integrating first with respect to x, and

b) integrating first with respect to y.

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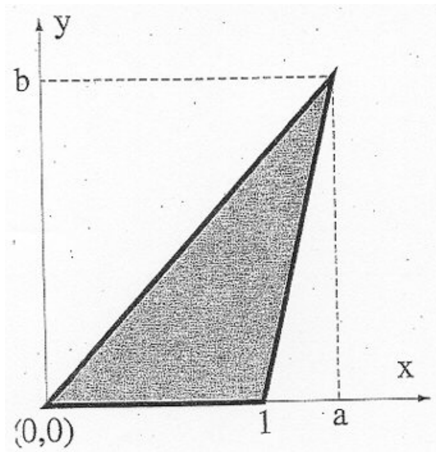
4.1.5 Let R be the interior of the triangle with vertices at (0,0), (1,0) and (a, b). Find the y coordinate of its centroid. [If you happen to know the answer without calculation you may use this as a check. No partial credit for just quoting the result and plugging it in, however.]



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4.1.6 Consider the integral

$$\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2} dydx$$

a) Sketch the region for which this integral gives the area.
b) Convert the integral to polar coordinates.
c) Evaluate the integral.

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4.1.7 Consider the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \cos{(y^3)} dydx$$

a) Draw the region of integration in the x-y plane.
b) Write an equivalent integral with the order of integration reversed.
c) Evaluate the integral.