## 4.5 Surface Area

MATH 294FALL 1982FINAL# 8294FAS2FQ8.tex4.5.1Find the area of the surface cut from the plan

$$x + y + z = 1$$

by the cylinder

$$x^2 + y^2 = 1.$$

MATH 294 SPRING 1983 FINAL # 8 294SP83FQ8.tex

**4.5.2** Find the surface area of the paraboloid  $z - x^2 - y^2 = 0$  over that part of the surface for which x and y are inside the curve  $x = \cos(t)$ ,  $y = \sin(t)$ ,  $0 \le t \le 2\pi$ .

MATH 294 SPRING 1987 PRELIM 1 # 7 294SP87P1Q7.tex

**4.5.3** The base of a box is the unit square in the first quadrant of the (x, y) plane with corners at (0,0,0), (0,1,0), (1,1,0), and (1,0,0). Points on the crooked top of the otherwise rectangular box satisfy 3z - 4x - 12y = 3. What is the area of the top of the box?



**MATH 294** FALL 1987 MAKE UP PRELIM 1 # 1 294FA87MUP1Q1.tex **4.5.4** Let S denote the portion of the sphere  $x^2 + y^{23} + z^2 = 25$  that is above the cone  $z = 2\sqrt{2}\sqrt{x^2 + y^2}$ . Evaluate

$$\int \int_S x d\sigma$$

MATH 294 SPRING 1988 PRELIM 1 # 2 294SP88P1Q2.tex

- **4.5.5** The region of interest is the inside of some ellipse drawn on the plane  $z = \frac{4x}{3}$ . Its projection onto the x y plane (is also an ellipse, of course, and) has area  $9\pi$ . What is the area of the region of interest?
- MATH 294 FALL 1989 PRELIM 1 # 2 294FA89P1Q2.tex
- **4.5.6** Find the surface area of the part of the paraboloid  $z = 1 x^2 y^2$  that lies above the x y plane.

MATH 294FALL 1989FINAL# 1294FA89FQ1.tex4.5.7Consider the surface given by

$$z^2 = 2xy, \ z > 0,$$

lying over the square  $0 \le x \le 1$ ,  $0 \le y \le 1$  in the x - y plane. Find its surface area.

- MATH 294 SPRING 1990 PRELIM 2 # 4 294SP90P2Q4.tex
- **4.5.8** Determine the surface area of the portion of the cone  $z^2 = x^2 + y^2$  between the planes z = 1 and z = 2.
- MATH 294 SPRING 1990 FINAL # 6 294SP90FQ6.tex
- **4.5.9** Compute the surface area of the portion of the saddle,  $z = x^2 y^2$ , that is contained in the cylinder  $x^2 + y^2 = 1$ .
- MATH 294 SUMMER 1990 PRELIM 1 # 5 294SU90P1Q5.tex
- **4.5.10** Sketch the part of the surface  $z + x^2 + y^2 = 2$  in the first octant and calculate its surface area.
- MATH 294 SPRING 1991 PRELIM 3 # 1 294SP91P3Q1.tex
- **4.5.11** Consider the conical surface  $4y^2 + 4z^2 x^2 = 0$ 
  - **a**) Determine a field of unit vectors normal to this surface.
  - b) Determine the area of the portion of the surface between the planes x = 0 and x = 2.
- MATH 294 FALL 1991 PRELIM 3 # 2 294FA91P3Q2.tex
- **4.5.12** a) The total charge Q on a surface S is given by the formula

$$Q = \int \int_{S} \rho(x, y, z) d\sigma$$

where  $\rho$  is the surface charge density. Find Q if S is the hemispherical surface

$$x^{2} + y^{2} + z^{2} = 4$$
, with  $z \ge 0$ ,

 $\operatorname{and}$ 

$$\rho = z(1 - \frac{z^2}{4})^{\frac{1}{2}}$$

**b**) The average charge  $\vec{\rho}$  is defined in terms of Q and the area A of S by  $\vec{\rho} = \frac{Q}{A}$ . Determine the average charge on S.

MATH 294SPRING 1992PRELIM 3# 3294SP92P3Q3.tex4.5.13Evaluate the surface integral

$$\int \int_{S} (1+4z) d\sigma$$

where S is the portion of the paraboloid  $z = x^2 + y^2$  between the two planes z = 1and z = 4. MATH 294 SPRING 1992 FINAL # 7 294SP92FQ7.tex

**4.5.14** Find the surface area of the portion of the graph of the function f(x, y) = xy which lies inside the cylinder  $x^2 + y^2 = 1$ .

MATH 294 FALL 1992 FINAL # 9a 294FA92FQ9a.tex 4.5.15 Find the surface area,

$$A = \int \int_S d\sigma,$$

where S is the portion of the plane 2x + 3y + 4z = 12 in the first octant.

MATH 294FALL 1993PRELIM 1# 3294FA93P1Q3.tex4.5.16Find the area of the saddle-like surface

$$z = x^2 - y^2, \qquad x^2 + y^2 \le b^2$$

where b is a constant.



MATH 294SUMMER 1995QUIZ 2# 2294SU95P2Q2.tex4.5.17Evaluate the surface integral

$$\int \int_{S} (y\hat{i} - z\hat{j} + z\hat{k}) \cdot \hat{n} d\sigma,$$

where *S* is the sphere  $x^{2} + y^{2} + (z - 2)^{2} = 9$ .

MATH 294 FALL 1995 PRELIM 1 # 3 294FA95P1Q3.tex

- **4.5.18** a) Find a normal vector field to the surface S given by  $z x^2 5y^2 = 0$ ,  $z \le 20$ .
  - **b**) Make a sketch showing the relationship between area elements,  $d\sigma$  on S and area elements dA on R, the projection of S onto the (x, y) plane, and use this sketch to explain the equation relating  $d\sigma$  to dA
  - c) Set up, but do not evaluate, a double integral for the surface area of S.

**MATH 294** SPRING 1996 PRELIM 1 # 2a 294SP96P1Q2a.tex **4.5.19** Find the total area of the surface S given by  $z = x^2 + y^2$ ,  $z \le 25$ . Sketch S.

MATH 293 FALL 1996 FINAL # 5 293FA96FQ5.tex

4.5.20 Surface. Find the surface area of the planar surface ABD shown by any means except MATLAB.



4