4.6 3-D Flux and Divergence

MATH 294 SPRING 1984 FINAL # 7 294SP84FQ7.tex

4.6.1 Evaluate $\int_S \vec{F}\cdot\vec{N}dudv$ where $\vec{F} = -y\hat{\imath} + x^2\hat{\jmath} + z^2\hat{k}$ and S is the closed surface which consists of the hemisphere $x^2 + y^2 + z^2 = 1,\ z > 0$, together with its base $x^2 + y^2 \le 1,\ z = 0$.

MATH 294 SPRING 1985 FINAL # 2 294SP85FQ2.tex

4.6.2 For the case $\vec{b} = \vec{0}$, the vector $\vec{x} = \vec{0}$

a) Is always a solution.
b) May or may not be a solution depending on $\underline{\underline{A}}$.
c) Is always the only solution.
d) Is never a solution.

MATH 294 FALL 1986 FINAL # 7 294FA86FQ7.tex

4.6.3 What is the value of the surface integral $\int_S\int \vec{F}\cdot\hat{n}d\sigma$ where $\vec{(F)} = (yz^2 + y)\hat{\imath} + (x^2z^2 + y)\hat{\jmath} + (x^2y + y^4 + z)\hat{k},\ S$ is the unit hemisphere, $x^2 + y^2 + z^2 = 1,\ z \ge 0$, which is "capped" from below by the disk $z = 0,\ x^2 + y^2 \le 1$ and $\hat{n}$ is the unit outward normal to S?

MATH 294 FALL 1986 FINAL # 11 294FA86FQ11.tex

4.6.4 Let $\vec{F}$ be the vector field $\vec{F}(x, y, z) = x^3\hat{\imath}+y^3\hat{\jmath}+3z\hat{k}$. Let S be the complete boundary of the solid region inside the cylinder $x^2 + y^2 = 4$, between the planes $z = 0$ and $z = 3$. Let $\hat{n}$ denote the outward unit normal on S. Evaluate $\int_S\int \vec{F}\cdot\hat{n}d\sigma$.

MATH 294 SPRING 1987 PRELIM 1 # 5 294SP87P1Q5.tex

4.6.5 Consider the three dimensional vector field:

$$\mathbf{F} = (2x - y)\mathbf{i} + (x + z)\mathbf{j} + z^2\mathbf{k}$$

a) Evaluate $div(\mathbf{F})$ at (1,2,3).
b) Is $\int\int_S \mathbf{F}\cdot\mathbf{n}dS$ greater than, less than, or equal to zero for: $\mathbf{F}$ given above, and $\mathbf{n}$ the out facing unit normal to the unit sphere centered at (7,7,7)? (Hint: Don't do any difficult calculations.)

MATH 294 SPRING 1987 FINAL # 5 294SP87FQ5.tex

4.6.6 Evaluate the integrals below by any means. In each case, $\mathbf{F} = (ye^z + x)\mathbf{i} + (2y - z)\mathbf{j} + 7k$, S is the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. D is the interior of the sphere, and $\mathbf{n}$ is the outward pointing unit normal of the sphere's surface. Hints: Each one of the integrals below is equal to at least one of the others. Volume of a sphere $= (4/3)\pi r^3$, surface area of a sphere $= 4\pi r^2$.

a) $\displaystyle\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma.$

b) $\displaystyle\int\int_S d\sigma.$

c) $\displaystyle\int\int_D\int zdV.$

d) $\displaystyle\int\int_D\int div(curl(\mathbf{F}))dV.$

e) $\displaystyle\int\int_S curl(\mathbf{F}) \cdot \mathbf{n} d\sigma.$

MATH 294 FALL 1987 PRELIM 1 # 2 294FA87P1Q2.tex

4.6.7 Find the flux of the vector field $\mathbf{F}(x, y, z) = xz\mathbf{i} + yz\mathbf{j} + k$ across the hemisphere $S : x^2 + y^2 + z^2 = 4,\ z \geq 0$. Use the outward unit normal $\mathbf{n}$ (from the origin).

MATH 294 FALL 1987 PRELIM 1 # 4 294FA87P1Q4.tex

4.6.8 Let D be the solid bounded above by the surface $x^2 + y^2 + z^2 = 8$ and below by the cone $z = \sqrt{x^2 + y^2}$. For $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$, compute the flux of F across the complete boundary surface of D (which looks like an ice cream cone).

MATH 294 FALL 1987 FINAL # 2 294FA87FQ2.tex

4.6.9 Let D be the solid in the first octant that is bounded above by the paraboloid $x^2 + y^2 + z = 1$ and bounded by the coordinate planes. For $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + x\cos(xy)\mathbf{k}$, compute the flux $\displaystyle\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$, where S is the boundary surface of D.

MATH 294 FALL 1987 MAKE UP FINAL # 3 294FA87MUFQ3.tex

4.6.10 D is the solid ring bounded between the spheres $\rho = 2$ and $\rho = 3$ and between the cones $\phi = \frac{\pi}{3}$ and $\phi = \frac{\pi}{4}$.(ρ and ϕ are two of the three spherical coordinates, (ρ, θ, ϕ)). (Draw a picture!)

Let S denote the boundary surface of D. Compute the flux $\displaystyle\int\int_S \vec{F} \cdot \hat{n} d\sigma$, where $\vec{F} = x^3\hat{i} = y^3\hat{j} + z^3\hat{k}$.

MATH 294 SPRING 1988 PRELIM 1 # 6 294SP88P1Q6.tex

4.6.11 Find the outward flux of the vector field

$$\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + \mathbf{k}$$

through the portion of the sphere $x^2 + y^2 + z^2 = a^2$ that lies in the first octant $(x > 0,\ y > 0,\ z > 0)$.

MATH 294 SPRING 1988 PRELIM 2 # 2 294SP88P2Q2.tex

4.6.12 Find the outward flux of the vector field

$$\mathbf{F} = (x - y\sin z)\mathbf{i} + (2y + \sin z)\mathbf{j} + (3z - \sin x)\mathbf{k}$$

through the sphere $x^2 + y^2 + z^2 = 25$.

MATH 294 FALL 1988 PRELIM 3 # 4 294FA88P3Q4.tex

4.6.13 **a**) Using the Divergence Theorem, evaluate $\int\int_S \nabla \text{ x } \mathbf{F} \cdot \mathbf{n} d\sigma$, where S is any closed surface.(Hint: This can be done without specifying either the surface S or the vector F. Think!)

b) Show that the volume of a bubble, i.e., the surface 'blown up' from a plane curve C is given by the expression $\int\int_S \frac{\mathbf{n} \text{ x } \mathbf{R}}{3} d\sigma$, where S is the surface of the bubble R is the position vector to any point on the bubble, and n is the unit vector to the surface at that point.

MATH 294 SUMMER 1990 PRELIM 1 # 6 294SU90P1Q6.tex

4.6.14 Determine the outward flux of the vector field $\mathbf{F} = 6x\mathbf{i}$ over the volume bounded by the conical surface $x = \sqrt{y^2 + z^2}$ and the plane $x = 1$ in two ways:

a) By direct calculation of the fluxes over the two surfaces.

b) Using the Divergence theorem.

MATH 294 FALL 1989 PRELIM 1 # 1 294FA89P1Q1.tex

4.6.15 Calculate the outward flux of the vector field

$$\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$$

through the sides of the cube in the first octant bounded by the coordinate planes and the planes $x = 1,\ y = 1$, and $z = 1$.

MATH 294 SPRING 1990 PRELIM 2 # 3 294SP90P2Q3.tex

4.6.16 Given $\mathbf{F}(x, y, z) = (e^x + x^3)\mathbf{i} + (z^2 + y^3)\mathbf{j} - ze^x\mathbf{k}$, compute the flux of F across the boundary of the solid enclosed by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$, employing the outward unit normal vector $\mathbf{n}$ (pointing out of the solid).

MATH 294 SPRING 1990 FINAL # 4 294SP90FQ4.tex

4.6.17 Let D denote the hemisphere $x^2 + y^2 + z^2 \leq 9,\ z \geq 0$. Compute the flux of $\mathbf{F} = (x^3 + 7)\mathbf{i}(y^3 - \sin x)\mathbf{j} + (z^3 + \sin x)\mathbf{k}$ across the boundary surface S of D (employ the unit normal that points out of the solid D.)

MATH 294 FALL 1990 PRELIM 2 # 1 294FA90P2Q1.tex

4.6.18 **a**) Set up, but do not evaluate, the double integrals in the xy plane for:

i) the area of that part of the surface $y = z^2$ that is above the triangle bounded by$x = 0$, $y = 0$, and $y = 1 - x$,

ii) the flux of the vector field $\mathbf{F} = xy\mathbf{j}$ away from the origin over the surface $2x + y + 2z = 2$ in the first octant.

Take care to show the limits of integrations.

b) Use the divergence theorem to calculate the outward flux of the vector field $\mathbf{F} = yz\mathbf{i}+xy\mathbf{j}+xz\mathbf{k}$ from the volume in the first octant bounded by the coordinate planes and the surface $z = 1 - x^2 - y^2$.

MATH 294 FALL 1990 FINAL # 1 294FA90FQ1.tex

4.6.19 Given the surface $z = x^2 + 2y^2$. At the point (1,1) in the xy plane:

a) determine the direction of greatest increase of z.

b) determine a unit normal to the surface.

Given the vector field $\mathbf{F} = 6xy^2z\mathbf{i} - 2y^3z\mathbf{j} + 4z\mathbf{k}$,

a. calculate its divergence.

b. use the divergence theorem to calculate the outward flux of the vector field over the surface of a sphere of a unit radius centered at the origin.

MATH 294 FALL 1990 MAKE UP FINAL # 2 294FA90MUFQ2.tex

4.6.20 Given the plane $x+2y+3z = 6$ in the first octant and the vector field $\mathbf{F} = x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$,

a) calculate the flux of $\mathbf{F}$ through the plane in the direction away from the origin.

b) use the divergence theorem to calculate the total outward flux through the surface of the region bounded by the inclined plane and the coordinate planes.

MATH 294 SPRING 1991 PRELIM 3 # 2 294SP91P3Q2.tex

4.6.21 Given the closed region cut from the first octant by the coordinate planes and sphere $x^2+y^2+z^2 = 4$. Determine the outward flux of the vector field $\mathbf{F} = xz\mathbf{i}+yz\mathbf{j}+z^2\mathbf{k}$ through the surface in two different ways:

a) by direct calculation of the fluxes over the bounding surfaces;

b) using the Divergence Theorem.

MATH 294 SPRING 1991 FINAL # 1 294SP91FQ1.tex

4.6.22 Consider the vector field $\mathbf{F} = 2(x + y + 2z)\mathbf{k}$ and the region in the first octant bounded by the coordinate planes and the plane $x + y + 2z = 2$. Calculate the outward flux of the vector field from the region using the Divergence Theorem. Hint: The volume of the region is $\frac{2}{3}$.

MATH 294 FALL 1991 PRELIM 3 # 4 294FA91P3Q4.tex

4.6.23 **a**) Calculate the outward flux of

$$\mathbf{F} = (x^2 + z)\mathbf{i} + (y^2 + z)\mathbf{j} + (x^2 + y^2 + z)\mathbf{k}$$

over the surface S consisting of the paraboloid $z = x^2 + y^2$ closed at the top by the plane $z = 4$.

b) Determine the outward flux over the paraboloid portion of S alone.

MATH 294 FALL 1991 FINAL # 1 294FA91FQ1.tex

4.6.24 Let S be the part of the cylindrical surface $x^2 + z^2 = 1$ for $z \geq 0$ and $0 \leq y \leq 2$.

a) Determine the flux of the vector field $\mathbf{F} = y\mathbf{k}$ through S in a direction away from the y axis.

b) Determine the net outward flux of $\mathbf{F}$ from the closed region bounded by S and the planes $z = 0,\ y = 0,\ z = 2$.

MATH 294 SPRING 1992 PRELIM 3 # 4 294SP92P3Q4.tex

4.6.25 Let R be the unit cube $0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq z \leq 1$. Consider the vector field

$$\vec{F} = (x^3 + 2e^y)\hat{i} + (y^3 + \sin x)\hat{j} + (z^2 + y)\hat{k}.$$

a) Calculate the outward flux of $\vec{F}$ across the surface of R.

b) Calculate the outward flux of $\vec{F}$ across the base $z = 0,\ 0 \leq x \leq 1,\ 0 \leq y \leq 1$ of the cube R and then find the total outward flux across the remaining 5 sides.

MATH 294 SPRING 1992 FINAL # 8 294SP92FQ8.tex

4.6.26 Let S be the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

a) Compute the outward flux of the vector field

$$\vec{F}(x, y, z) = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$$

across the surface S.

b) Compute the circulation of $\vec{F}$ around the circle $x^2 + y^2 = 4,\ z = 4$ in the counterclockwise direction as viewed from above.

MATH 294 SPRING 1993 FINAL # 7 294SP93FQ7.tex

4.6.27 Use the Divergence Theorem to evaluate

$$\int\int_S (x\mathbf{i} + 2y\mathbf{j} - 25xy\mathbf{k}) \cdot \mathbf{n} d\sigma$$

where S is the sphere $x^2 + y^2 + z^2 = 1$ and $\mathbf{n}$ is the outer normal.

MATH 294 FALL 1993 PRELIM 1 # 4 294FA93P1Q4.tex

4.6.28 Find the flux of $\mathbf{F} = yz^4\mathbf{i} + 7y\mathbf{j} + 8\mathbf{k}$ outward through the surface of the cube

$$0 \leq x,\ y,\ z \leq 2$$

You may use the Divergence Theorem.

MATH 294 FALL 1993 FINAL # 3b 294FA93FQ3b.tex

4.6.29 The Divergence Theorem in three dimensions is $\int\int_S \mathbf{G} \cdot \hat{n} d\sigma = \int\int\int_D \nabla \cdot \mathbf{G} dV$, where $\hat{n}$ is a unit normal to the surface S which encloses the volume D. Choose a vector G such that the theorem "calculates" the volume of an arbitrary solid from a surface integral.

Test your result for a sphere of radius a, for which the surface area is $4\pi a^2$.

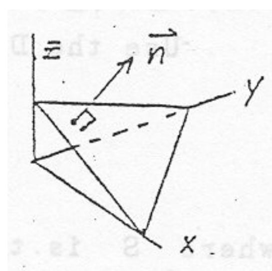
MATH 294 SPRING 1994 FINAL # 1 294SP94FQ1.tex
4.6.30 Find the upward flux of the vector field

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j}$$

across the portion of the surface $z = 4 - x^2 - y^2$ for which $z \geq 0$. Here "upward flux" means flux with respect to the upward unit normal to the surface.

MATH 294 SPRING 1994 FINAL # 3 294SP94FQ3.tex
4.6.31 Use the Divergence theorem to find the outward flux of the vector field $\mathbf{F} = 9y\mathbf{i} + 2xy\mathbf{j} - 2z\mathbf{k}$ across the boundary of the region D which is the portion of the cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$.

MATH 294 FALL 1994 PRELIM 1 # 2 294FA94P1Q2.tex
4.6.32 S is the triangular surface $x + y + 2z = 1$, $(x,\ y,\ z \geq 0)$ with unit normal $\hat{n}$ as shown.
a) Find $\hat{n}$.
b) Find an expression for $d\sigma$.
c) Find the flux of $\vec{F} = x\hat{i} - z\hat{j}$ through S.

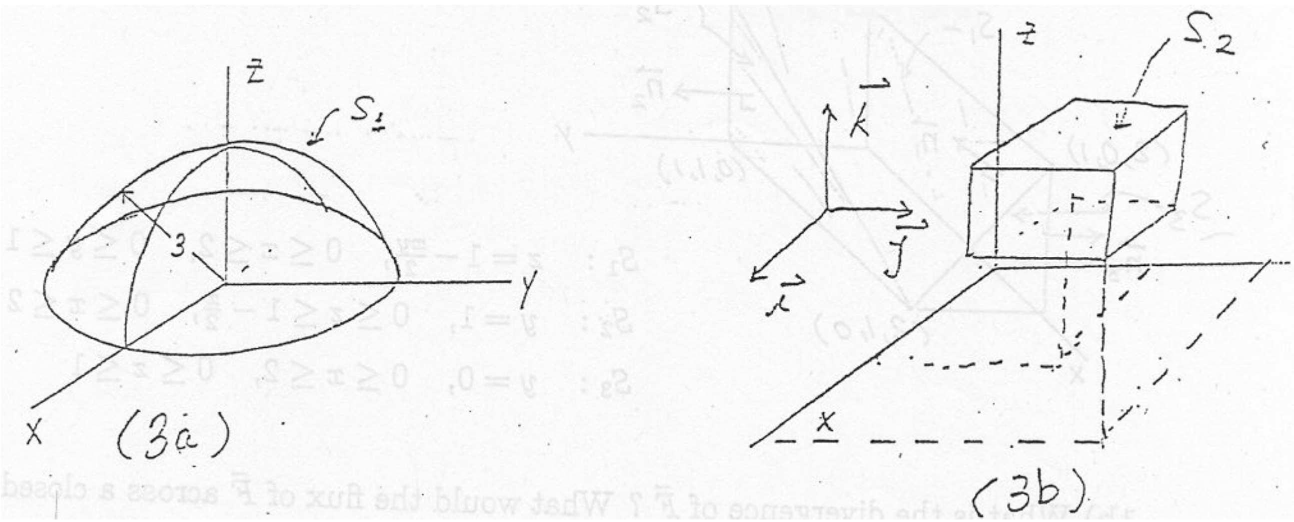


MATH 294 FALL 1994 PRELIM 1 # 3 294FA94P1Q3.tex
4.6.33 Evaluate $\displaystyle\int\int_{x^2+y^2+z^2=5} (\hat{i} + y\hat{j} - x\hat{k}) \cdot \hat{n} d\sigma$ where $\hat{n}$ is the outward normal. You may use the Divergence Theorem.

MATH 294 FALL 1994 FINAL # 3 294FA94FQ3.tex

4.6.34 Let S_1 = surface of the hemisphere $x^2 + y^2 + z^2 = 9,\ z \geq 0$

$$S_2 = \text{ surface of the cube} \begin{cases} 191 \leq x \leq 294 \\ 191 \leq y \leq 294 \\ 191 \leq z \leq 294 \end{cases}$$

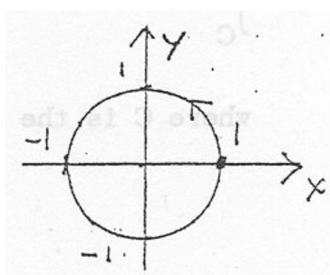


a) Evaluate the surface integral $\int\int_{S_2} z d\sigma$ directly.

b) Evaluate the surface integral $\int\int_{S_2} (2x\hat{\imath} - 5\hat{\jmath}) \cdot \hat{n} d\sigma$ where $\hat{n}$ is the outer normal to the cube.

MATH 294 SPRING 1995 PRELIM 1 # 1 294SP95P1Q1.tex

4.6.35 a) Evaluate the flux $\vec{F} = V_o z \hat{\jmath}$ across the surface S_1 and S_2 with outward unit normal vectors oriented as shown.



b) What is the divergence of $\vec{F}$? What would the flux $\vec{F}$ across a closed surface be?

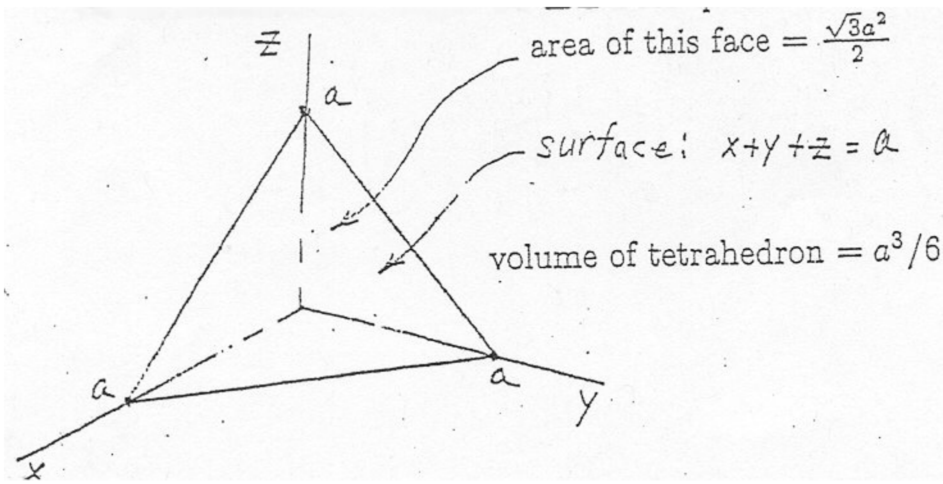
c) From your answers to part (a) and (b) deduce the flux across S_3 in terms of the fluxes across S_1 and S_2.

MATH 294 FALL 1995 PRELIM 1 # 4 294FA95P1Q4.tex

4.6.36 **a)** Evaluate the flux of $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ over each of the four faces of the tetrahedral surface sketched. (Hint: on 3 faces no calculations are needed.)

b) Use the divergence theorem to calculate the flux over the entire surface.

c) Think about the results of part (a) and (b). Are they consistent? Why or why not?



MATH 294 FALL 1995 FINAL # 6 294FA95FQ6.tex

4.6.37 Here S is the surface $x^2 - y^2 + z = 0$, where $x^2 + y^2 \leq 25$.

a) Find an expression for a normal unit vector field $\hat{n}$ on S. How many correct answers are there for $\hat{n}$.

b) Find an expression for the area element $d\sigma$ on S.

c) Find the area of S.

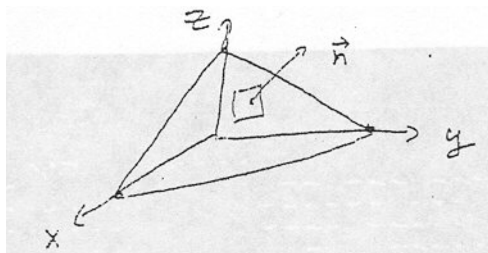
d) Find the flux of the vector field $300\,\vec{k}$ through S in the direction of your normal.

MATH 294 SPRING 1996 PRELIM 1 # 3 294SP96P1Q3.tex

4.6.38 **a)** Find the outward flux $\int\int_S \vec{F} \cdot \hat{n} d\sigma$ where $\vec{F} = 5x\hat{\imath} + ze^x\hat{\jmath} - 2\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 16$.

MATH 294 SPRING 1996 MAKE UP FINAL # 2 294SP96MUFQ2.tex

4.6.39 S is the triangular surface $2x + 3y + 4z + 12$ in the first octant with the unit normal vector field $\hat{n}$ as shown.



a) Find a formula for the unit normal vector field.

b) Find an expression for the area element $d\sigma$ on S.

c) Find the flux of $\vec{F} = \frac{3}{5}yz|hati - yz\hat{j}$ through S.

d) Let R be the solid cut off the first octant by S. Express the volume integral.

$$\int\int\int_R \vec{\nabla} \cdot \vec{F} dxdydz$$

in terms of a surface integral over the triangular face of R lying in the (y, z) plane. (You are not asked to evaluate it.)

MATH 294 FALL 1996 PRELIM 1 # 3 294FA96P1Q3.tex

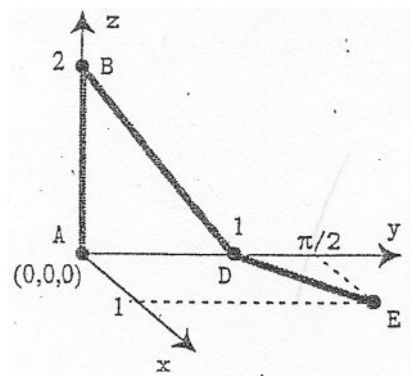
4.6.40 The divergence Theorem in three dimensions is $\int\int_S \mathbf{F} \cdot \hat{n} d\sigma = \int\int\int_D \nabla \cdot \mathbf{F} dv$, where $\hat{n}$ is the unit normal to the surface S which encloses the volume D. This may be used to "calculate" the volume of an arbitrary solid from a surface integral by choosing an appropriate $\mathbf{F}$. Do so and explicitly test your result for a sphere of a radius a, for which the surface area is $4\pi a^2$.

MATH 293 FALL 1996 FINAL # 3 293FA96FQ3.tex

4.6.41 Integrals. Any method allowed except MATLAB.

a) Evaluate $\int\int_S \mathbf{F}\cdot\mathbf{n}d\sigma$ with $\mathbf{F}=\mathbf{r}$ and S the sphere of radius 7 centered at the origin. [$\mathbf{n}$ is the outer normal to the surface, $\mathbf{r}\equiv x\hat{\imath}+y\hat{\jmath}+z\hat{k}$].

b) Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ with $\mathbf{F}=\sin(y)e^z\hat{\imath}+x\cos(y)e^z\hat{\jmath}+x\sin(y)e^z\hat{k}$ and the path C is made up of the sequence of three straight line A to B, B to D, and D to E.



MATH 293 FALL 1998 FINAL # 2 293FA98FQ2.tex

4.6.42 Find, by direct calculation, the outward flux of the vector field

$$\mathbf{F}=xz\mathbf{i}+yz\mathbf{j}+\mathbf{k}$$

through that part of the sphere $x^2+y^2+z^2=25$ that is above the plane $z=3$.

MATH 293 FALL 1998 FINAL # 5 293FA98FQ5.tex

4.6.43 Use the Divergence Theorem to calculate the outward flux of the vector field

$$\mathbf{F}=xz\mathbf{i}+yz\mathbf{j}+\mathbf{k}$$

through that part of the sphere $x^2+y^2+z^2=25$ that is above the plane $z=3$.