Chapter 6

Outliers

6.1 Some Geometry and Kinematics

MATH 294 UNKNOWN FINAL # 2

6.1.1 Let $R(t) = e^t \cos t\vec{i} + t\vec{j} + te^t\vec{k}$ be position of a particle moving in space at time t.

a) Set up, but do not evaluate, a definite integral equal to the distance traveled by the particle from $t = 0$ to $t = \pi$.

b) Find all points on the curve where the velocity vector is orthogonal to the acceleration vector.

MATH 293 UNKNOWN PRELIM 2 # 2

6.1.2 If $\vec{r}(t) = \cos t\vec{i} - 3\sin t\vec{j}$ gives the position of a particle

a) find the velocity and acceleration

b) sketch the curve, and sketch the acceleration and velocity vectors at one point of the curve (you choose the point)

c) what is the torsion (if you can do this without computation, that is acceptable - but please give reasons for your answer).

MATH 293 UNKNOWN FINAL # 1

6.1.3 **a**) Find an equation for the plane containing the points $(1, 0, 1), (-1, 2, 0), (1, 1, 1)$.

b) Find the cosine of the angle between the plane in a) and the plane $x-2y+z-5 = 0$.

MATH 294 SPRING 1984 FINAL # 1

6.1.4 Prove that for any vector $\vec{F}$:

$$\vec{F} = \left(\vec{F} \cdot \vec{i}\right)\vec{i} + \left(\vec{F} \cdot \vec{j}\right)\vec{j} + \left(\vec{F} \cdot \vec{k}\right)\vec{k}.$$

MATH 294 FALL 1985 FINAL # 1

6.1.5 Find a unit vector in $\Re^3$ which is perpendicular to both $\vec{i} + \vec{j}$ and $\vec{k}$.

MATH 294 SPRING 1985 FINAL # 5

6.1.6 Find a solution defined in the right half-plane $\{(x, y)|x > 0\}$ whose gradient is the vector field $\frac{-y}{x^2+y^2}\vec{i} + \frac{x}{x^2+y^2}\vec{j}$.

MATH 294 FALL 1985 FINAL # 7

6.1.7 Let S be the surface with equation $x^2 + xy = z^2 + 2y$.

a) Find the equation of the plane tangent to S at the point $(1, 0, 1)$

b) Find all points on S at which the tangent plane is parallel to the xy plane.

MATH 294 FALL 1986 FINAL # 8

6.1.8 Consider the curve $C : x = t, y = \frac{1}{t}, z = \ln t$, and the line $L : x = 1 + \tau, y = 1 + 2\tau, z = -\tau$. The curve and the line intersect at the point $P = (1, 1, 0)$. Let $\vec{v}$ be a unit vector tangent to C at $P, \vec{w}$ a unit vector tangent to L at P. Compute the cosine of the angle θ between $\vec{v}$ and $\vec{w}$.

MATH 294 SPRING 1987 PRELIM 1 # 4

6.1.9 Consider the function $f(x, y, z) = 1 - 2x^2 - 3y^4$.

a) Find a unit vector that points in the direction of maximum increase of f at the point $R = (1, 1, 1)$.

b) Find the outward unit normal to the surface $f = -4$ at any point of your choice (clearly indicate your choice near your answer).

MATH 294 SPRING 1987 FINAL # 6

6.1.10 A particle moves with velocity $\vec{v}$ that depends on position $(x, y).\vec{v} = (a + y)\vec{i} + (-x + y)\vec{j}$. At $t = 0$ the particle is at $x = 1, y = 0$. Where is the particle at $t = 1$?

MATH 294 SPRING 1988 PRELIM 2 # 1

6.1.11 Given $f = xy \sin z$ and $\vec{F} = (xy)\vec{i} + (e^{yz}\vec{j} + (z^2)\vec{k}$, evaluate:

a) $\vec{\nabla} f = grad(f)$ at $(x, y, z) = (1, 2, 3)$

b) $\vec{\nabla} \cdot \vec{F}$ at $(x, y, z) = (1, 2, 3)$

MATH 294 FALL 1988 PRELIM 3 # 1

6.1.12 Curve C is the line of intersection of the paraboloid$z = x^2 + y^2$ and the plane $z = x + \frac{3}{4}$. The positive direction on C is the counterclockwise direction direction viewed from above, i.e. from a point (x, y, z) with $z > 0$. Calculate the length of the curve C.

MATH 293 SUMMER 1990 PRELIM 1 # 1

6.1.13 A parallelogram $ABCD$ has vertices at $A(2, -1, 4), B(1, 0, -1), C(1, 2, 3)$ and D.

a) Find the coordinates of D.

b) Find the cosine of the interior angle at B.

c) Find the vector projection of $\overrightarrow{BA}$ onto $\overrightarrow{BC}$

d) Find the area of $ABCD$.

e) Find an equation for the plane in which $ABCD$ lies.

MATH 294 SUMMER 1990 PRELIM 1 # 1

6.1.14 Given the function $f(x, y) = e^{-x^2} + y - e^y$:

a) Compute the directional derivative at $(1, -1)$ in the direction of the origin;

b) Find all relative extreme points and classify them as maximum, minimum, or saddle points;

c) Give the linearization of f about $(1, -1)$

MATH 293 SUMMER 1990 PRELIM 1 # 5 3

6.1.15 The position vector of a particle

$$\vec{R}(t) \text{ is given by } \vec{R}(t) = t\cos t\vec{i} + t\sin t\vec{j} + \left(\frac{2\sqrt{2}}{3}\right) t^{\frac{3}{2}}\vec{k}$$

a) Find the velocity and acceleration of the particle at $t = \pi$

b) Find the total distance travelled by the particle in space from $t = 0$ to $t = \pi$.

MATH 293 SUMMER 1990 PRELIM 1 # 4

6.1.16 Find $\vec{T}, \vec{N}, \vec{B}$ and κ at $t = 0$ for the space curve defined by

$$\vec{R}(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + t\vec{k}$$

MATH 294 SPRING 1990 FINAL # 10

6.1.17 Find t e shortest distance from the plane $3x + y - z = 5$ to the point $(1, 1, 1)$.

MATH 293 FALL 1990 PRELIM 1 # 1

6.1.18 **a**) Find the equation of the plane P which contains the point $R = (2, 1, -1)$ and is perpendicular to the straight line $L : x = -1 + 2t, y = 5 - 4t, z = t$.

b) Find the point of intersection of the lint L and the plane P.

c) Use b) to find the distance of the point R from the line L.

MATH 294 UNKNOWN 1990 UNKNOWN # ?

6.1.19 **a**) Determine the rate of change of the function $f(x, y, z) = e^x \cos yz$ in the direction of the vector $A = 2\vec{i} + \vec{j} - 2\vec{k}$ at the point $(0, 1, 0)$.

b) Determine the equation of the plane tangent to the surface $e^x \cos yz = 1$ at the point $(0, 1, 0)$.

MATH 293 FALL 1990 PRELIM 1 # 2

6.1.20 **a**) Find a <u>unit</u> vector which lies in the plane of $\vec{a}$ and $\vec{b}$ and is orthogonal to $\vec{c}$ if

$$\vec{a} = 2\vec{i} - \vec{j} + \vec{k}, \vec{b} = \vec{i} + 2\vec{j} - \vec{k}, \vec{c} = \vec{i} + \vec{j} - 2\vec{k}$$

b) Find the vector projection of $\vec{b}$ onto $\vec{a}$.

MATH 293 FALL 1990 PRELIM 1 # 3

6.1.21 Show that the following are true

a) $\left(\vec{a} \cdot \vec{i}\right)^2 + \left(\vec{a} \cdot \vec{j}\right)^2 + \left(\vec{a} \cdot \vec{k}\right)^2 = |\vec{a}|^2$

b) $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$
(hint: use the angle between them)

c) $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ is orthogonal to $|\vec{a}|\vec{b} + |\vec{b}|\vec{a}$

MATH 293 FALL 1990 PRELIM 1 # 4

6.1.22 **a**) Find $\vec{v}$ and $\vec{a}$ for the motion

$$\vec{R}(t) = t\vec{i} + t^3\vec{j}$$

b) Sketch the curve including $\vec{v}, \vec{a}$.

c) Find the <u>speed</u> at $t = 2$.

MATH 293 FALL 1990 PRELIM 1 # 5

6.1.23 Let $\vec{R}(t) = (\cos 2t)\vec{i} + (\sin 2t)\vec{j} + t\vec{k}$.

a) Find the length of the curve from $t = 0$ to $t = 1$.

b) Find the unit tangent $\vec{T}$, the principal unit normal $\vec{N}$ and the curvature κ at $t = 1$.

MATH 294 FALL 1990 FINAL # 1

6.1.24 Given the surface $z = x^2 + 2y^2$. At the point $(1, 1)$ in the x-y plane:

a) determine the direction of greatest increase of z

b) determine a unit normal to the surface.

Given the vector field $\vec{F} = 6xy^z\vec{i} - 2y^3z\vec{j} + 4z\vec{k}$,

c) calculate its divergence

d) use the divergence theorem to calculate the outward flux of the vector field over the surface of a sphere of unit radius centered at the origin.

MATH 293 SPRING 1992 PRELIM 1 # 1

6.1.25 Given the vectors

$$\vec{A} = \vec{i} + \vec{j} + \vec{k}$$

$$\vec{B} = \vec{i} + 2\vec{j} + 3\vec{k}$$

$$\vec{C} = \vec{i} - 2\vec{j} + \vec{k}$$

where $\vec{i}, \vec{j}$ and $\vec{k}$ are mutually perpendicular unit vectors.

Evaluate

a) $\vec{A} \cdot \vec{B}$

b) $\vec{A} \times \vec{B}$

c) $\left(\vec{A} \times \vec{B}\right) \cdot \vec{C}$

d) $\left(\vec{A} \times \vec{B}\right) \times \vec{C}$

MATH 293 SPRING 1992 PRELIM 1 # 4

6.1.26 Consider the plane $x + 2y + 3z = 17$ and the line through the points $P : (0, 3, 4)$ and $Q : (0, 6, 2)$.

a) Is the line parallel to the plane? Five clear reasons for your answer.

b) Find the point of intersection, if any, of the line and the plane.

MATH 293 SPRING 1992 PRELIM 1 # 2

6.1.27 The acceleration of a point moving on a curve in space is given by $\vec{a} = -\vec{i}b\cos t - \vec{j}c\sin t + 2d\vec{k}$ where $\vec{i}, \vec{j}$, and $\vec{k}$ are mutually perpendicular unit vectors and b,c and d are scalars. Also, the position vector $\vec{R}(t)$ and velocity vector $\vec{v}(t)$ have the initial values

$$\vec{R}(0) = \vec{i}(b+1), \vec{v}(0) = \vec{j}c$$

Find $\vec{R}(t)$ and $\vec{v}(t)$

MATH 293 SPRING 1992 PRELIM 1 # 5

6.1.28 Consider the curve

$$\vec{R}(t) = 3\vec{i} + \vec{j}\cos t + \vec{k}\sin t, 0 \leq t \leq 2\pi$$

where $\vec{i}, \vec{j}$ and $\vec{k}$ are mutually perpendicular unit vectors.

a) Sketch and describe the curve in words.

b) Determine the unit tangent, principal normal and binormal vectors ($\vec{T}, \vec{N}$ and $\vec{B}$) to the curve at the point $t = \frac{\pi}{2}$

c) Sketch the vectors $\vec{T}, \vec{N}$ and $\vec{B}$ at $t = \frac{\pi}{2}$.

MATH 293 SPRING 1992 PRELIM 1 # 6

6.1.29 The position vector of a point moving along a curve is

$$\vec{R}(t) = t\vec{i} + e^{2t}\vec{j}$$

where $\vec{i}$ and $\vec{j}$ are mutually perpendicular unit vectors and t is time. The acceleration vector $\vec{a}$ at the time $t = 0$ can be written as

$$\vec{a}(0) = c\vec{T} + d\vec{N}$$

where $\vec{T}$ and $\vec{N}$ are the unit tangent and principal normal vectors to the curve at the time $t = 0$.

Find the scalars c and d

MATH 293 SPRING 1992 FINAL # 1

6.1.30 A point is moving on a spiral given by the equation

$$\vec{R}(t) = e^t\cos t\vec{i} + e^t\sin t\vec{j}$$

where $\vec{i}$ and $\vec{j}$ are the usual mutually perpendicular unit vectors. Find

a) The speed (the magnitude of the velocity) of the point at $t = 0$.

b) The curvature of the spiral at $t = 0$.

MATH 293 SUMMER 1992 PRELIM 6/30 # 1

6.1.31 Find the equation of the plane which passes through the points

$$A(0,0,0), B(-1,1,0) \text{ and } C(-1,1,1).$$

MATH 293 SUMMER 1992 PRELIM 6/30 # 5

6.1.32 A point P, starting at the origin $(0,0,0)$ is moving along a smooth curve. At any time, the distance s travelled by the point from the origin, is observed to be

$$s = 2t$$

Also, the unit tangent vector to the curve, as this point, is

$$\vec{T} = -\frac{\sin t}{2}\vec{i} + \frac{\cos t}{2}\vec{j} + \frac{\sqrt{3}}{2}\vec{k}$$

a) Find the acceleration $\vec{a}$ of P as a function of time.

b) Find the position vector $\vec{R}(t)$ of P.

MATH 293 SUMMER 1992 FINAL # 6

6.1.33 A point P is moving on a curve defined as

$$x(t) = \cos \alpha t$$
$$y(t) = 2t$$
$$z(t) = 3\cos t + 6t + 3(\alpha - 1)t^2$$

Find value(s) of α such that the curve defined above lies in a plane for all $0 \le t \le \infty$.
Hint: The idea of torsion of a curve should be useful here!

MATH 293 FALL 1992 PRELIM 1 # 3

6.1.34 Let $P_1(-1,0,-1), P_2(1,1,-1)$ and $P_3(1,-1,1)$ be three points and let $\vec{A} = \overrightarrow{P_1P_2} = 2\vec{i} + \vec{j}$ and $\vec{B} = \overrightarrow{P_1P_3} = 2\vec{i} - \vec{j} + 2\vec{k}$.

a) Find a vector perpendicular to the plane containing $\vec{A}$ and $\vec{B}$.

b) Find the area of the parallelogram whose edges are $\vec{A}$ and $\vec{B}$.

c) Find the equation of the plane passing through the points P_1, P_2 and P_3.

MATH 293 FALL 1992 PRELIM 1 # 4

6.1.35 Let $P_1(-1,0,-1), P_2(1,1,-1)$ and $P_3(1,-1,1)$ be three points and let $\vec{A} = \overrightarrow{P_1P_2} = 2\vec{i} + \vec{j}$ and $\vec{B} = \overrightarrow{P_1P_3} = 2\vec{i} - \vec{j} + 2\vec{k}$.

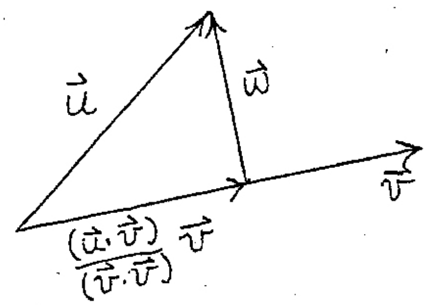
a) Find the distance from the point $(1,1,1)$ to the plane passing through the points P_1, P_2 and P_3.

b) Find the equation of the line passing through the point P_3 and parallel to the line passing through P_1 and P_2.

c) Find the vector projection of $\vec{A}$ in the direction of $\vec{B}$ and the scalar component of $\vec{A}$ in the direction of $\vec{B}$.

MATH 293 FALL 1992 PRELIM 1 # 5

6.1.36 Let $\vec{u}$ and $\vec{v}$ be two given vectors. The vector projection of $\vec{u}$ in the direction of $\vec{v}$ is $\frac{(\vec{u}\cdot\vec{v})}{(\vec{v}\cdot\vec{v})}\vec{v}$. Consider the vector $\vec{w} = \vec{u} - \frac{(\vec{u}\cdot\vec{v})}{(\vec{v}\cdot\vec{v})}\vec{v}$. By taking the scalar product of $\vec{w}$ with $\vec{v}$ show that $\vec{w}$ is perpendicular (orthogonal) to $\vec{v}$.



MATH 293 FALL 1992 PRELIM 2 # 2

6.1.37 Find all points (x, y, z) which lie on the intersection of the planes

$$x + y + z = 6, -x + 2z = 1, y + 3z = 7$$

Is this set of points a single point, a line or a plane?

MATH 293 FALL 1992 PRELIM 2 # 3

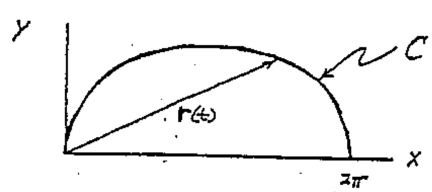
6.1.38 A point move on a space curve with the position vector

$$\vec{v}(t) = e^t \cos t\vec{i} + e^t \sin t\vec{j} + 2\vec{k}$$

Find the velocity $\vec{v}$, speed, unit tangent vector $\vec{T}$, unit principal normal $\vec{N}$, acceleration $\vec{a}$ and curvature κ as functions of time. Also check that $\vec{N}$ is perpendicular to $\vec{T}$.

MATH 294 FALL 1992 PRELIM 2 # 3?

6.1.39 Determine the arc-length, $\int_C ds$ of the curve C (a cycloid) given by: $r(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, 0 \leq t \leq 2\pi$ (see figure below).



MATH 293 FALL 1993 PRELIM 1 # 4

6.1.40 The four corners of a parallelopiped are given as $(1, 1, 1), (1, 4, 2), (4, 2, 3)$ and $(1, 1, 4)$ in xyz-space. Using $(1, 1, 4)$ as the common point of three vectors lying along the parallelopiped's edges, calculate the volume of the parallelopiped.

MATH 293 FALL 1992 FINAL # 2

6.1.41 A particle is moving along the positive branch of the curve $y = 1 + x^2$ and its x coordinates is controlled as a function of time according to $x(t) = 2t$. Find

a) The tangential component of the particle's acceleration, a_T, at time $t = 0$.

b) The normal component of the particle's acceleration, A_N, at time $t = 0$.

c) The radius of curvature ρ of the curve, along which the particle is moving, at the point $(0, 1)$. Hint: $|\tilde{a}|^2 = a_N^2 + a_T^2, a_T = \frac{d\tilde{v}}{dt}, a_N = \frac{|\tilde{v}|^2}{\rho}$.

MATH 293 FALL 1993 PRELIM 1 # 6

6.1.42 Find the equation of the plane that contains the intersecting lines L_1 and L_2 given by:

$$L_1 : \left| \begin{array}{l} x = 1 + t \\ y = 2 + t \\ z = 1 + t \end{array} \right. \quad L_2 : \left| \begin{array}{l} x = 1 - t \\ y = 2 - t \\ z = 1 \end{array} \right.$$

Sketch the plane.

MATH 293 FALL 1993 PRELIM 1 # 5

6.1.43 A line contains the two points $(1, 2, 3)$ and $(-2, 1, 4)$. Find parametric equations of the line and calculate the distance from the line to the point $(5, 5, 5)$.

MATH 293 FALL 1993 PRELIM 1 # 3

6.1.44 Calculate the volume of the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

by imaging it to be comprised of a set of thin elliptical disks, of thickness dz, oriented parallel to the x-y plane.

MATH 293 SPRING 1993 PRELIM 1 # 2

6.1.45 **a**) Solve the initial value problem

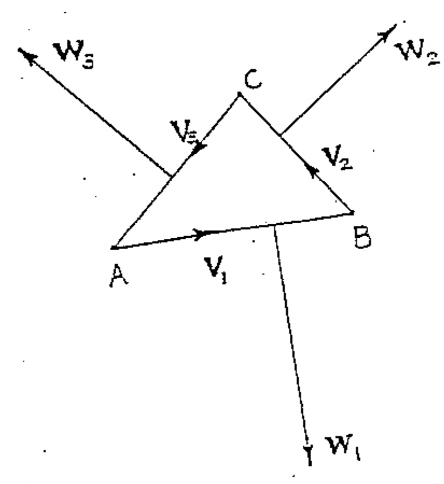
$$\frac{dy}{dx} + \frac{y}{x} = x^3$$

if $y = 0$ when $x = 1$

b) Consider a triangle ABC with three vectors defined as

$$\vec{v}_1 = \overrightarrow{AB}, \vec{v}_2 = \overrightarrow{BC}, \vec{v}_3 = \overrightarrow{CA}$$

From three points, one on each side of the triangle, draw vectors $\vec{w}_1, \vec{w}_2,$ and $\vec{w}_3$ in plane of the triangle. Each of these vectors is perpendicular to its side (i.e. $\vec{w}_1$ is perpendicular to $\overrightarrow{AB}$ and so on) with length equal to the length of the side and pointing out of the triangle.



i) Find $\vec{w}_1, \vec{w}_2,$ and $\vec{w}_3$ in terms of the components of $\vec{v}_1, \vec{v}_2,$ and $\vec{v}_3$.

c) Show that $\vec{w}_1 + \vec{w}_2 + \vec{w}_3 = 0$

MATH 294 FALL 1993 PRELIM 1 # 2

6.1.46 C is the curve given by

$$\vec{r}(t) = e^{-t}\cos t\vec{i} + e^{-t}\sin t\vec{j} + \sqrt{1 - e^{-2t}}\vec{k},\ (0 \leq t < \infty).$$

Show that C lies on the sphere $x^2 + y^2 + z^2 = 1$ and describe the curve with words and a sketch.
You may use the fact that $\cos t\vec{i} + \sin t\vec{j}$ $(0 \leq t \leq 2\pi)$ is a parametrization of the unit circle.

MATH 293 SPRING 1993 PRELIM 1 # 3

6.1.47 Find the equation of the plane that contains the intersecting lines L_1 and L_2 where:

$$L_1 : \left| \begin{array}{l} x = 1 + t \\ y = 1 + t \\ z = 1 + t \end{array} \right. \quad L_2 : \left| \begin{array}{l} x = 1 - t \\ y = 1 - t \\ z = 1 \end{array} \right.$$

MATH 293 SPRING 1993 PRELIM 1 # 4

6.1.48 Find the equation of the plane through the points $(2, 2, 1)$ and $(-1, 1, -1)$ that is perpendicular to the plane $2x - 3y + z = 3$.

MATH 293 SPRING 1993 PRELIM 1 # 5

6.1.49 Consider a point (x, y). Let d_1 be the distance from (x, y) to the line $x + y = 0$ and d_2 be the distance from (x, y) to the line $x - y = 0$.
Given $d_1 d_2 = 1$, find the locus of all such points, i.e., say what the curve is and find its equation.

MATH 293 SPRING 1993 PRELIM 2 # 1

6.1.50 A point P is moving along a plane curve. The unit tangent and principal normal vectors of this curve are, (for $t \geq 0$),

$$\vec{T}(t) = -\vec{i}\sin(t) + \vec{j}\cos(t)$$

$$\vec{N}(t) = -\vec{i}\cos(t) - \vec{j}\sin(t)$$

(where $\vec{i}$ and $\vec{j}$ are the usual mutually perpendicular unit vectors), and the tangential component of the velocity vector of P, (the speed), is

$$\vec{v}_T = t.$$

a) Find the velocity vector $\vec{v}(t)$ of P.
b) Find the acceleration vector $\vec{a}(t)$ of P.
c) Find the tangential ($\vec{a}_T$) and normal ($\vec{a}_n$) components of the acceleration vector.
d) Find the radius of curvature $\rho(t)$ of the curve.

MATH 293 SPRING 1994 PRELIM 1 # 1

6.1.51 Find the distance from the point $(2, 1, 3)$ to the plane which contains the points $(2, 1, 0), (0, 1, 1), (0, 0, 2)$.

MATH 293 SPRING 1994 PRELIM 1 # 2

6.1.52 Find the point on the segment from $P_1 = (1, 0, -1)$ to $P_2 = (4, 3, 2)$ which is twice as far from P_2 as it is from P_1.

MATH 293 SPRING 1994 PRELIM 1 # 3

6.1.53 A particle moves on the sphere of radius a centered at the origin. Its position vector $\vec{r}(t)$ is a differentiable function of the time, t. Show that the velocity vector $\vec{v}(t)$ of the particle is always perpendicular to its position vector, $\vec{r}(t)$.

MATH 293 SPRING 1994 PRELIM 1 # 4

6.1.54 A parallelogram, P, is determine by the two vectors $\vec{i}+\vec{j}+\vec{k}$ and $2\vec{i}-\vec{j}-\vec{k}$.

a) What is the area of P?
b) What is the area of the orthogonal projection of P in the xy-plane?
c) What is the area of the orthogonal projection of P in the xz-plane?
d) What is the area of the orthogonal projection of P in the yz-plane?
e) What is the area of the orthogonal projection of P in the plane $x+y-z=0$?

MATH 293 SPRING 1994 PRELIM 2 # 1

6.1.55 A point P is moving along the spiral

$$x = e^t \cos(t)$$

$$y = e^t \sin(t).$$

a) Find the curvature of the given spiral at $t=0$.
b) The acceleration of P is written as

$$\vec{a} = a_T\vec{T} + a_N\vec{N}.$$

Find a_T and a_N at $t=0$.

MATH 293 FALL 1994 PRELIM 1 # 1

6.1.56 Let

$$\vec{A} = 2\vec{i}-\vec{j}+\vec{k}$$

$$\vec{B} = \vec{i}+\vec{j}+\vec{k}$$

$$\vec{C} = \vec{i}+2\vec{j}+\vec{k}$$

a) Find the vector projection of A onto the direction of $\vec{B}$.
b) Show that $\vec{A} - proj_{\vec{B}}\vec{A}$ is perpendicular to $\vec{B}$.
c) Find the area of the parallelogram with edges $\vec{A}$ and $\vec{B}$.
d) Find the volume of the box with edges $\vec{A}, \vec{B}$ and $\vec{C}$.
e) Find the parametric equation of the line through $(0,0,0)$ and parallel to the intersection of the planes with normals $\vec{A}$ and $\vec{B}$.

MATH 293 FALL 1994 PRELIM 1 # 2

6.1.57 Let $\vec{a}$ and $\vec{b}$ be vectors. Show that

a) $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$, and
b) that $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$ is perpendicular to $|\vec{a}|\vec{b} - |\vec{b}|\vec{a}$

MATH 293 FALL 1994 PRELIM 1 # 3

6.1.58 Graph $z = x^2 + y^2 + 1$ and label any intersection the surface may have with any axis. Describe the curves that are the intersections of the surface with the planes $z =$ constant ($z > 1$).

MATH 293 FALL 1994 PRELIM 2 # 1

6.1.59 Consider the path traversed by a particle given parametrically by $\vec{r}(t) = (e^t \cos t)\vec{i} + (e^t \sin t)\vec{j} + e^t\vec{k}$. Find the

a) velocity vector
b) speed
c) acceleration vector
d) length of the path from $t = 0$ to $t = \ln 4$

MATH 293 FALL 1994 FINAL # 1

6.1.60 The level curves of the function $f(x, y, z) = z + x^2 + y^2 + 1$ are:

a) Hyperboloids
b) Planes
c) Cones
d) Paraboloids
e) Spheres

MATH 293 FALL 1994 FINAL # 3

6.1.61 The vector projection of $(1, 0, 1, 0)$ in the direction of $(1, 1, 1, 1)$ is:

a) $\left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$,
b) $(0, 1, 0, 1)$,
c) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$,
d) $(0, -1, 0, -1)$
e) $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$

MATH 293 FALL 1994 FINAL # 5

6.1.62 Any non-zero vector perpendicular to the vectors $\vec{i} + \vec{j} + \vec{k}$ and $\vec{i} + 2\vec{k}$ is

a) Perpendicular to $2\vec{i} + \vec{j} + \vec{k}$,
b) Parallel to $\vec{i} + \vec{j} + \vec{k}$,
c) Perpendicular to $2\vec{i} - \vec{j} - \vec{k}$,
d) Parallel to $2\vec{i} + \vec{j} + \vec{k}$,
e) Parallel to $2\vec{i} - \vec{j} - \vec{k}$

MATH 293 SPRING 1995 PRELIM 1 # 2

6.1.63 Consider the planar curve

$$y^2 = 4x.$$

Find parametric equations of the following lines.

a) Tangent to the above curve at $P(1, 2)$.
b) Normal to the above curve at $O(0, 0)$.

MATH 293 SPRING 1995 PRELIM 1 # 3

6.1.64 **a**) Show that the points $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$ are the vertices of a parallelogram.

b) What is the area of this parallelogram?

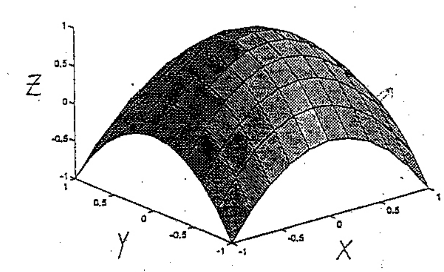
MATH 294 SPRING 1995 PRELIM 1 # 5

6.1.65 The surface S drawn below can be described in two ways, i.e.

as $z = f(x, y) = 1 - x^2 - y^2, -1 \le x \le 1, -1 \le y \le 1$

or $g(x, y, z) = z + x^2 + y^2 = 1, -1 \le x \le 1, -1 \le y \le 1$

Evaluate and sketch the gradient fields $\vec{\nabla} f$ and $\vec{\nabla} g$. Explain the relationship between these two vector fields.



MATH 293 SPRING 1995 PRELIM 2 # 2

6.1.66 Let

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

be a space curve; let $\vec{v}(t)$ be the velocity vector and $\vec{a}(t)$ the acceleration vector.

a) Give the formula which gives the curvature of the curve in terms of $\vec{v}$ and $\vec{a}$.

b) By differenting $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$, find a formula for $\frac{d\vec{v}}{dt}$ in terms of $\vec{v}$ and $\vec{a}$

c) If at some instant we have

$$|\vec{v}| = 3\ m/s, \frac{d|\vec{v}|}{dt} = 4\ m/s^2, |\vec{a}| = 5\ m/s^2$$

what is the radius of curvature in meters.

MATH 293 FALL 1995 PRELIM 1 # 1 *

6.1.67 This is a two-dimentional problem. Consider the parabola

$$y^2 = 4x \text{ and the point } P(1,2) \text{ on it.}$$

a) Find an unit vector $\vec{t}$ that is tangential to the parabola at P.
b) Find the equation of the tangent line to the parabola at P. Any correct form of the equation is acceptable.
c) Find an unit vector $\vec{n}$ that is normal to the parabola at P.
d) Find the equation of the normal line to the parabola at P. Any correct form of the equation is acceptable.

MATH 294 FALL 1995 PRELIM 1 # 1 *

6.1.68 **a**) For $f = x^2 + 8y^2$, show that $(4, 2)$ lies on the level curve $f(x, y) = 48$. Sketch this level curve.
b) Find the vector field $\vec{\nabla} f$
c) Evaluate $\vec{\nabla} f$ at $(x, y) = (\sqrt{48}, 0), (4, 2), (4, -2), (0, \sqrt{6})$ and sketch these vectors, showing very clearly their relation to the level curve.

MATH 293 FALL 1995 PRELIM 1 # 2

6.1.69 Consider two straight lines in space given by the equations:

$$L_1 : \left| \begin{array}{l} x = 2 + t \\ y = 2 + t \\ z = -t \end{array} \right. \quad -\infty \leq t \leq \infty$$

$$L_2 : \left| \begin{array}{l} x = 3 + u \\ y = -2u \\ z = 1 + u \end{array} \right. \quad -\infty \leq u \leq \infty$$

a) Do these lines intersect? If so, find the coordinates of the point of intersection.
b) Find a vector $\vec{u}$ along L_1 and a vector $\vec{v}$ along L_2.
c) Find, if possible, the equation of the plane that contains the lines L_1 and L_2.

MATH 293 FALL 1995 PRELIM 1 # 4a *

6.1.70 Describe the set of points defined by the equations

$$\begin{array}{rcl} x^2 + y^2 + z^2 & \leq & 4 \\ z & \leq & 1 \end{array}$$

Also, draw a sketch showing this set of points.

MATH 293 FALL 1995 FINAL # 5 *

6.1.71 A point P is moving on a plane curve with the position vector

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}, t \geq 0$$

where t is time and $\vec{i}$ and $\vec{j}$ are the usual orthogonal Cartesian unit vectors. The position components $x(t)$ and $y(t)$ satisfy the equations

$$t\frac{dx}{dt} + x = t^2, x(0) = 0$$

$$\text{and } \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 0, y(0) = 0, \frac{dy}{dt}(0) = 1$$

a) Find $x(t)$ as an explicit function of time.
b) Find $y(t)$ as an explicit function of time.
c) Find $\vec{v}(t) = \frac{d\vec{r}}{dt}$, the velocity of P as a function of time.

MATH 293 SPRING 1996 FINAL # 17 *

6.1.72 A bug flies around the room so that at time t, the position of the bug is given by $x = t^2, y = t^{\frac{3}{2}}, z = t^2$. The velocity at time $t = 1$ is
a) 10.25
b) $2\vec{i} + \frac{3}{2}\vec{j} + 2\vec{k}$
c) $\vec{i} + \vec{j} + \vec{k}$
d) 3
e) none of the above

MATH 293 SPRING 1996 FINAL # 18 *

6.1.73 The speed of the bug above at time $t = 1$ is
a) 40
b) 19
c) 3
d) $\vec{i} + 3\vec{j} + 8\vec{k}$
e) none of the above

MATH 293 SPRING 1996 FINAL # 19 *

6.1.74 The position of the bug above at time $t = 1$ is
a) $\sqrt{3}$
b) 40
c) 19
d) 3
e) none of the above

MATH 293 SPRING 1996 FINAL # 25 *

6.1.75 A cannon fires a cannonball at an angle of 45 degrees from horizontal. The cannonball lands 1000 meters away. Taking Newton's gravitational constant g to be 10 meters per second squared, the speed of the cannonball when leaving the cannon in meters per second is

a) 10
b) $10\sqrt{10}$
c) 100
d) $\frac{2000}{\sqrt{2}}$
e) none of the above

MATH 293 SPRING 1996 FINAL # 20 *

6.1.76 The projection of the vector (1,0,1,0) in the direction of (1,1,1,1) is

a) $\left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$
b) (0,1,0,1)
c) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$
d) (0,-1,0,1)
e) none of the above

MATH 293 SPRING 1996 FINAL # 24 *

6.1.77 Let $P = (1,1,1), Q = (1,0,0), R = (0,1,0)$. Then the equation of the plane in $\Re^3$ containing the triangle PQR is

a) $x + y - z = -1$
b) $x - y + z = 1$
c) $-x + y - z = -1$
d) $x + y - z = 1$
e) none of the above

MATH 293 SPRING 1996 FINAL # 30 *

6.1.78 If $\vec{u}, \vec{v}, \vec{w}$ are vectors in $\Re^3$, then $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{w} \times \vec{v}) \cdot \vec{u}$ (T/F)

MATH 294 SPRING 1996 FINAL # 1 MAKE-UP

6.1.79 In this problem $f(x, y) = x - y^2$.

a) Sketch the level curve $f(x, y) = -7$
b) Evaluate $\vec{\nabla} f(2,3)$ and sketch it on the graph, showing the relation to the level curve.
c) Find the to the right flux of $\vec{\nabla} f$ across the segment $0 \leq y \leq 5, x = 0$

MATH 293 FALL 1997 PRELIM 3 # 4

6.1.80 Evaluate the line integral

$$\int_C \frac{zydx + zxdy + (z - xy)dz}{z^2}$$

where C is the curve given by parametric equations $x(t) = \cos(\pi t), y(t) = \sin(\pi t), z(t) = t, (1 \leq t \leq 2)$.

MATH 293 SUMMER 1992 PRELIM 6/30 # 3

6.1.81 A point is moving along a curve given by the parametric equations

$$x(t) = t$$

$$y(t) = 2t^2$$

Find, as functions of time t

a) The velocity of the point, $\vec{v}$

b) The acceleration of the point, $\vec{a}$

c) The curvature κ of the curve

d) If $\vec{a} = a_N\vec{N} + a_T\vec{T}$, where $\vec{N}$ is the principal unit normal and $\vec{T}$ is the unit tangent vector to the curve at some point on it, find a_N and a_T.

MATH 293 SPRING 1995 PRELIM 2 # 1

6.1.82 Consider the spiral parametrized by

$$t \mapsto \begin{bmatrix} e^{-t}\cos t \\ e^{-t}\sin t \end{bmatrix} \quad 0 \leq t < \infty.$$

a) Sketch the curve.

b) Find its length or show that it has infinite length.

MATH 293 FALL 1994 PRELIM 2 # 1

6.1.83 Find the arc length parametrization of the space curve:

$$\vec{r}(t) = \cos(2t)\vec{i} + \sin(2t)\vec{j} + \frac{2}{3}t^{\frac{3}{2}}\vec{k}, \text{ with } 0 \leq t \leq 5.$$

MATH 294 SUMMER 1995 PRELIM (1) # 1

6.1.84 The surface

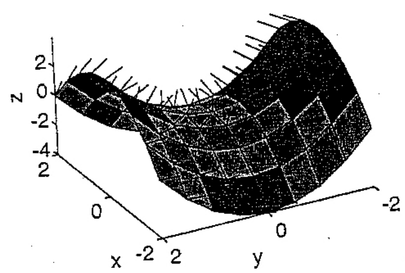
$$z = f(x, y) = y^2 - x^2; -2 \leq x \leq 2, -2 \leq y \leq 2$$

is shown below along with its normal vectors.

a) Sketch the contour lines of the surface in the (x, y) plane, i.e. draw the curves such that $z =$ constant, for example $z = -2, -1, 0, +1, +2$.

b) On your sketch for part (a) sketch the vector field $\vec{\nabla} f$.

c) Find an expression for the unit normal vectors $\vec{n}$ of the surface.

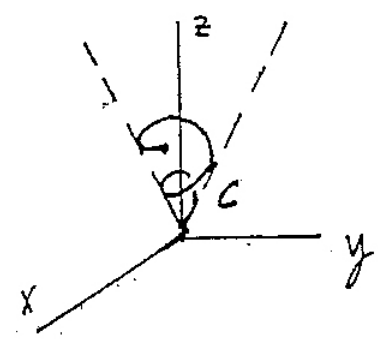


MATH 294 FALL 1992 FINAL # 2

6.1.85 Consider the curve $C : \vec{r}(t) = t\cos t\vec{i} + t\sin t\vec{j} + t\vec{k}, 0 \leq t \leq 4\pi$, which corresponds to the conical spiral shown below.

a) Set up, but so not evaluate, the integral yielding the arc-length of C.

b) Compute $\int_C (y + z)dx + (z + x)dy + (x + y)dz$.



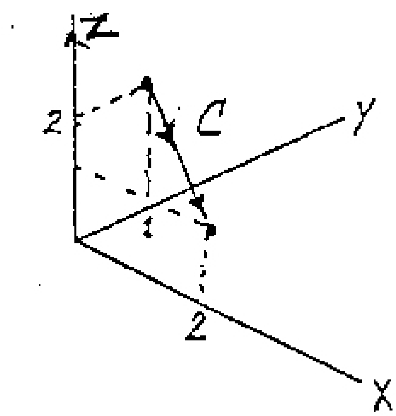
MATH 293 FALL 1994 FINAL # 2

6.1.86 A bug flies around the room along a path parametrized by $x = t^2, y = t^{\frac{3}{2}}, z = t^2$. If the temperature at any point (x,y,z) is given by $T(x, y, z) = x^2y + z^2$, the rate at which the bug feels the temperature change when $t = 1$ is

a) 3
b) -3
c) $\frac{19}{2}$
d) 0
e) $\frac{15}{2}$

MATH 294 FALL 1994 PRELIM 1 # 1 *

6.1.87 C is the line segment from (0,1,2) to (2,0,1).



a) which of the following is a parametrization of C?
i) $x = 2t, y = 1 - t, z = 2 - t, 0 \leq t \leq 1$
ii) $x = 2 - 2t, y = -2t, z = 1 - 2t, 0 \leq t \leq \frac{1}{2}$
iii) $x = 2\cos t, y = \sin t, z = 1 + \sin t, 0 \leq t \leq \frac{\pi}{2}$
b) evaluate $\int_C 3z\vec{j} \cdot d\vec{r}$

MATH 294 SPRING 1996 PRELIM 1 # 1 *

6.1.88 **a**) Evaluate $\int_{(0,0,0)}^{(4,0,2)} 2xz^3dx + 3x^2z^2dz$ on any path.
b) Write parametric equations for the line segment from (1,0,3) to (2,5,0).

MATH 293 SPRING 1992 PRELIM 1 # 3

6.1.89 Given a plane $x - 5y + z = 21$ and a point R with coordinates (1,2,3), find
a) The parametric equations of a line perpendicular to the plane and passing through R.
b) The point of intersection of the line and the plane.
c) The distance from R to the plane.

MATH 293 FALL 1997 PRELIM 3 # 3

6.1.90 Consider the sphere $x^2 + y^2 + z^2 = 25$.

a) Express the equation of the sphere in cylindrical coordinates (r, θ, z) and find volume inside it by evaluating a triple integral in cylindrical coordinates.

b) Now consider the region that you get by starting with the solid interior of the sphere as before, and removing the points which are contained inside the cone $z = \sqrt{x^2 + y^2}$. This means that our new region consists of points having $x^2 + y^2 + z^2 \leq 25, z \leq \sqrt{x^2 + y^2}$. Find the volume of this region by evaluating a triple integral spherical coordinates (ρ, ϕ, θ).

MATH 293 SUMMER 1992 PRELIM 6/30 # 6

6.1.91 A consider is described by the parametric equations

$$x = 2\cos t$$

$$y = 2\sin t$$

A point P inside the circle has coordinates (1,1). The line, normal to the circle, through P, intersects the circle at two points Q_1 and Q_2.Q_1 is the point nearer to P.

a) Find the vector $\vec{N}$ along the line PQ_1

b) Find the parametric equations of the line segment PQ_1.

c) Find the distance PQ_1.

MATH 293 SUMMER 1990 PRELIM 1 # 2

6.1.92 **a**) Find the distance between the point $P = (0, 0, 0)$ and the line L defined parametrically by

$$X = t + 1$$

$$Y = t + 1$$

$$Z = t$$

b) Find an equation of the line through P that is perpendicular to the line L.