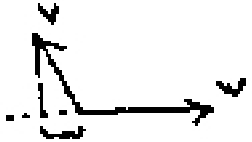
# Orthogonal Projection

Section 2.10

M293 PI FA95 #3

1)

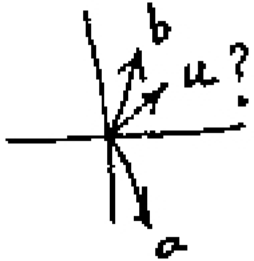
(a)  or 

scalar projection $= \dfrac{\vec{v}\cdot\vec{w}}{|\vec{w}|} = \dfrac{1\cdot5+1\cdot12}{\sqrt{5^2+12^2}} = \boxed{\dfrac{17}{13}}$ , so the 1st picture

(b) $a = 3\vec{i} - 4\vec{j}$

$b = 3\vec{i} + 4\vec{j}$

$u = u_1\vec{i} + u_2\vec{j}$




$\text{proj}_{\vec{a}}\,\vec{u} = \dfrac{\vec{a}\cdot\vec{u}}{|\vec{a}|} = \dfrac{3u_1 - 4u_2}{\sqrt{3^2+4^2}} = \frac{3}{5}u_1 - \frac{4}{5}u_2$

$= -\frac{1}{5}$

$\text{proj}_{\vec{b}}\,\vec{u} = \frac{3}{5}u_1 + \frac{4}{5}u_2 = \frac{7}{5}$

adding, $\frac{6}{5}u_1 = \frac{6}{5}$ so $u_1 = 1$

$u_2 = 1$

$\boxed{u = \vec{i} + \vec{j}}$

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7) (25 pt) Consider the following three vectors in $\mathcal{R}^3$:

$$\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.$$

[Note: $\mathbf{u}_1$ and $\mathbf{u}_2$ are orthogonal.].

⇐ *Please put scrap work for problem 1 on the page to the left* ⇐.

⇓ *Put neat work to be graded for problem 1 below.* ⇓

( *If you need the space, clearly mark work to be graded on the scrap page.*)

$$\langle u_1, u_1 \rangle = [1\ 1\ 1]\begin{bmatrix}1\\1\\1\end{bmatrix} = 3$$

$$\langle u_1, u_2 \rangle = [1\ 1\ 1]\begin{bmatrix}1\\-1\\0\end{bmatrix} = 0$$

$$\langle u_2, u_2 \rangle = [1\ -1\ 0]\begin{bmatrix}1\\-1\\0\end{bmatrix} = 2$$

$$\langle y, u_1 \rangle = [1\ 0\ 1]\begin{bmatrix}1\\1\\1\end{bmatrix} = 2$$

$$\langle y, u_2 \rangle = [1\ 0\ 1]\begin{bmatrix}1\\-1\\0\end{bmatrix} = 1$$

a) Find the orthogonal projection of y onto the subspace of $\mathcal{R}^3$ spanned by $\mathbf{u}_1$ and $\mathbf{u}_2$.

$$\text{Proj}_{\text{span}\{u_1, u_2\}}\, y = \frac{\langle y, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 + \frac{\langle y, u_2 \rangle}{\langle u_2, u_2 \rangle} u_2 \quad (\S)$$

$$= \underbrace{\frac{2}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\-1\\0\end{bmatrix}} = \boxed{\frac{1}{6}\begin{bmatrix}7\\1\\4\end{bmatrix}}$$

plug into (§) to get

b) What is the distance between y and span$\{\mathbf{u}_1, \mathbf{u}_2\}$?

$$d = \left\| y - \frac{1}{6}\begin{bmatrix}7\\1\\4\end{bmatrix} \right\| = \frac{1}{6}\left\| \begin{bmatrix}6\\0\\6\end{bmatrix} - \begin{bmatrix}7\\1\\4\end{bmatrix} \right\| = \frac{1}{6}\left\| \begin{bmatrix}-1\\1\\2\end{bmatrix} \right\|$$

$$= \frac{1}{6}(1+1+4)^{\frac{1}{2}} = \frac{\sqrt{6}}{6} = \boxed{\frac{1}{\sqrt{6}}}$$

c) In terms of the standard basis for $\mathcal{R}^3$, find the matrix of the linear transformation that orthogonally projects vectors onto span$\{\mathbf{u}_1, \mathbf{u}_2\}$.

We find the matrix of a linear transformation by

$$A = [\,T(e_1)\ \ T(e_2)\ \ T(e_3)\,]$$

projections of basis vectors of $\mathbb{R}^3$.

Replace $y$ in eq (§) above to get

$$\boxed{A = \frac{1}{6}\begin{bmatrix}5 & -1 & 2\\ -1 & 5 & 2\\ 2 & 2 & 2\end{bmatrix}}$$

Can check this by verifying that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$, $A\begin{bmatrix}1\\0\\1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}7\\1\\4\end{bmatrix}$

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9) (a) $\begin{bmatrix} c_1 - c_2 \\ c_1 \\ 2c_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

call $b_1$, $b_2$ then $\{b_1, b_2\}$ is a basis

Use $u_1 = b_1$, and choose $\alpha$ so that $u_2 = b_2 - \alpha b_1$ is $\perp u_1$: $u_1 \cdot u_2 = b_1 \cdot b_2 - \alpha b_1 \cdot b_1 = -1 - \alpha \cdot 2$

so $\alpha = -1/2$, $u_2 = b_2 + \frac{1}{2} b_1 = \begin{bmatrix} -1/2 \\ 1/2 \\ 2 \end{bmatrix}$

then $\frac{u_1}{\|u_1\|} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$, $\frac{u_2}{\|u_2\|} = \frac{1}{\sqrt{\frac{1}{4} + \frac{1}{4} + 4}} \begin{bmatrix} -1/2 \\ 1/2 \\ 2 \end{bmatrix}$ is an orthonormal basis

(b) $c_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ \sqrt{5} \end{bmatrix} \cdot b_2 = 1/2$, and $1 = \|b_2\|^2 = \frac{1}{4} + \alpha^2$ so $\alpha = \pm \frac{\sqrt{3}}{2}$