294 FA81 PI # 2b

$$(A|I) = \left(\begin{array}{ccc|ccc} 2 & 4 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \underset{R_1 \leftrightarrow R_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 4 & 1 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\underset{\substack{R_2 \to R_2 + (-2)R_1 \\ R_3 \to R_3 + (-2)R_1}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & -2 & 1 \end{array}\right) \underset{R_2 \leftrightarrow R_3}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & -2 & 1 \\ 0 & 2 & -1 & 1 & -2 & 0 \end{array}\right)$$

$$\underset{\substack{R_1 \to R_1 + (-1)R_2 \\ R_3 \to R_3 + (-2)R_2}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 3 & -1 \\ 0 & 1 & -1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{array}\right)$$

$$\underset{\substack{R_1 \to R_1 + (-2)R_3 \\ R_2 \to R_2 + (1)R_3}}{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -1 & 3 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 & -2 \end{array}\right) \sim (I|A^{-1})$$

$$\therefore A^{-1} = \boxed{\begin{pmatrix} -2 & -1 & 3 \\ 1 & 0 & -1 \\ 1 & 2 & -2 \end{pmatrix}}$$

$\boxed{B^{-1} \text{ does not exist}}$ because $\det B = 0$

$$\det A = 2\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} - 4\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \quad \leftarrow \text{choose row 1}$$

$$= 2(1-3) - 4(1-2) + (3-2) = -4 + 4 + 1 = \boxed{1}$$

$\det B = \boxed{0}$ because the 4^{th} column is twice the 2^{nd} column

294 SP82 P1 #1

a) $\boxed{\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}}$

b)

$$\det A = 1 \cdot \begin{vmatrix} 3 & 4 \\ 4 & 6 \end{vmatrix} - 2 \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$$

$$= 2 - 2 \cdot 0 + 3(-1) = \boxed{-1}$$

c) $\boxed{\text{yes}}$ because $\det A \neq 0$

d) $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & -2 \\ 3 & 4 & 6 & 0 \end{array}\right) \underset{\substack{R_2 \to R_2 + (-2)R_1 \\ R_3 \to R_3 + (-3)R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & -4 \\ 0 & -2 & -3 & -3 \end{array}\right)$

$\underset{R_3 \to R_3 + 2R_2}{\sim} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -1 & -2 & -4 \\ 0 & 0 & 1 & 5 \end{array}\right)$

$\therefore x_3 = 5$

$x_2 = \dfrac{-4 + 2x_3}{-1} = 4 - 2 \cdot 5 = -6$

$x_1 = 1 - 3x_3 - 2x_2 = 1 - 3 \cdot 5 - 2(-6) = -2$

$$\boxed{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2 \\ -6 \\ 5 \end{pmatrix}}$$

e) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 7 & -1 & 4 \\ 10 & -1 & 6 \\ 15 & -2 & 9 \end{pmatrix}}$

19. FA91 PIII #4

(a)

$$\det\begin{pmatrix} 2 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \det\begin{pmatrix} 2 & 3 \\ 3 & 5 \end{pmatrix} = \boxed{1}$$

(b)

$$= 2\begin{vmatrix} 5 & 0 & 0 \\ 0 & -11 & -3 \\ 0 & 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & 0 & 0 \\ 0 & -11 & -3 \\ 0 & 4 & 1 \end{vmatrix} = 2\cdot 5\begin{vmatrix} -11 & -3 \\ 4 & 1 \end{vmatrix} - 3\cdot 3\begin{vmatrix} -11 & -3 \\ 4 & 1 \end{vmatrix}$$

$$= (2\cdot 5 - 3\cdot 3)(-11\cdot 1 - (-3)(4)) = (1)(1) = \boxed{1}$$

(c)

$$= \frac{1}{2\cdot 3\cdot(-1)\cdot 4} = \boxed{-\frac{1}{24}}$$

(d) False because if one row of A is multipiplied by k to produce B then $\det B = k(\det A)$

and $|\det B| = |k|\,|\det A| \neq |\det A|$ for $|k| \neq 1$

(e) True

(k) True

(ℓ) False. For example, if $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $\det A = 1$

$$B = \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & 1 \end{pmatrix}, \quad \det B = -\frac{1}{2}$$

$$\det B \neq \frac{1}{\det A}$$

(m) True

(n) False. $\operatorname{Det}(kA_{n\times n}) = k^n \operatorname{Det}(A_{n\times n})$

(o) True.

293 SP92 P III ※1

(a) $= -2 \begin{vmatrix} 1 & 3 \\ 5 & 8 \end{vmatrix} = (-2)(-7) = \boxed{14}$

(choose the 2nd column)

(b) $= \cos^2\theta + \sin^2\theta = \boxed{1}$

(c) $= \boxed{0}$ because 1st column = 2nd column

293 SU92 P (7/21) #2

(a)

$$\begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 2 & 3 & 2 & 3 \\ -3 & -5 & 0 & -1 \end{vmatrix} \underset{\substack{R_3 = R_3 + (-2)R_1 \\ R_4 = R_4 + (3)R_1}}{=} \begin{vmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -1 \\ -1 & 0 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$$= -1 \begin{vmatrix} -1 & -1 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = (-1)(1) + {}^-2 = \boxed{{}^-3}$$

(b)

$$= (3-\lambda) \begin{vmatrix} 1-\lambda & -4 \\ 0 & -1-\lambda \end{vmatrix} + \begin{vmatrix} 2 & 1-\lambda \\ 1 & 0 \end{vmatrix} = (3-\lambda)(1-\lambda)(-1-\lambda) - (1-\lambda)$$

(choose row 1)

$$= (1-\lambda)\left(\lambda^2 - 2\lambda - 3 - 1\right) = \boxed{(1-\lambda)(\lambda^2 - 2\lambda - 4)}$$

293 FA92 P$\boxed{\text{II}}$ ※1

$$\begin{vmatrix} b & a & a & a & a \\ b & b & a & a & a \\ b & b & b & a & a \\ b & b & b & b & a \\ b & b & b & b & b \end{vmatrix} \underset{\substack{R_1 \to R_1 - R_5 \\ R_2 \to R_2 - R_5 \\ R_3 \to R_3 - R_5 \\ R_4 \to R_4 - R_5}}{=} \begin{vmatrix} 0^{+} & a-b & a-b & a-b & a-b \\ 0^{-} & 0 & a-b & a-b & a-b \\ 0^{+} & 0 & 0 & a-b & a-b \\ 0^{-} & 0 & 0 & 0 & a-b \\ b^{+} & b & b & b & b \end{vmatrix}$$

$$= b \begin{vmatrix} a-b & a-b & a-b & a-b \\ 0 & a-b & a-b & a-b \\ 0 & 0 & a-b & a-b \\ 0 & 0 & 0 & a-b \end{vmatrix} = \boxed{b\,(a-b)^{4}}$$

293 FA92 P III #4

a) $\det A = (3-s)(1-s)(-(1+s)) - (1-s)$

$= (1-s)\left(s^2 - 2s - 3 - 1\right) = \boxed{(1-s)(s^2-2s-4)}$

b) $(1-s)(s^2-2s-4) = 0$

$s = 1,\ \dfrac{2 \pm \sqrt{4+16}}{2} = 1,\ \boxed{1 \pm \sqrt{5}}$

293 FA92 F ※3

b) $\det A = 0$

$\because$ A has a nontrivial null space

$\equiv$ A has linearly dependent rows and columns

$\equiv \det A = 0$

293 FA92 F ※3a

$$A \cdot A^{-1} = I \Rightarrow \det(A \cdot A^{-1}) = \det(I) = 1$$

$$\det(A \cdot A^{-1}) = \det(A) \cdot \det(A^{-1})$$

$$\det(I) = \det(A) \cdot \det(A^{-1})$$

$$1 = \det(A) \cdot \det(A^{-1})$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)} \quad ※$$

294 SP93 P3 #1

a)

$$\begin{array}{ccccc} & & -36 & -12 & -14 \\ -2 & 1 & 2 & -2 & 1 \\ -2 & 2 & 2 & -2 & 2 \\ 9 & 3 & 7 & 9 & 3 \\ & & -28 & -18 & -12 \end{array}$$

$$= (-28-18-12)-(-36-12-14)$$

$$= \boxed{4}$$

294 SP94 P2 #6

c)

$$\begin{array}{ccccccc} & & & 0 & 1-\lambda & (2-\lambda)2 \\ \left|\begin{array}{ccc} 1-\lambda & 1 & 0 \\ 2 & 2-\lambda & 1 \\ 0 & 1 & 2-\lambda \end{array}\right. & \begin{array}{c} 1-\lambda \\ 2 \\ 0 \end{array} & \begin{array}{c} 1 \\ 2-\lambda \\ 1 \end{array} & & & \\ & & & (1-\lambda)(2-\lambda)^2 & 0 & 0 \end{array}$$

$$= (1-\lambda)(2-\lambda)^2 - (1-\lambda+4-2\lambda) = (1-\lambda)(\lambda^2-4\lambda+4) - (5-3\lambda)$$

$$= -\lambda^3+5\lambda^2-8\lambda+4-5+3\lambda = \boxed{-\lambda^3+5\lambda^2-5\lambda-1}$$

d)

$$\det A(0) = \begin{array}{ccccccc} & & & 0 & 1 & 4 \\ \left|\begin{array}{ccc} 1 & 1 & 0 \\ 2 & 2 & 1 \\ 0 & 1 & 2 \end{array}\right. & \begin{array}{c} 1 \\ 2 \\ 0 \end{array} & \begin{array}{c} 1 \\ 2 \\ 1 \end{array} & & & \\ & & & 4 & 0 & 0 \end{array} = 4-(1+4) = \boxed{-1}$$

This value is $\boxed{\text{equal}}$ to the value of the function found in c) when $\lambda = 0$.

293 FA94 P2 #3

a) $A = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 5 & -8 \\ 2 & 2 & 5 \end{pmatrix} \underset{\substack{R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - 2R_1}}{\sim} \begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 1 \\ 0 & 6 & -1 \end{pmatrix} \underset{R_3 \to R_3 + 6R_2}{\sim} \begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 5 \end{pmatrix} = B$

$$\det(A) = \det(B) = (1)(-1)(5) = \boxed{-5}$$

b) $\det(A) = 1\begin{vmatrix} 5 & -8 \\ 2 & 5 \end{vmatrix} - (-2)\begin{vmatrix} -3 & -8 \\ 2 & 5 \end{vmatrix} + 3\begin{vmatrix} -3 & 5 \\ 2 & 2 \end{vmatrix}$

$= (25+16) + 2(-15+16) + 3(-6-10) = \boxed{-5}$

293 FA94 P2 \#5

a) A is singular if and only if $\det(A) = 0$

b) $$\det\begin{pmatrix} \lambda-1 & 3 \\ 2 & \lambda-2 \end{pmatrix} = (\lambda-1)(\lambda-2)-6 = \lambda^2-3\lambda+2-6 = \lambda^2-3\lambda-4 = 0$$

$$(\lambda-4)(\lambda+1) = 0$$

$$\therefore \lambda = \boxed{-1, 4}$$

c) $\det(AB) = (\det A)(\det B)$ if $\det(A)$ and $\det(B)$ exist

d)
$$AA^{-1} = I$$
$$\det(AA^{-1}) = \det(I) = 1$$
$$(\det A)(\det A^{-1}) = 1$$
$$\therefore \det(A^{-1}) = \frac{1}{\det A}$$

293 FA94 P2 #6

use cofactors of entries in the first column

$$= +2 \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & 1 \\ 3 & 0 & 0 \end{vmatrix} - 3 \begin{vmatrix} 0 & -1 & 3 \\ 1 & 2 & 1 \\ -2 & 5 & 2 \end{vmatrix}$$

$$= 2 \cdot 3 \cdot (-7) - 3\left(2 + 15 + 12 + 2\right) = \boxed{-135}$$

294 FA94 P III #3

b), e), g)

Note that a) $\det A = 0$

c) the rows of A are linearly dependent

d) rank $A < n$

f) $A\vec{x} = \vec{b}$ has infinitely many solutions for each $\vec{b}$

299 FA94 F ※3

$$\begin{vmatrix} a & b & b & b \\ a & a & b & b \\ a & a & a & b \\ a & a & a & a \end{vmatrix} \underset{\substack{R_1 \to R_1 - R_4 \\ R_2 \to R_2 - R_4 \\ R_3 \to R_3 - R_4}}{=} \begin{vmatrix} 0 & b-a & b-a & b-a \\ 0 & 0 & b-a & b-a \\ 0 & 0 & 0 & b-a \\ a & a & a & a \end{vmatrix}$$

$$= \boxed{-a(b-a)^3}$$

293 FA94 F ~~X~~ 12

$$\det\left(\frac{1}{2}A^{-1}\right) = \left(\frac{1}{2}\right)^3 \cdot \frac{1}{\det A} = \frac{1}{8}\cdot\frac{1}{3} = \boxed{\frac{1}{24}}$$

293 FA94 F ※13

$\det(AB) = (\det A)(\det B) = 0 \quad \Rightarrow \quad \boxed{e.}$

293 FA94 F #14

$$y = \frac{\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}} = \frac{r}{p} \Rightarrow \boxed{(b)}$$

293 SP95 P2 * 5

a) use cofactors

$$\boxed{a_{13}a_{22}a_{36}a_{45}a_{51}a_{64} \; , \; a_{16}a_{25}a_{34}a_{43}a_{52}a_{61}}$$

will appear in det A

because they have a_{ij}'s from different rows and different columns

$a_{15}a_{21}a_{36}a_{45}a_{52}a_{63}$ will not appear in det A

because it has a_{15} and a_{45} which come from the same column.

$i+j$

b) $\left.\begin{array}{l} (-1)^4(-1)^4(-1)^9(-1)^9(-1)^6(-1)^{10} = 1 \\ (-1)^7(-1)^7(-1)^7(-1)^7(-1)^7(-1)^7 = 1 \end{array}\right\}$ $\boxed{\text{positive}}$

c) $6! = \boxed{720}$ terms

because $\left\{ \begin{array}{l} \text{for } a_{1j_1}a_{2j_2}a_{3j_3}a_{4j_4}a_{5j_5}a_{6j_6}, \\ \text{can choose } j_1 \text{ to be any of the six columns} \\ \quad " \quad j_2 \quad " \quad " \quad \text{remaining 5 columns} \\ \quad " \quad j_3 \quad " \quad " \quad \quad " \quad 4 \text{ columns} \\ \quad \vdots \qquad\qquad\qquad\qquad\qquad \vdots \\ \text{and } j_6 \text{ has to be the } \quad " \quad 1 \text{ columns} \end{array} \right.$

293 SP95 PII #6

a) $\begin{vmatrix} 2 & 0 & 1 & -1 \\ 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ -2 & -2 & 1 & 0 \end{vmatrix} \underset{c \to C_4 + C_3}{=} \begin{vmatrix} 2 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ -2 & -2 & 1 & 1 \end{vmatrix}$

(cofactor expansion down the 4^{th} column)

$= 0 + 0 + 0 + \begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = (2)(2)(1) + 0 \cdot (-1)(0) + (1)(1)(1)$
$- (0)(2)(1) - (1)(-1)(2) - (1)(1)(0)$

$= 4 + 1 + 2 = \boxed{7}$

b) The equation $A\vec{x} = 0$ has only the trivial solution

293 SP96 PIII ※9

1, 2, 4

Note that

For 3 $\text{Null } A = \{\vec{0}\}$ (not empty)

For 5 Linear transformation $\vec{x} \to A\vec{x}$ is invertible, 1:1, onto

293 SP96 F #12

Since $R_2 - R_4 = 5R_1$, $\det A = \boxed{0}$

294 sp97 F #2

a) $\det A^{-1} = \dfrac{1}{\det A} = \boxed{\dfrac{1}{2}}$

$\det A^{T} = \det A = \boxed{2}$

294 FA97 F #6

(a) For $(x, y, z) = (a_1, a_2, a_3)$, $R_1 = R_2 \Rightarrow \det = 0$

$\therefore$ point $\vec{a}$ $\boxed{\text{lies on}}$ S

(b) For $(x, y, z) = (b_1, b_2, b_3)$, $R_1 = R_3$

For $(x, y, z) = (c_1, c_2, c_3)$, $R_1 = R_4$ $\Big\}$ $\det = 0$

$\therefore$ points $\vec{b}$ and $\vec{c}$ $\boxed{\text{lie on}}$ S

(c)

$$\det \begin{vmatrix} 0 & 0 & 0 & 1 \\ a_1 & a_2 & a_3 & 1 \\ b_1 & b_2 & b_3 & 1 \\ c_1 & c_2 & c_3 & 1 \end{vmatrix} = -\det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = -\det\left([\vec{a}\ \vec{b}\ \vec{c}]^T\right) = 0$$

$$\therefore \boxed{\det\left([\vec{a}\ \vec{b}\ \vec{c}]\right)} = 0$$

or $\vec{a}, \vec{b}, \vec{c}$ are linearly independent

294 SP98 P2 *5

a) $\begin{vmatrix} 1 & -2 & 5 & -2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 0 & 0 & 4 & 4 \end{vmatrix} = -3 \begin{vmatrix} 1 & -2 & -2 \\ 2 & -6 & 5 \\ 0 & 0 & 4 \end{vmatrix} = -3 \cdot 4 \cdot \begin{vmatrix} 1 & -2 \\ 2 & -6 \end{vmatrix} = (-3)(4)(-2) = \boxed{24}$

b) i) $\boxed{\text{False}}$ For example $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \neq 1$

ii) $\boxed{\text{False}}$ They're equal

294 FA98 P II # 2

d)

$$\begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & 0 \\ 2 & 0 & -2 & 0 \end{vmatrix} = 3\begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & 3 \\ 0 & -2 & 0 \end{vmatrix} = 3(2)\begin{vmatrix} 2 & 4 \\ 2 & 3 \end{vmatrix} = 3(2)(-2) = \boxed{-12}$$

293 SP? PII ☆3

(a)

$$\begin{vmatrix} 4 & -7 & 2 \\ 5 & 2 & 0 \\ 3 & 0 & 0 \end{vmatrix} = 3\begin{vmatrix} -7 & 2 \\ 2 & 0 \end{vmatrix} = 3(-2)(2) = \boxed{-12}$$

(b) Try cofactor expansion across the first row

$$\det\begin{vmatrix} 1 & x & x^2 & x^3 \\ 1 & 2 & 2^2 & 2^3 \\ 1 & 3 & 3^2 & 3^3 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 1\cdot a + x\cdot b + x^2\cdot c + x^3\cdot d$$

when a, b, c, d are some real numbers.

This means $F(x)=0$ has 3 roots maximum.

However, can see easily that $\underset{R_1=R_4}{F(1)} = \underset{R_1=R_2}{F(2)} = \underset{R_1=R_3}{F(3)} = 0$

$\therefore \underbrace{1, 2, 3}_{\text{already three}}$ are roots of $F(x)=0$

Therefore, $F(737) \neq 0$