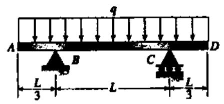
TAM 202 Spring 03 HW14 Soln by Peeyush & Tian

4.5-9, 4.5-10, 5.4-2, 5.5-2, 5.5-4, 5.5-6, 5.5-8, 5.5-12 (due 04/29)

4.5-9 Beam *ABCD* is simply supported at *B* and *C* and has overhangs at each end (see figure). The span length is *L* and each overhang has length *L*/3. A uniform load of intensity *q* acts along the entire length of the beam.

Draw the shear-force and bending-moment diagrams for this beam.

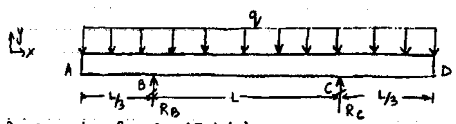


PROB. 4.5-9

Solution

Span length = distance between B and C = L
length of each overhang = L/3

FBD for beam ABCD.



By symmetry $R_B = R_C = (5qL/6)$
In gory detail:

$$\Sigma F_y = 0 = -q\left(L + \frac{L}{3} + \frac{L}{3}\right) + R_B + R_C$$

or $$R_B + R_C = \frac{5L}{3}q$$

To find other equation to solve for reactions, do moment balance about B.

(Anticlockwise moments (+))

$\Sigma M_B = 0$

$$+q\left(\frac{L}{3}\right)\left(\frac{L}{6}\right) + R_C(L) - q\left(\frac{L}{3}+L\right)\left(\frac{L}{3}+L\right)\frac{1}{2} = 0$$

$$\Rightarrow R_C \cdot L = q\left(\frac{15}{18}L^2\right)$$

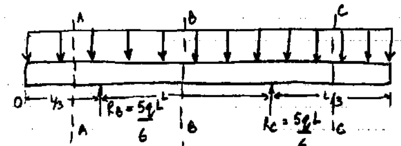
or $$\boxed{R_C = \frac{5}{6}qL}$$

As you could do by inspection

$$\boxed{R_B = \frac{5L}{3}q - \frac{5L}{6}q = \frac{5L}{6}q}$$

[Note: the moment due to a distributed load with constant magnitude is $\frac{qx^2}{2}$ where q is the magnitude & x is the distance from the point about which moment is taken]

To draw shear force diagram, start from either end and take cuts [i.e method 1: FBD's]

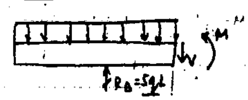


Cut A-A

$\Sigma F_y = 0 \Rightarrow V = -qx$ (Linear function of x with negative slope) — ①

$M = -\frac{qx^2}{2}$ (quadratic function of x with (−) slope). — ②

Cut B-B



$\Sigma F_y = 0 \quad V = R_B - qx = \frac{5}{6}qL - qx$

$$= q\left(\frac{5}{6}L - x\right) \quad -③$$

[See that $q = 0$ when $x = \frac{5}{6}L$
the distance of this point from B
$= \frac{5}{6}L - \frac{L}{3} = \frac{L}{2}$]

$\Sigma M = 0$

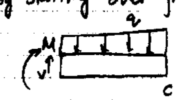
$$\Rightarrow M = \frac{5}{6}qL\left(x - \frac{L}{3}\right) - \frac{qx^2}{2}$$

$$\boxed{M = -\frac{qx^2}{2} + \frac{5}{6}qLx - \frac{5}{18}qL^2} \quad -④$$

M = 0 will have 2 roots.

cut C-C can continue the same way by taking all the forces and moments

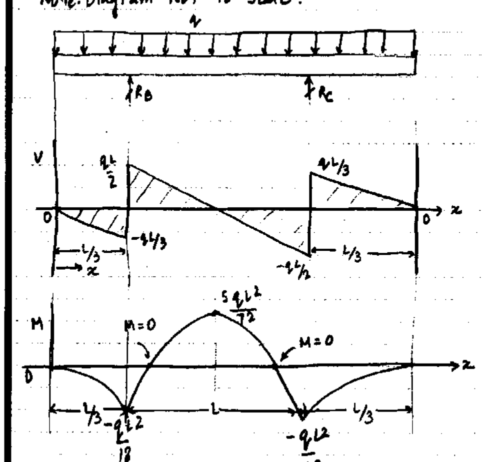
Or by starting over from the other side.



$M = -qx^2/2 \quad -⑤$

$V = +qx. \quad -⑥$

Using ①, ③, ⑥, Draw shear force diagram.
Note: Diagram Not to Scale.



Use ②, ④, ⑤ to draw Bending moment diagram
Diagram Not to Scale.

Method 2: Use the equations $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = V$, integrate, find out V and M in terms of x and V in terms of x.

In this problem $q = q_0$ (a constant).

$\frac{dV}{dx} = -q$ $\quad 0 \le x \le L/3$ [x measured from Left hand side].

Integrate,

$$V = V(0) - \int_0^x q\,dx = V(0) - qx$$

To calculate $V(0)$, use $V=0$ at $x=0$, so $V_0 = 0$.

So $V = -qx$ $\quad 0 \le x \le L/3$ — 1.

Similarly

$$\frac{dM}{dx} = V \Rightarrow M = \int_0^x V\,dx + M_0.$$

use $V = -qx \Rightarrow M = M(0) + \int_0^x (-qx)\,dx$

or $\boxed{M = M(0) - \frac{qx^2}{2}}$

To calculate $M(0)$ use $M=0$ at $x=0$.

$\Rightarrow M(0) = 0$.

So $\boxed{M = -\frac{qx^2}{2}}$ $\quad 0 \le x \le L/3$ — 2

See that the equation we got here is the same as when we took the cut A-A & calculated V & M.

Now take $\frac{L}{3} < x < \frac{4L}{3}$. [have to take x in the region where there is no discontinuity in load or moment (no concentrated load or moment]

So using the same equation $\left[\frac{dV}{dx} = -q, \frac{dM}{dx} = V\right]$ get

$$\left.\begin{aligned}\frac{dV}{dx} &= -q\\ \frac{dM}{dx} &= +V\end{aligned}\right\} \quad \frac{L}{3} < x < \frac{4L}{3}$$

Integrate for V

$$V = V\left(\tfrac{L}{3}\right)^+ - \int_{L/3}^{x} q\,dx$$

or $V = V\left(\frac{L}{3}\right)^+ - q\left(x - L/3\right)$

Now $V\left(\frac{L}{3}\right)^+ = \underbrace{-\frac{qL}{3}}_{V\left(\frac{L}{3}\right)^-} + \underbrace{\frac{5qL}{6}}_{\text{jump in V due to reaction at B } (R_B)}$.

$V\left(\frac{L}{3}\right)^+ = \frac{qL}{2}$.

So $V = V\left(\frac{L}{3}\right)^+ - qx + qL/3$

$V = \frac{qL}{2} + \frac{qL}{3} - qx$

or $\boxed{V = \frac{5qL}{6} - qx}$ — 3.

Integrate for M

$$M = M\left(\tfrac{L}{3}\right)^+ + \int_{L/3}^{x} \left(\frac{5qL}{6} - qx\right)dx$$

or $M = M\left(\frac{L}{3}\right)^+ + \frac{5qL}{6}\left(x - \frac{L}{3}\right) - \frac{q}{2}\left(x - \frac{L}{3}\right)^2$

$M\left(\frac{L}{3}\right)^+ = -\frac{qL^2}{18}$.

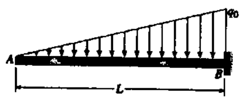
So $M = -\frac{qL^2}{18} + \frac{5qL}{6}\left(x - L/3\right) - \frac{q}{2}\left(x - L/3\right)^2$

or $\boxed{M = -\frac{qx^2}{2} + \frac{5qLx}{6} - \frac{5}{18}qL^2}$

Similarly V & M can be determined for $\frac{4L}{3} < x < \frac{5L}{3}$, or can start over from the other side as done for method 1. Plot the bending moment & the shear force diagrams using equations found out. [of course, would be the same].

4-5-10

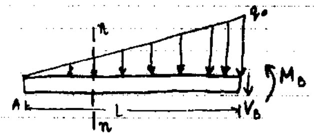
4.5-10 Draw the shear-force and bending-moment diagrams for a cantilever beam AB supporting a linearly varying load of maximum intensity q_0 (see figure).



PROB. 4.5-10

Solution

FBD for the cantilever beam



$\Sigma F_y = 0 \Rightarrow -V_B - (q_0 \cdot L)\frac{1}{2} = 0$

$\frac{1}{2}(q_0 L)$ = Area of triangular region = magnitude of force.

$\Rightarrow \boxed{V_B = -\frac{q_0 L}{2}}$ - (1).

$\Sigma M_B = 0$

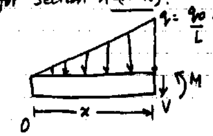
$\Rightarrow M_B - \underbrace{\left(\frac{q_0 L}{2}\right)\left(\frac{L}{3}\right)}_{\text{Magnitude of moment due to distributed load}} = 0$

$\Rightarrow \boxed{M_B = \frac{q_0 L^2}{6}}$ — (2).

Now to draw shear force & bending moment diagram take a cut (n-n) & look at V & M.

Method 1

FBD for Section A-(N-N).



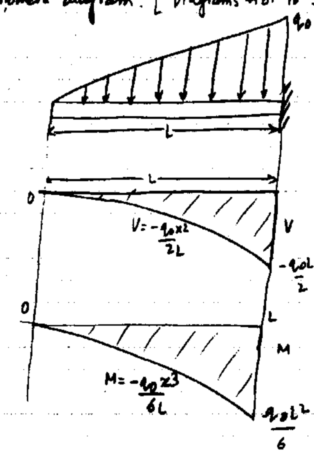
$\Sigma F_y = 0 \Rightarrow V + \frac{1}{2}\left(\frac{q_0 x}{L}\right)(x) = 0$

or $\boxed{V = -\frac{q_0 x^2}{2L}}$ - (3)

$\Sigma M = 0 \Rightarrow M + \frac{1}{2}\left(\frac{q_0 x}{L}\right)(x)\left(\frac{x}{3}\right) = 0$

$\Rightarrow \boxed{M = -\frac{q_0 x^3}{6L}}$ - (4)

Using (3) & (4), Draw shear force & Bending moment diagram. [Diagrams not to scale]



Method 2.

Use $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = +V$.

Here q is a function of x.

$q = \frac{q_0 x}{L}$.

$\frac{dV}{dx} = -\frac{q_0 x}{L}$.

$V = V(0) - \int_0^x \frac{q_0 x}{L}\, dx$

or $V = V(0) - \frac{q_0 x^2}{2L}$

$V(0) = 0 \quad \because V = 0$ at $x = 0$.

So $\boxed{V = -\frac{q_0 x^2}{2L}}$. - (1)

$M = M(0) + \int_0^x V\, dx$

So $M = M(0) - \int_0^x \frac{q_0 x^2}{2L}\, dx$

or $M = -\frac{q_0 x^3}{6L} + M(0)$

$M(0) = 0 \quad \because M = 0$ at $x = 0$.

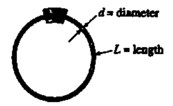
$\boxed{M = -\frac{q_0 x^3}{6L}}$ - (2)

Use (1) & (2) to plot Bending moment & Shear force diagrams

#5.4-2

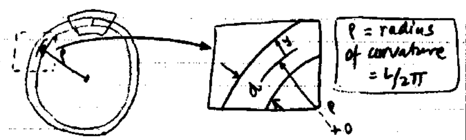
(5-4-2)

5.4-2 A copper wire having diameter $d = 3$ mm mm is bent into a circle and held with the ends just touching (see figure). If the maximum permissible strain in the copper is $\epsilon_{max} = 0.004$, what is the shortest length L of wire that can be used?



PROB. 5.4-2

Solution: $d = 3$ mm $= 3 \times 10^{-3}$ m.
$\epsilon_{max} = 0.004$



Now

$$\epsilon_{max} = \frac{y}{\rho} = \frac{d/2}{L/2\pi} = \frac{\pi d}{L}$$

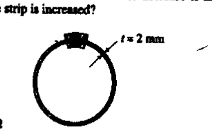
$$\Rightarrow L_{min} = \frac{\pi d}{\epsilon_{max}} = \frac{\pi(3 \times 10^{-3}\,m)}{0.004}$$

or $\boxed{L_{min} = 2.36\ m}$

(5-5-2)

5.5-2 A thin strip of hard copper ($E = 113$ GPa) having length $L = 2$ m and thickness $t = 2$ mm is bent into a circle and held with the ends just touching (see figure).

(a) Calculate the maximum bending stress σ_{max} in the strip. (b) Does the stress increase or decrease if the thickness of the strip is increased?



PROB. 5.5-2

#5.5-2

Solution: $E = 113$ GPa.
$L = 2$ m.
$t = 2$ mm $= 2 \times 10^{-3}$ m.

a) Calculate σ_{max}.

$$\epsilon = \frac{y}{\rho} \Rightarrow \sigma = E\epsilon = \frac{Ey}{\rho}.$$

ρ = radius of curvature $= \frac{L}{2\pi}$ and $y = t/2$

$$\Rightarrow \sigma_{max} = \frac{E(t/2)}{(L/2\pi)} = \frac{\pi E t}{L}$$

or $\sigma_{max} = \frac{\pi(113\,e\,9\ GPa)(2\,e(-3)\,m)}{2\,m}$

or $\sigma_{max} = 355$ MPa.

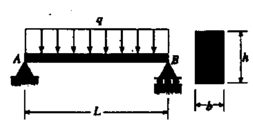
b) $\sigma_{max} \propto t$.

If t increases, σ_{max} increases.

(5-5-4)

5.5-4 A simply supported wood beam AB with span length $L = 3.75$ m carries a uniform load of intensity $q = 6.4$ kN/m (see figure).

Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 150$ mm and height $h = 300$ mm.

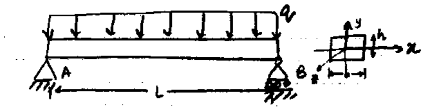


PROB. 5.5-4

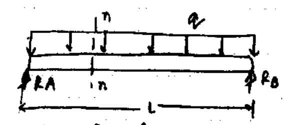
#5.5-4

Solution: The maximum bending stress occurs where the bending moment is maximum.

So $\sigma_{max} = \frac{M_{max}\,y_{max}}{I}$



FBD for the beam.

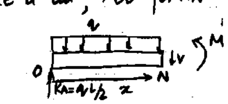


$\Sigma F_y = 0 \quad R_A + R_B = qL$

$\Sigma M_B = 0 \quad \Rightarrow -R_A \cdot L + \frac{qL^2}{2} = 0$

or $\boxed{R_A = \frac{qL}{2} \ \& \ R_B = \frac{qL}{2}}$

take a cut, FBD for AN



$\Sigma F_y = 0 \Rightarrow V = \frac{qL}{2} - qx$

$\Sigma M = 0 \quad \Rightarrow M = \frac{qL}{2}x - \frac{qx^2}{2}$

M_{max} occurs at $\frac{dM}{dx} = 0$ or $V = 0$

when $V = 0$ $x = (L/2)$.

$M_{max} = M(@\ x = L/2) = \frac{qL^2}{8}$.

$$I = I_x = \frac{1}{12} bh^3 \quad [\text{From Appendix, D, pg 877}]$$

$$y_{max} = h/2.$$

$$\Rightarrow \sigma_{max} = \frac{M_{max} y_{max}}{I_x} = \frac{\frac{qL^2}{8} * (h/2)}{\frac{1}{12}(bh^3)} = \frac{3qL^2}{4bh^2}.$$

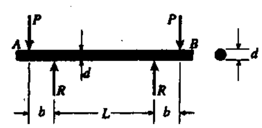
$$\text{or } \sigma_{max} = \frac{3(6.4\ kN/m)(3.75\ m)^2}{4(150 * 10^{-3}\ m)(300 * 10^{-3}\ m)^2}$$

$$\text{or } \boxed{\sigma_{max} = 5\ MPa}$$

5-5-6

5.5-6 A freight-car axle *AB* is loaded as shown in the figure, with the forces *P* representing the car loads (transmitted through the axle boxes) and the forces *R* representing the rail loads (transmitted through the wheels). The diameter of the axle is $d = 80$ mm, the wheel gauge is $L = 1.45$ m, and the distance between the forces *P* and *R* is $b = 200$ mm.

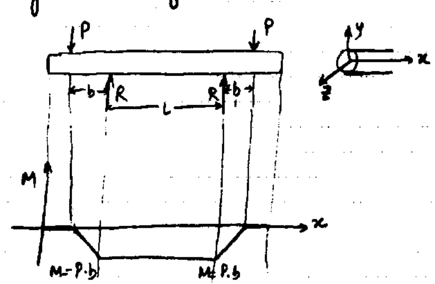
Calculate the maximum bending stress σ_{max} in the axle if $P = 46.5$ kN.



PROB. 5.5-6

Solution: $R = P = 46.5\ kN$.

The Bending moment diagram looks like this:-



$$M_{max} = P.b.$$

$$y_{max} = d/2$$

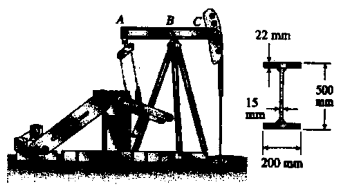
$$I = I_x = \frac{\pi d^4}{64} \quad [\text{From Appendix D, 9 pg 879}]$$

$$\text{So } \sigma_{max} = \frac{M_{max} y_{max}}{I_x} = \frac{P.b.d/2}{(\pi d^4/64)} = \frac{32Pb}{\pi d^3}.$$

$$\text{or } \boxed{\sigma_{max} = \frac{32(46.5kN)(200 * 10^{-3}\ m)}{\pi(80 * 10^{-3}\ m)^3} = 185.0\ MPa}$$

5.5-8

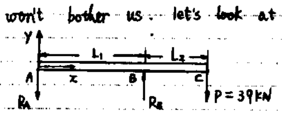
5.5-8 The horizontal beam *ABC* of an oil-well pump has the cross section shown in the figure. If the vertical pumping force acting at end *C* is 39 kN, and if the distance from the line of action of that force to point *B* is 4.5 m, what is the maximum bending stress in the beam due to the pumping force?



PROB. 5.5-8

① Find M_{max}

In this problem, we are not given the length AB, but this won't bother us. let's look at FBD of ABC:

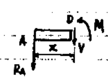


L_1 is not given
$L_2 = 4.5m$

$\Sigma M_{/A} = 0 = R_B L_1 - P(L_1 + L_2) \Rightarrow R_B = P(L_1+L_2)/L_1$

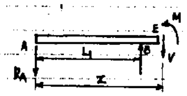
$\Sigma F_y = 0 = R_B - R_A - P \Rightarrow R_A = PL_2/L_1$

Now we can find bending moment distribution in ABC.



$\Sigma M_{/D} = 0 = M + R_A x$

$\Rightarrow M = -Px L_2/L_1 \quad (0 \le x \le L_1)$

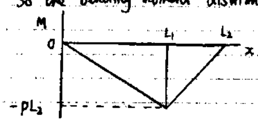


$\Sigma M_{/E} = 0 = M + R_A x - R_B(x - L_1)$

$\Rightarrow M = P(x - L_1 - L_2)$

$(L_1 \le x \le L_2)$

So the bending moment distribution are simply straight lines

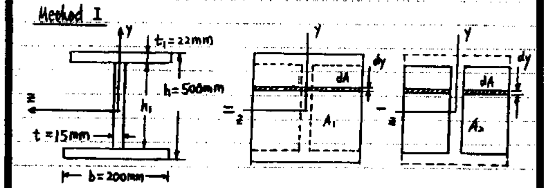


$\Rightarrow |M_{max}| = PL_2$ occurs at $x = L_1$, and L_2 is given in this problem.

5.5-8 (Cont'd)

② Find I for an I-beam

Method I



$$I = I_1 - I_2$$

where

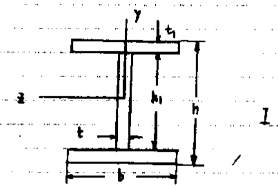
$$I_1 = \int_{A_1} y^2\,dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \cdot b\,dy = \frac{bh^3}{12}$$

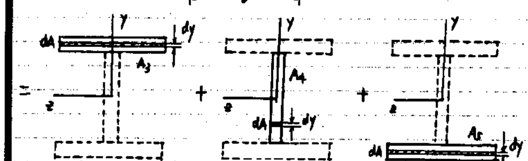
$$I_2 = \int_{A_2} y^2\,dA = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} y^2 \cdot (b-t)\,dy = \frac{(b-t)h_1^3}{12}$$

$$= \frac{(b-t)(h-2t_1)^3}{12}$$

$$\Rightarrow I = I_1 - I_2 = \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

Method II





$$= I_3 + I_4 + I_5$$

where

$$I_3 = \int_{A_3} y^2\,dA = \int_{\frac{h_1}{2}}^{\frac{h}{2}} y^2 \cdot b\,dy = \frac{b}{24}[h^3 - h_1^3]$$

$$I_4 = \int_{A_4} y^2\,dA = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} y^2 \cdot t\,dy = \frac{th_1^3}{12}$$

$$I_5 = \int_{A_5} y^2\,dA = \int_{-\frac{h}{2}}^{-\frac{h_1}{2}} y^2 \cdot b\,dy = \frac{b}{24}[h^3 - h_1^3]$$

(Continued)

5.5-8 (Cont'd)

$$\Rightarrow I = I_3 + I_4 + I_5$$

$$= \frac{b}{24}[h^3 - h_1^3] + \frac{th_1^3}{12} + \frac{b}{24}[h^3 - h_1^3]$$

$$= \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12}$$

$$= \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

Substitute numbers into I

$$\Rightarrow I = \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

$$= \frac{1}{12}[(200mm)(500mm)^3 - (200mm - 15mm)(500mm - 2\times 22mm)^3]$$

$$= 6.21 \times 10^8\ mm^4$$

③ Find σ_{max}

$$|\sigma_{max}| = \left|\frac{M_{max}\,y_{max}}{I}\right|$$

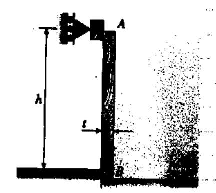
$$= \frac{PL_2(\frac{h}{2})}{I}$$

$$= \frac{(39\,kN)(4.5m)(\frac{500mm}{2})}{6.21\times 10^8\ mm^4}$$

$$= \boxed{70.6\ MPa}$$

5.5-12

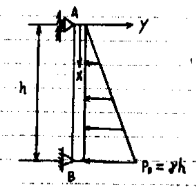
5.5-12 A small dam of height $h = 2.4$ m is constructed of vertical wood beams *AB* of thickness $t = 150$ mm, as shown in the figure. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is $\gamma = 9.81\ kN/m^3$.

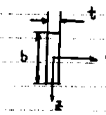


PROB. 5.5-12

(Continued)

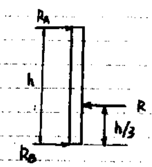
① water pressure on the beam





② Find R_A or R_B (reaction forces).

FBD:



where $R = \bar{P}(bh) = \frac{P_0}{2}(bh)$

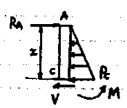
$= \frac{\gamma b h^2}{2}$

$\Sigma M_{/B} = 0 = R\left(\frac{h}{3}\right) - R_A(h)$

$\Rightarrow R_A = \gamma b h^2/6$

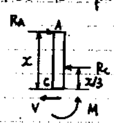
③ Find M_{max}

FBD:



$P_c = \frac{P_0 x}{h} = \gamma x$

This FBD is equivalent to the following one:



$R_c = \frac{P_c}{2}(bx)$

$= \frac{\gamma b}{2}x^2$

$\Sigma M_{/C} = 0 = M + R_c\left(\frac{x}{3}\right) - R_A \cdot x$

$\Rightarrow M = R_A x - R_c x/3 = \gamma b h\left[\frac{hx}{6} - \frac{x^3}{6h}\right]$

To find M_{max}, we set $\frac{dM}{dx} = 0$.

$\frac{dM}{dx} = 0 = \gamma b h\left[\frac{h}{6} - \frac{x^2}{2h}\right] \Rightarrow x = h/\sqrt{3}$

$\Rightarrow M_{max} = M\left(x = \frac{h}{\sqrt{3}}\right) = \gamma b h\left[\frac{h}{6}\frac{h}{\sqrt{3}} - \frac{1}{6h}\left(\frac{h}{\sqrt{3}}\right)^3\right] = \frac{\gamma b h^3}{9\sqrt{3}}$

(Continued)

④ Find σ_{max}

$|\sigma_{max}| = \left|\frac{M_{max}\, y_{max}}{I}\right| = \frac{\gamma b h^3}{9\sqrt{3}} \cdot \frac{\frac{t}{2}}{\frac{bt^3}{12}} = \frac{2\gamma h^3}{3\sqrt{3}\, t^2}$

$= \frac{2(9.81\ kN/m^3)(2.4m)^3}{3\sqrt{3}\ (150mm)^2}$

$= \boxed{2.32\ MPa}$