"SOLUTIONS"

ENGRD 202 Quiz 5

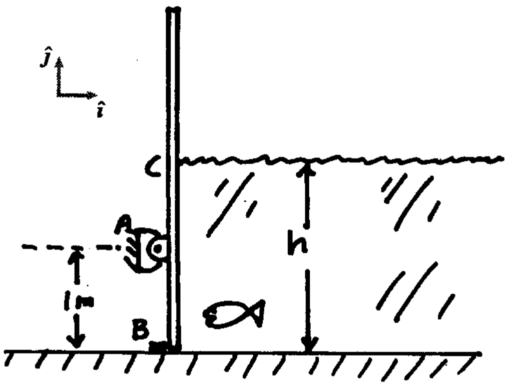
April 4, 2003

Your Name: RUINA

Section day & time: __________

TA name & section #: __________

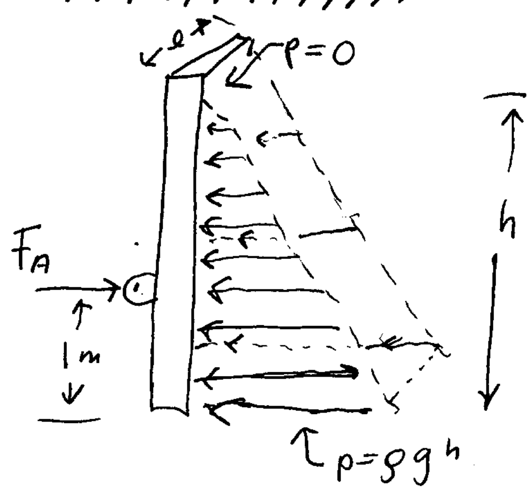
9) (7 pts) Water is held in a reservoir by a board with negligible weight that is 5 meters long. It is hinged 1 meter off the bottom at A and kept from leaking by a seal at B. What is h when the board starts to pull away from the stop at B? At that h what is the force of the hinge on the board? Assume $\rho = 1000\,\mathrm{kg/m^3}$, $g = 10\,\mathrm{N/kg}$.



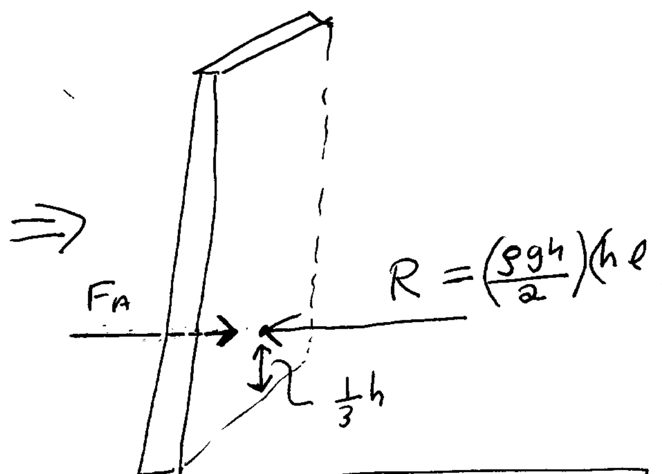
Just at pull away there is no reaction at B.

For no moment about A, R must act at A, $\Rightarrow \frac{h}{3} = 1\,m$

$\boxed{h = 3\,m}$



$\Rightarrow$



$$\boxed{\begin{aligned}&\sum F_x = 0 \Rightarrow F_A = R\\ &\Rightarrow \underline{F}_A = F_A\,\hat{\imath}\end{aligned}}$$

$$\begin{aligned}R &= (\text{ave pressure})\cdot(\text{area})\\ &= \left(\frac{\rho g h}{2}\right)\cdot(h\ell)\\ &= \frac{(1000\,\mathrm{kg/m^3})\cdot(10\,\mathrm{N/kg})\cdot(3\,m)}{2}\cdot(3\,m)\cdot(5\,m)\\ &= \frac{1000\cdot 10\cdot 3\cdot 3\cdot 5}{2}\,N\\ &= \frac{450\,000}{2}\,N = 225000\,N\end{aligned}$$

$h = 3\,m$

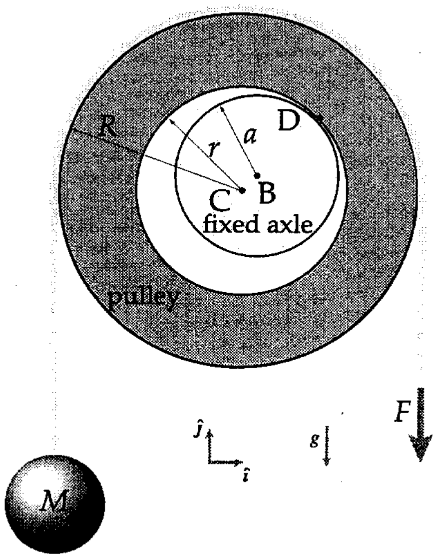
$\underline{\mathbf{F}}_A = 225\,000\,N\,\hat{\imath}$

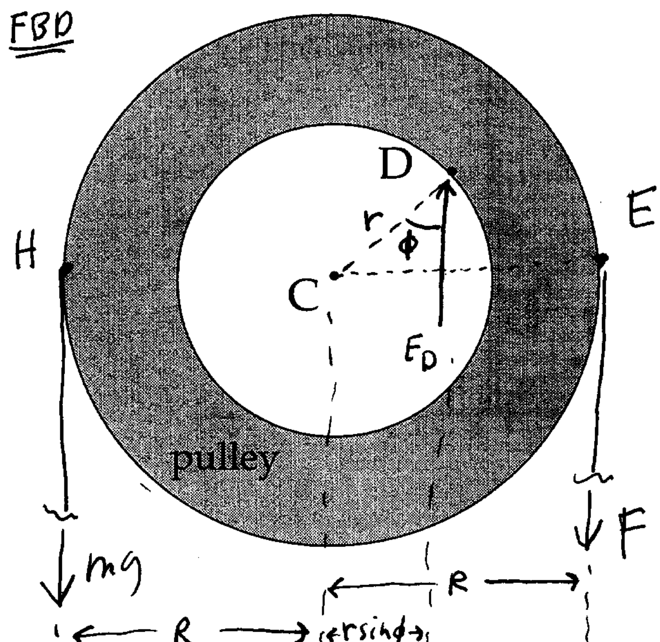
10) **(10 pts)** A mass M is steadily raised by pulling with a force F on a rope going over a negligible-mass pulley on an unlubricated journal bearing (no ball bearings). The friction coefficient between the pulley and its axle is $\mu = \tan\phi$. (The figure at right is the start of a drawing for one useful FBD.)

a) Find F in terms of M, g, R, r, a and μ (or ϕ or $\sin\phi$ or $\cos\phi$ — whichever is most convenient, for example $\cos(\tan^{-1}(\mu))$ is just $\cos\phi$). [Hint: Finding the location of the contact point D is probably part of your solution.]

b) Evaluate F in the special case that $M = 100\,\text{kg}, g = 10\,\text{N/kg}, r = 1\,\text{cm}, R = 2\,\text{cm}$, and $\mu = \sqrt{3}/3$ (so $\phi = \pi/6, \sin\phi = 1/2, \cos\phi = \sqrt{3}/2$).

c) What happens instead if μ is very large, say the limit $\mu \to \infty$? Does the needed force F go to ∞ or what?





Because F and mg both act vertically, force at D must also. Because of friction F_D makes angle ϕ with CD.

$$\sum M_{/D} = 0 \Rightarrow mg(R + r\sin\phi) - F(R - r\sin\phi) = 0$$

$$\Rightarrow \boxed{F = \frac{R + r\sin\phi}{R - r\sin\phi}\, mg} \quad \text{(a)}$$

b) $F = \dfrac{(2 + 1/2)\text{cm}}{(2 - 1/2)\text{cm}} 1000\,\text{N} = \boxed{\dfrac{5000}{3}\,\text{N}}$ (b)

c) Formula in (a) goes to, when $\mu \to \infty$ [$\mu \to \infty \Rightarrow \tan\phi \to \infty \Rightarrow \phi \to \pi/2 \Rightarrow \sin\phi \to 1$]

$$\underset{\mu\to\infty}{\uparrow}\; F = \frac{R + r}{R - r}\, mg \Rightarrow$$

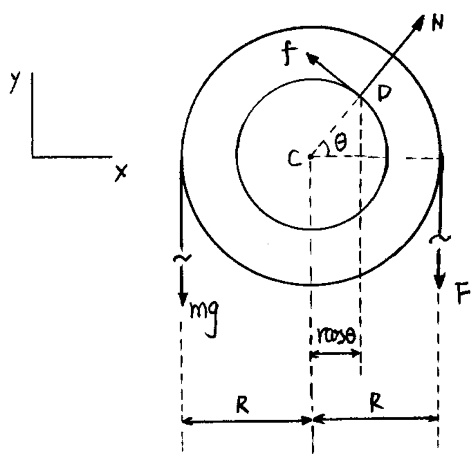
F only slightly greater than mg if $R \gg r$ no matter what the value of μ.

a) $F = \dfrac{R + r\sin\phi}{R - r\sin\phi}\, mg$ b) $F = \dfrac{5000}{3}\,\text{N}$ c) $\underset{\mu\to\infty}{\uparrow}\; F = \dfrac{R+r}{R-r}\, mg$, does not blow up. Not even $F \gg mg$ so long as $r \ll R$.

Alternative Approach to Problem 10 (Brute Force)

(a)

① FBD



Note that the position of D is not determined yet. i.e. θ is still an unknow. So in this FBD, we totally have 4 unknowns:

f, N, F, θ.

and we should be able to find 4 eqns to solve for them

② Equilibrium eqns

$\Sigma F_x = 0 = N\cos\theta - f\sin\theta$ (1)

$\Sigma F_y = 0 = N\sin\theta + f\cos\theta - mg - F$ (2)

$\Sigma M_{/D} = 0 = mg(R + r\cos\theta) - F(R - r\cos\theta)$ (3)

Slip condition $f = \mu N$ (4)

These are 4 eqns for 4 unknowns. Actually you don't have to use all of them since you are only asked to find F

sub (4) into (1) $\Rightarrow$ $N\cos\theta - \mu N\sin\theta = 0 \Rightarrow \tan\theta = \frac{1}{\mu}$

notice that $\tan\phi = \mu$. where ϕ is the friction angle $\Rightarrow$ $\theta = \frac{\pi}{2} - \phi$

$\Rightarrow \cos\theta = \cos(\frac{\pi}{2} - \phi) = \sin\phi$ (5)

Sub (5) into (3) $\Rightarrow$ $F = \dfrac{R + r\sin\phi}{R - r\sin\phi} mg$ (6)

(b) Directly sub numbers into (6)

$$\Rightarrow F = \frac{(2cm) + (1cm)\cdot\frac{1}{2}}{(2cm) - (1cm)\cdot\frac{1}{2}} \cdot (100kg)(10N/kg) = \frac{5000}{3} N$$

(c) as $\mu \to \infty$, $\tan\phi \to \mu$ $\Rightarrow \phi \to \frac{\pi}{2}$ $\Rightarrow$ $\sin\phi \to 1$

$$\Rightarrow F = \frac{R + r\sin\phi}{R - r\sin\phi} mg \to \frac{R+r}{R-r} mg$$ which doesn't go to ∞.