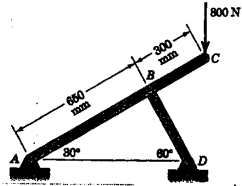


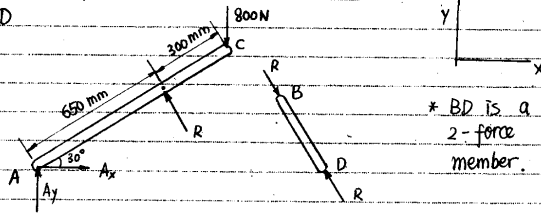
HW 6 Solution provided by Tian Tang

4.65, 4.67, 4.81, 4.91, 4.143 (due 02/25/03)

4/65 Determine the magnitudes of all pin reactions for the frame loaded as shown.  
Ans. A = 612 N, B = D = 1018 N



⊙ FBD



$$\sum M_A = 0 = R \cdot 650 \text{ mm} - 800 \text{ N} \cdot (650 + 300) \text{ mm} \cdot \cos 30^\circ$$

$$\Rightarrow R = 1013 \text{ N}$$

$$\sum F_x = 0 = A_x - R \sin 30^\circ$$

$$\Rightarrow A_x = 506 \text{ N}$$

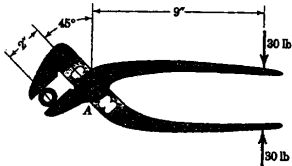
$$\sum F_y = 0 = A_y + R \cos 30^\circ - 800 \text{ N}$$

$$\Rightarrow A_y = -77 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = 512 \text{ N}$$

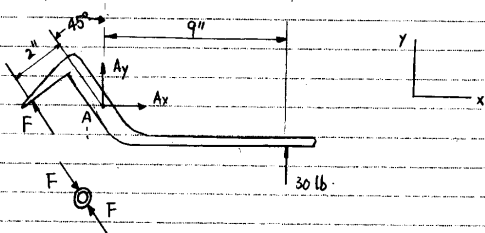
# 4.67

4/67 Compute the force supported by the pin at A for the slip-joint pliers under a grip of 30 lb.  
Ans. A = 187.6 lb



Problem 4/67

⊙ FBD



(Continued)

# 4.67 (Cont'd)

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$$\sum M_A = 0 = 30 \text{ lb} \cdot 9 \text{ in} - F \cdot 2 \text{ in}$$

$$\Rightarrow F = 135 \text{ lb}$$

$$\sum F_x = 0 = A_x - F \sin 45^\circ$$

$$\Rightarrow A_x = 95.5 \text{ lb}$$

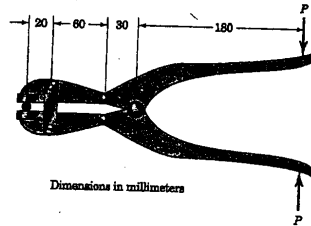
$$\sum F_y = 0 = A_y + F \cos 45^\circ + 30 \text{ lb}$$

$$\Rightarrow A_y = -125.5 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = 157.6 \text{ lb}$$

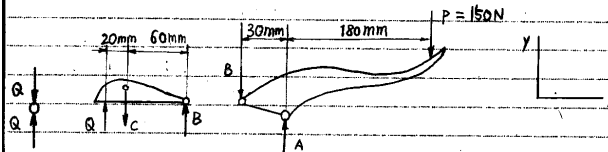
# 4.81

4/81 A small bolt cutter operated by hand for cutting small bolts and rods is shown in the sketch. For a hand grip P = 150 N, determine the force Q developed by each jaw on the rod to be cut.  
Ans. Q = 2.7 kN



Problem 4/81

⊙ FBD



Note that the force at A does not have x component because of symmetry. Thus forces at B and C don't have x components either.

⊙ Equilibrium of the handle

$$\sum M_A = 0 = B \cdot 30 \text{ mm} - P \cdot 180 \text{ mm}$$

$$\Rightarrow B = 900 \text{ N}$$

⊙ Equilibrium of the jaw

$$\sum M_C = 0 = B \cdot 60 \text{ mm} - Q \cdot 20 \text{ mm}$$

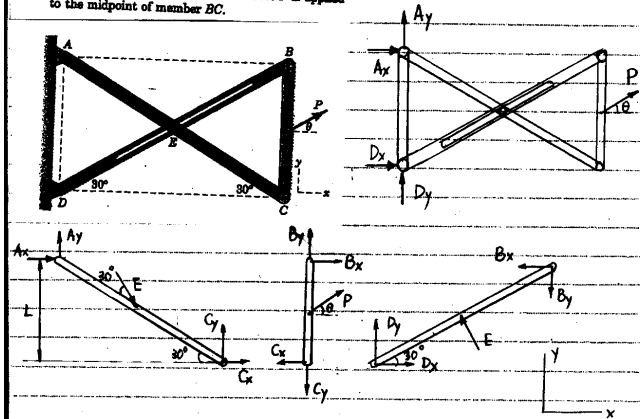
$$\Rightarrow Q = 2700 \text{ N}$$

# 4.91

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4/91 Determine the x- and y-components of all forces acting on each member of the loaded frame for the conditions (a)  $\theta = 0$  and (b)  $\theta = 30^\circ$ . Force P is applied to the midpoint of member BC.

⊙ FBD



⊙ Equilibrium of AC

$$\sum F_x = 0 = A_x + C_x + E \cos 60^\circ \quad (1)$$

$$\sum F_y = 0 = A_y + C_y - E \sin 60^\circ \quad (2)$$

$$\sum M_C = 0 = E [\sin 60^\circ \cdot L \cos 30^\circ - \cos 60^\circ \cdot L \sin 30^\circ] - A_x \cdot L - A_y \cdot 2L \cos 30^\circ \quad (3)$$

⊙ Equilibrium of BC

$$\sum F_x = 0 = B_x - C_x + P \cos \theta \quad (4)$$

$$\sum F_y = 0 = B_y - C_y + P \sin \theta \quad (5)$$

$$\sum M_C = 0 = -B_x \cdot L - P \cos \theta \cdot \frac{L}{2} \quad (6)$$

⊙ Equilibrium of DB

$$\sum F_x = 0 = -B_x + D_x - E \cos 60^\circ \quad (7)$$

$$\sum F_y = 0 = -B_y + D_y + E \sin 60^\circ \quad (8)$$

$$\sum M_D = 0 = B_x \cdot L - B_y \cdot 2L \cos 30^\circ + E [\cos 60^\circ \cdot L \sin 30^\circ + \sin 60^\circ \cdot L \cos 30^\circ] \quad (9)$$

Here we have 9 equations and 9 unknowns ( $A_x, A_y, B_x, B_y, C_x, C_y, D_x, D_y, E$ ). We can solve for these 9 unknowns either by hand or by using computer. (Continued)

5) Solve the 9 equations by hand.

(6)  $\Rightarrow B_x = -\frac{P}{2} \cos \theta$  (10)

Sub (10) into (4)  $\Rightarrow C_x = \frac{P}{2} \cos \theta$  (11)

Sub (10) into (7)  $\Rightarrow D_x = \frac{1}{2} E - \frac{P}{2} \cos \theta$  (12)

Sub (10) into (9)  $\Rightarrow B_y = \frac{1}{\sqrt{3}} [E - \frac{P}{2} \cos \theta]$  (13)

Sub (11) into (1)  $\Rightarrow A_x = -\frac{1}{2} E - \frac{P}{2} \cos \theta$  (14)

Sub (13) into (5)  $\Rightarrow C_y = \frac{1}{\sqrt{3}} [E - \frac{P}{2} \cos \theta] + P \sin \theta$  (15)

Sub (13) into (8)  $\Rightarrow D_y = -\frac{1}{2\sqrt{3}} [E + P \cos \theta]$  (16)

Sub (15) into (2)  $\Rightarrow A_y = \frac{1}{2\sqrt{3}} [E + P \cos \theta] - P \sin \theta$  (17)

Sub (14) & (17) into (3)

$\Rightarrow E [\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2}] - [-\frac{1}{2} E - \frac{P}{2} \cos \theta]$

$-[\frac{1}{2\sqrt{3}} [E + P \cos \theta] - P \sin \theta] \cdot 2 \cdot \frac{\sqrt{3}}{2} = 0$

$\Rightarrow E = -2\sqrt{3} P \sin \theta$  (18)

Sub (18) into (12)  $\Rightarrow D_x = -[\sqrt{3} \sin \theta + \frac{1}{2} \cos \theta] P$

Sub (18) into (13)  $\Rightarrow B_y = -[2 \sin \theta + \frac{\sqrt{3}}{2} \cos \theta] P$

Sub (18) into (14)  $\Rightarrow A_x = [\sqrt{3} \sin \theta - \frac{1}{2} \cos \theta] P$

Sub (18) into (15)  $\Rightarrow C_y = -[\sin \theta + \frac{\sqrt{3}}{2} \cos \theta] P$

Sub (18) into (16)  $\Rightarrow D_y = [\sin \theta - \frac{\sqrt{3}}{2} \cos \theta] P$

Sub (18) into (17)  $\Rightarrow A_y = -[2 \sin \theta - \frac{\sqrt{3}}{2} \cos \theta] P$

Summary

$A_x = [\sqrt{3} \sin \theta - \frac{1}{2} \cos \theta] P$   
 $A_y = -[2 \sin \theta - \frac{\sqrt{3}}{2} \cos \theta] P$   
 $B_x = -\frac{P}{2} \cos \theta$  (a)

$B_y = -[2 \sin \theta + \frac{\sqrt{3}}{2} \cos \theta] P$

$C_x = \frac{P}{2} \cos \theta$

$C_y = -[\sin \theta + \frac{\sqrt{3}}{2} \cos \theta] P$  (Continued)

$D_x = -[\sqrt{3} \sin \theta + \frac{1}{2} \cos \theta] P$   
 $D_y = [\sin \theta - \frac{\sqrt{3}}{2} \cos \theta] P$   
 $E = -2\sqrt{3} P \sin \theta$

Now if  $\theta = 0^\circ$ ,  $\sin \theta = 0$ ,  $\cos \theta = 1$ , then

$A_x = -\frac{1}{2} P = -0.5 P$

$A_y = \frac{\sqrt{3}}{2} P = 0.289 P$

$B_x = -\frac{1}{2} P = -0.5 P$

$B_y = -\frac{\sqrt{3}}{2} P = -0.289 P$

$C_x = \frac{P}{2} = 0.5 P$

$C_y = -\frac{\sqrt{3}}{2} P = -0.289 P$

$D_x = -\frac{1}{2} P = -0.5 P$

$D_y = -\frac{\sqrt{3}}{2} P = -0.289 P$

$E = 0$

(b)

If  $\theta = 30^\circ$ ,  $\sin \theta = \frac{1}{2}$ ,  $\cos \theta = \frac{\sqrt{3}}{2}$ , then

$A_x = \frac{\sqrt{3}}{4} P = 0.433 P$

$A_y = -\frac{3}{4} P = -0.75 P$

$B_x = -\frac{\sqrt{3}}{4} P = -0.433 P$

$B_y = -\frac{5}{4} P = -1.25 P$

$C_x = \frac{\sqrt{3}}{4} P = 0.433 P$

$C_y = -\frac{3}{4} P = -0.75 P$

$D_x = -\frac{3\sqrt{3}}{4} P = -1.299 P$

$D_y = \frac{1}{4} P = 0.25 P$

$E = -\sqrt{3} P = -1.732 P$

(c)

(Continued)

6) Solve the 9 equations again using MATLAB

First, write down eqn (1) - (9) into matrix form:

1	0	0	0	1	0	0	0	$\frac{1}{2}$	$A_x$	0
0	1	0	0	0	1	0	0	$-\frac{\sqrt{3}}{2}$	$A_y$	0
-1	$-\sqrt{3}$	0	0	0	0	0	0	$\frac{1}{2}$	$B_x$	0
0	0	1	0	-1	0	0	0	0	$B_y$	$-P \cos \theta$
0	0	0	1	0	-1	0	0	0	$C_x$	$-P \sin \theta$
0	0	-1	0	0	0	0	0	0	$C_y$	$P \cos \theta / 2$
0	0	-1	0	0	0	1	0	$-\frac{1}{2}$	$D_x$	0
0	0	0	-1	0	0	0	1	$\frac{\sqrt{3}}{2}$	$D_y$	0
0	0	1	$-\sqrt{3}$	0	0	0	0	1	E	0

We can solve this set of linear equations

in two ways: using MATLAB Symbolic Toolbox

to get symbolic solution, or using numerical

calculation to get numerical solution for a given

$\theta$ .

(a) Method 1: Symbolic Calculation

% Meriam and Kraige 4.91;

% Solution by Tian Tang.

% Solve Ax=b using symbolic toolbox;

% See Pratap "Getting Started with Matlab 5" Chapter 8.

% Set up coefficient matrix A;

```
A=[ 1 0 0 0 1 0 0 0 0 1/2
    0 1 0 0 0 1 0 0 0 -sqrt(3)/2
    -1 -sqrt(3) 0 0 0 0 0 0 0 1/2
    0 0 1 0 -1 0 0 0 0 0
    0 0 0 1 0 -1 0 0 0 0
    0 0 -1 0 0 0 0 0 0 0
    0 0 -1 0 0 0 1 0 -1/2
    0 0 0 -1 0 0 0 1 sqrt(3)/2
    0 0 1 -sqrt(3) 0 0 0 0 1];
```

% Set up right hand side b;

syms theta P; % symbolic variables;

b=[0; 0; 0; -cos(theta); -sin(theta); cos(theta)/2; 0; 0; 0; 0]\*P;

% find the solution x=[Ax Ay Bx By Cx Cy Dx Dy E];

x=A\b % solve symbolic equations.

(Continued)

Output of the program on P98 is

```
x =
[ -1/2*P*cos(theta)+3^(1/2)*P*sin(theta)
[ 1/6*3^(1/2)*P*cos(theta)-2*P*sin(theta)
[ -1/2*P*cos(theta)
[ -1/6*3^(1/2)*P*cos(theta)-2*P*sin(theta)
[ 1/2*P*cos(theta)
[ -1/6*3^(1/2)*P*cos(theta)-P*sin(theta)
[ -3^(1/2)*P*sin(theta)-1/2*P*cos(theta)
[ -1/6*3^(1/2)*P*cos(theta)+P*sin(theta)
[ -2*3^(1/2)*P*sin(theta)
```

i. e.

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \cos \theta + \sqrt{3} \sin \theta \\ \frac{\sqrt{3}}{6} \cos \theta - 2 \sin \theta \\ -\frac{1}{2} \cos \theta \\ -\frac{\sqrt{3}}{6} \cos \theta - 2 \sin \theta \\ \frac{1}{2} \cos \theta \\ -\frac{\sqrt{3}}{6} \cos \theta - \sin \theta \\ -\frac{1}{2} \cos \theta - \sqrt{3} \sin \theta \\ -\frac{\sqrt{3}}{6} \cos \theta + \sin \theta \\ -2\sqrt{3} \sin \theta \end{bmatrix} P$$

If  $\theta = 0^\circ$  we'll get again

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{6} \\ -\frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ \frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ -\frac{1}{2} \\ -\frac{\sqrt{3}}{6} \\ 0 \end{bmatrix} P = \begin{bmatrix} -0.5 \\ 0.289 \\ -0.5 \\ -0.289 \\ 0.5 \\ -0.289 \\ -0.5 \\ -0.289 \\ 0 \end{bmatrix} P$$

If  $\theta = 30^\circ$

$$\begin{bmatrix} A_x \\ A_y \\ B_x \\ B_y \\ C_x \\ C_y \\ D_x \\ D_y \\ E \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{4} \\ -\frac{3}{4} \\ -\frac{\sqrt{3}}{4} \\ -\frac{5}{4} \\ \frac{\sqrt{3}}{4} \\ -\frac{3}{4} \\ -\frac{3\sqrt{3}}{4} \\ \frac{1}{4} \\ -\sqrt{3} \end{bmatrix} P = \begin{bmatrix} 0.433 \\ -0.75 \\ -0.433 \\ -1.25 \\ 0.433 \\ -0.75 \\ -1.299 \\ 0.25 \\ -1.732 \end{bmatrix} P$$

(Continued)

(b) Method 2: Numerical Calculation

```
% Meriam and Kraige 4.91;
% Solution by Tian Tang.
% Solve Ax=b using numerical calculation, not symbolic calculation.

% Set up coefficient matrix A;
A=[ 1 0 0 0 1 0 0 0 0 1/2
0 1 0 0 0 1 0 0 0 -sqrt(3)/2
-1 -sqrt(3) 0 0 0 0 0 0 0 1/2
0 0 1 0 -1 0 0 0 0 0
0 0 0 1 0 -1 0 0 0 0
0 0 -1 0 0 0 0 0 0 0
0 0 -1 0 0 0 1 0 0 -1/2
0 0 0 -1 0 0 0 0 1 sqrt(3)/2
0 0 1 -sqrt(3) 0 0 0 0 0 1 ];
```

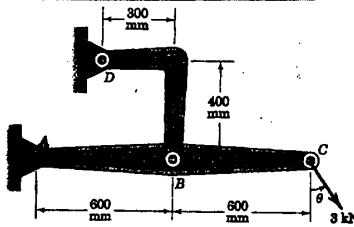
```
% Set up right hand side b;
% Set P=1, and the solution to the problem should be
% P times the solution obtained from this program;
P=1;
% Given theta;
theta=0;
b=[ 0; 0; 0; -cos(theta); -sin(theta); cos(theta)/2; 0; 0; 0]*P;

% find the solution x=[Ax Ay Bx By Cx Cy Dx Dy E];
x=A\b;
format short; % use format short to get x with less digits;
x % show x on the screen.
```

Output of the program above

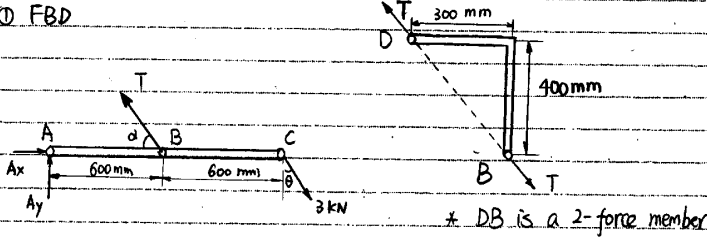
$\theta = 0$	$\theta = \frac{\pi}{6}$
x =	x =
-0.5000	0.4330
0.2887	-0.7500
-0.5000	-0.4330
-0.2887	-1.2500
0.5000	0.4330
-0.2887	-0.7500
-0.5000	-1.2990
-0.2887	0.2500
0	-1.7321

The structural members support the 3-kN load which may be applied at any angle  $\theta$  from essentially  $-90^\circ$  to  $+90^\circ$ . The pin at A must be designed to support the maximum force transmitted to it. Plot the force  $F_A$  at A as a function of  $\theta$  and determine its maximum value and the corresponding angle  $\theta$ .  
 Ans.  $F_{A_{max}} = 6 \text{ kN}$  at  $\theta = -26.6^\circ$



Problem 4/143

(10) FBD



$$\tan \alpha = \frac{400}{300} = \frac{4}{3} \quad \cos \alpha = \frac{3}{5} \quad \sin \alpha = \frac{4}{5}$$

$$\textcircled{2} \sum M/A = 0 = T \cdot \frac{4}{5} \cdot 600 \text{ mm} - 3 \text{ kN} \cdot \cos \theta \cdot 1200 \text{ mm}$$

$$\Rightarrow T = \frac{15}{2} \cos \theta \text{ kN}$$

$$\sum F_x = 0 = A_x + 3 \text{ kN} \cdot \sin \theta - T \cdot \frac{3}{5}$$

$$\Rightarrow A_x = \frac{9}{2} \cos \theta - 3 \sin \theta \text{ kN}$$

$$\sum F_y = 0 = A_y + T \cdot \frac{4}{5} - 3 \text{ kN} \cdot \cos \theta$$

$$\Rightarrow A_y = -3 \cos \theta \text{ kN}$$

$$\textcircled{3} F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{\left[\left(\frac{9}{2} \cos \theta - 3 \sin \theta\right) \text{ kN}\right]^2 + \left(-3 \cos \theta \text{ kN}\right)^2}$$

$$= \frac{3}{2} \sqrt{9 \cos^2 \theta - 12 \cos \theta \sin \theta + 4} \text{ kN} \quad -90^\circ \leq \theta \leq 90^\circ$$

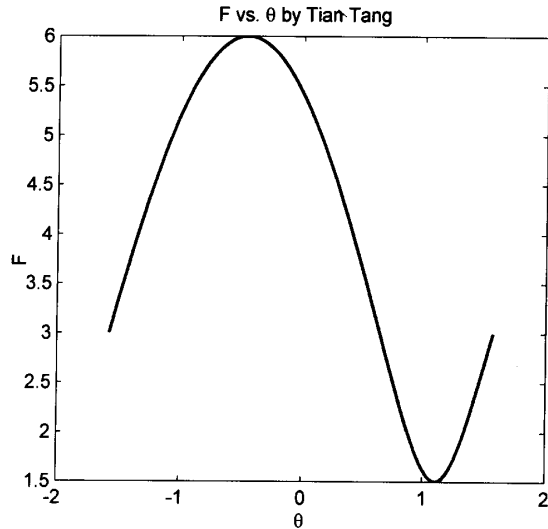
④ Now we can use MATLAB to plot  $F_A$  vs.  $\theta$  and find out the maximum value of  $F_A$  and the corresponding  $\theta$ .

(Continued)

% Meriam and Kraige 4.143;  
 % Solution by Tian Tang.  
 % Plot F as a function of theta;  
 % Find the maximum value of F and corresponding theta.

```
theta=linspace(-pi/2,pi/2,100); % define the variable theta; (100 points)
F=3/2*(9*cos(theta).^2-12*cos(theta).*sin(theta)+4).^(1/2); % find corresponding F;
plot(theta,F); % plot F vs.theta;
xlabel('\theta'); % label x axis with theta;
ylabel('F'); % label y axis with F;
[y,i]=max(F); % find maximum value of F and corresponding index of theta;
theta0=theta(i) % find corresponding theta;
theta0=theta0*180/pi % find theta which has unit of degrees;
Fmax=y % maximum value of F.
```

Plot of F vs.  $\theta$



Output of Fmax and corresponding  $\theta$

```
theta0 =
    -0.4601
theta0 =
   -26.3636
Fmax =
    6.0000
```