Solutions

Your Name: Tian, Peeyush and Vijay

Section day & time: ____________

TA name & section #: ____________

ENGRD 202 Final Exam

Wednesday May 14, 2003, 9:00 AM — 11:30 AM

This version last edited May 11, 2003.

9 ~~10~~ problems, 120 ~~130~~ points, and 150 minutes (no over time).

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed besides the one-sided formula sheet which is being passed out with this exam. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it.

b) Full credit if

- →free body diagrams← are drawn whenever force or moment balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well-defined with sketches and/or words;
- ⊣ reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems poorly defined;
- work is: I) Neat, II) Clear, and III) Well organized.
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "R_{Ax} = 18" instead of, say, "RAX = 18".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/10
Problem	2:	/10
Problem	3:	/10
~~Problem~~	~~4:~~	~~/10~~
Problem	5:	/10
Problem	6:	/10
Problem	7:	/20
Problem	8:	/20
Problem	9:	/~~20~~ 10
Problem	10:	/~~10~~ 20
TOTAL:		/~~130~~ 120

1) **(10 pts)** Give approximate values for the quantities below. Give units (any common units you like may be used) as part of your answer. No justification is needed. Your answer will get full credit if it is "in the ball park" (if the range of real materials have the property A roughly ranging as $A_1 < A < A_2$ your answer will count as correct if it is in the bigger range $A_1^{3/2}/A_2^{1/2} < A < A_2^{3/2}/A_1^{1/2}$). 7 points for a correct answer to any one of the questions below, 8 for two, 9 for three, and 10 for 4.

a) Young's Modulus E of the best steel used in high-tech bicycles or airplanes:

a) Fancy steel $E =$ 200 - 220 GPa
$\approx 3 \times 10^7$ psi

b) Yield stress σ_y of that fancy steel:

b) Fancy steel $\sigma_y =$ 650 - 2000 MPa
or 100,000 - 300,000 psi

c) E of steel used in paper clips:

c) cruddy steel $E =$ 200 - 220 GPa
$\approx 3 \times 10^7$ psi

1 ksi = 6.89 MPa
www.matweb.com

d) σ_y of paper-clip steel:

d) cruddy steel $\sigma_y =$ 135 - 270 MPa
20,000 - 40,000 psi

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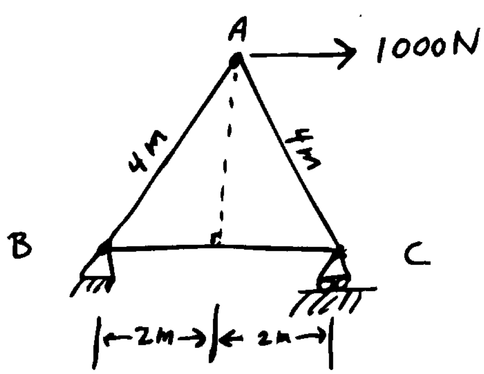
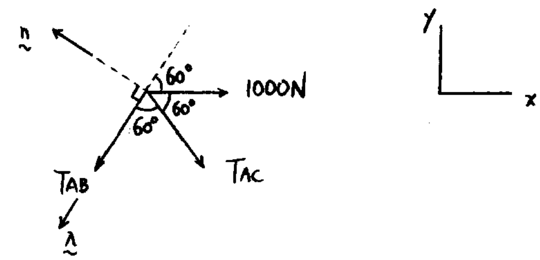
2) **(10 pts)** The triangular truss shown is loaded by the single force shown. What is the tension in bar AC?



① FBD of joint A:



② Force Balance:

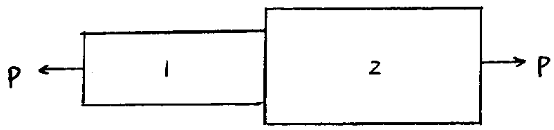
$$\Sigma \underset{\sim}{F} = \underset{\sim}{0} = (1000\,N)\left[(-\cos 60^\circ)\underset{\sim}{\lambda} - (\sin 60^\circ)\underset{\sim}{n}\right] + T_{AC}\left[(\cos 60^\circ)\underset{\sim}{\lambda} - (\sin 60^\circ)\underset{\sim}{n}\right] + T_{AB}\,\underset{\sim}{\lambda} \qquad (*)$$

$$\{*\} \cdot \underset{\sim}{n} \Rightarrow \quad 0 = -(1000\,N)\sin 60^\circ - T_{AC}\sin 60^\circ$$

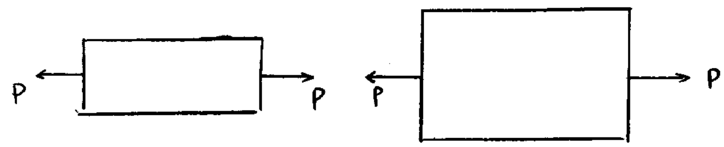
$$\Rightarrow \boxed{T_{AC} = -1000\,N}$$

$T_{AC} =$ - 1000 N

3) **(10 pts)** Two uniform elastic bars are welded end to end to make a longer bar. A tension P acts on the extreme right and left ends of that longer bar. The bars have given geometry and properties: $\ell_1, A_1, J_1, I_1, G_1, \nu_1, E_1$ and $\ell_2, A_2, J_2, I_2, G_2, \nu_2, E_2$. What is the change of length of that longer bar?



① FBD



② $\Delta \ell_1 = \varepsilon_1 \ell_1 = \dfrac{\sigma_1}{E_1} \ell_1 = \dfrac{(P/A_1)}{E_1} \ell_1 = \dfrac{P\ell_1}{E_1 A_1}$

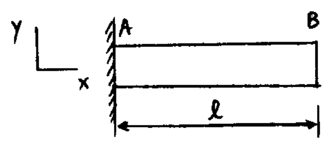
$\Delta \ell_2 = \varepsilon_2 \ell_2 = \dfrac{\sigma_2}{E_2} \ell_2 = \dfrac{(P/A_2)}{E_2} \ell_2 = \dfrac{P\ell_2}{E_2 A_2}$

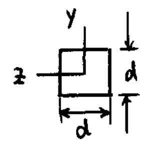
③ $\delta = \Delta \ell_1 + \Delta \ell_2 = \boxed{P\left[\dfrac{\ell_1}{E_1 A_1} + \dfrac{\ell_2}{E_2 A_2}\right]}$

$\delta = P\left[\dfrac{\ell_1}{E_1 A_1} + \dfrac{\ell_2}{E_2 A_2}\right]$

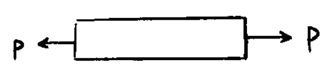
5) **(10 pts)** A square-cross-section long narrow bar is clamped (welded, built-in) at one end. It can be loaded in tension or bending with the same load P at the other end. Square side is d, length of bar is ℓ.

a) Which is larger, the maximum tension stress from tension or from bending? Explain using equations.





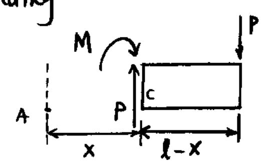
① tension



$$\sigma_{max} = \sigma_x = \frac{P}{A} = \frac{P}{d^2}$$

② bending

FBD:



$$\Sigma M_{/c} = 0 = M + P(\ell - x)$$

$$\Rightarrow M = P(x - \ell)$$

$\Rightarrow |M_{max}|$ occurs at A and $|M_{max}| = P\ell$

$$\sigma = -\frac{My}{I}$$

$$\Rightarrow \sigma_{max} = \frac{|M_{max}|\,|y_{max}|}{I} = \frac{(P\ell)(\frac{d}{2})}{\frac{d^4}{12}} = \frac{6P\ell}{d^3}$$

③ $$\frac{\sigma_{max}^{bending}}{\sigma_{max}^{tension}} = \frac{\frac{6P\ell}{d^3}}{\frac{P}{d^2}} = \frac{6\ell}{d} \gg 1$$

because the bar is long and narrow ($\ell \gg d$).

So the maximum tension stress in bending is much larger.

b) In bending, which is larger the maximum tension stress or the average shear stress on a section orthogonal to the axis of the beam? Explain using equations.

① maximum tension stress has already be obtained in (5a),

$$\sigma_{max} = \frac{6P\ell}{d^3}$$

② average shear stress

shear force V is everywhere equal to P (according to FBD in (a))

$$\Rightarrow \tau_{avg} = \frac{V}{A} = \frac{P}{d^2} = \sigma_{max}^{tension}$$

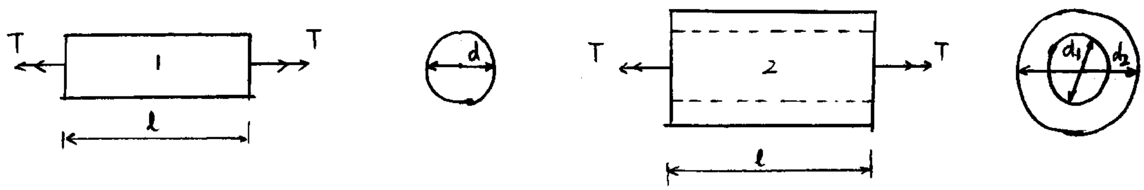
③ So again, in bending,

$$\frac{\sigma_{max}}{\tau_{avg}} = \frac{\frac{6P\ell}{d^3}}{\frac{P}{d^2}} = \frac{6\ell}{d} \gg 1 \quad \text{because } \ell \gg d$$

i.e. the maximum tension stress is much larger than the average shear stress.

6) **(10 pts)** Two round shafts are made of the same elastic material and have the same length and weight. One is hollow (and thus has a bigger outer diameter). The same torque is applied to both of them.

a) Which has a bigger twist. Explain with equations.



$$\phi_1 = \frac{Tl}{G I_p^1} \qquad \phi_2 = \frac{Tl}{G I_p^2} \qquad \text{same mass}: \quad \frac{\pi(d_2^2 - d_1^2)}{4} = \frac{\pi d^2}{4}$$

We can actually immediately see that $I_p^2 > I_p^1$ because the areas are distributed further from the centroid on section of 2 than on section of 1. A detailed calculation is:

$$I_p^2 = \frac{\pi(d_2^4 - d_1^4)}{32} = \frac{\pi(d_2^2 - d_1^2)(d_2^2 + d_1^2)}{32} = \left(\frac{\pi d^2}{32}\right)(d_2^2 + d_1^2)$$

$$> \left(\frac{\pi d^2}{32}\right) d^2 = \frac{\pi d^4}{32} = I_p^1 \qquad \Rightarrow \qquad \boxed{\phi_1 > \phi_2}$$

<u>A numerical example</u>: $d = 3\text{ cm}$, $d_1 = 4\text{ cm}$, $d_2 = 5\text{ cm}$ satisfying $d_2^2 - d_1^2 = d^2$

$$I_p^1 = \frac{\pi d^4}{32} = \frac{\pi(3\text{cm})^4}{32} = 7.95\text{ cm}^4 \Rightarrow \phi_1 = (Tl/G)/(7.95\text{ cm}^4) = (0.126\text{ cm}^{-4})(Tl/G)$$

$$I_p^2 = \frac{\pi(d_2^4 - d_1^4)}{32} = \frac{\pi[(5\text{cm})^4 - (4\text{cm})^4]}{32} = 36.23\text{ cm}^4 \Rightarrow \phi_2 = (Tl/G)/(36.23\text{ cm}^4) = (0.028\text{cm}^{-4})(Tl/G)$$

Obviously $\phi_1 > \phi_2$.

b) Which has a bigger maximum shear stress. Explain with equations.

$$\tau(\rho) = \frac{T\rho}{I_p}$$

$$\Rightarrow \tau_{max}^1 = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi d^4}{32}} = \frac{16T}{\pi}\left(\frac{1}{d^3}\right)$$

$$\tau_{max}^2 = \frac{T\left(\frac{d_2}{2}\right)}{\frac{\pi(d_2^4 - d_1^4)}{32}} = \frac{16T}{\pi}\left(\frac{d_2}{(d_2^2 - d_1^2)(d_2^2 + d_1^2)}\right)$$ Recall in (6a) same mass $\Rightarrow d_2^2 - d_1^2 = d^2$

$$\Rightarrow \tau_{max}^2 = \frac{16T}{\pi}\left(\frac{d_2}{d^2(d_2^2 + d_1^2)}\right) < \frac{16T}{\pi}\left(\frac{d_2}{d^2 d_2^2}\right)$$

$$= \frac{16T}{\pi}\left(\frac{1}{d^2 d_2}\right) < \frac{16T}{\pi}\left(\frac{1}{d^3}\right) \qquad \Rightarrow \qquad \boxed{\tau_{max}^2 < \tau_{max}^1}$$

<u>Same numerical example</u>: $\tau_{max}^1 = \frac{16T}{\pi}\left(\frac{1}{d^3}\right) = \frac{16T}{\pi}\left(\frac{1}{3\text{cm}}\right)^3 = (0.037\text{ cm}^{-3})\left(\frac{16T}{\pi}\right)$

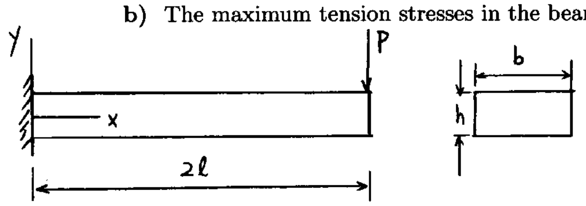
$$\tau_{max}^2 = \frac{16T}{\pi}\left(\frac{d_2}{d_2^4 - d_1^4}\right) = \frac{16T}{\pi}\left(\frac{5\text{ cm}}{(5\text{cm})^4 - (4\text{cm})^4}\right) = (0.0136\text{ cm}^{-3})\left(\frac{16T}{\pi}\right) < \tau_{max}^1$$

<u>Note</u>: The increase in τ_{max} due to the increase in radius for the hollow tube is more than made up for by the decrease from the increased polar moment of inertia.

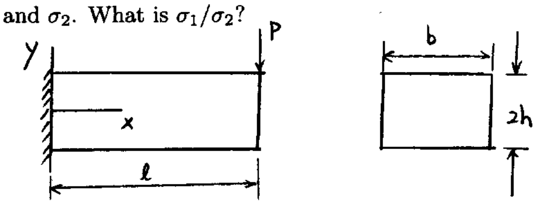
7) (20 pts) Two rectangular-cross-section cantilever beams (clamped at one end, load at the other) are made of the same elastic material and have the same volume. Beam 1 is twice as long as beam 2; beam 2 is twice as high as beam 1. They have the same width. (As always, more than minimal credit depends on full justification even if you somehow know the numerical answer.)

a) The deflections of the ends are δ_1 and δ_2. What is δ_1/δ_2?

b) The maximum tension stresses in the beams are σ_1 and σ_2. What is σ_1/σ_2?



beam 1



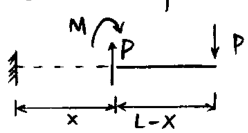
beam 2

a). For a cantilever beam  with length L height H, width B

we can find the deflection of the beam by solving the 2nd order ODE:

$$\frac{d^2v}{dx^2} = \frac{M(x)}{EI}$$

$M(x)$ can be found from FBD:



$$\Rightarrow \quad M(x) = -P(L-x)$$

$$\Rightarrow \frac{d^2v}{dx^2} = -\frac{P}{EI}(L-x) \Rightarrow v' = -\frac{P}{EI}\left(Lx - \frac{x^2}{2}\right) + C_1 \Rightarrow v = -\frac{P}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right) + C_1x + C_2$$

B.C. $v(0) = 0 \Rightarrow C_2 = 0$

$v'(0) = 0 \Rightarrow C_1 = 0$

$$\Rightarrow v = -\frac{P}{EI}\left(L\frac{x^2}{2} - \frac{x^3}{6}\right)$$

$$\Rightarrow \text{deflection at the end: } \delta = |v(x=L)| = \frac{PL^3}{3EI} = \frac{PL^3}{3E\cdot\frac{BH^3}{12}} = \frac{4PL^3}{EBH^3}$$

So, for beam 1: $L = 2l$, $H = h$, $B = b$ $\Rightarrow \delta_1 = \frac{4P(2l)^3}{Ebh^3} = \frac{32Pl^3}{Ebh^3}$

for beam 2: $L = l$, $H = 2h$, $B = b$ $\Rightarrow \delta_2 = \frac{4Pl^3}{Eb(2h)^3} = \frac{Pl^3}{2Ebh^3}$

$$\Rightarrow \boxed{\frac{\delta_1}{\delta_2} = 64}$$

As handed to you on a silver platter in lecture

b). For a cantilever beam, M_{max} occurs at the fixed end, and $|M_{max}| = PL$.

$$\Rightarrow \sigma_{max} = \frac{|M_{max}||y_{max}|}{I} = \frac{PL\frac{H}{2}}{\frac{BH^3}{12}} = \frac{6PL}{BH^2}$$

$$\Rightarrow \sigma^1_{max} = \frac{6P(2l)}{bh^2} = \frac{12Pl}{bh^2}$$

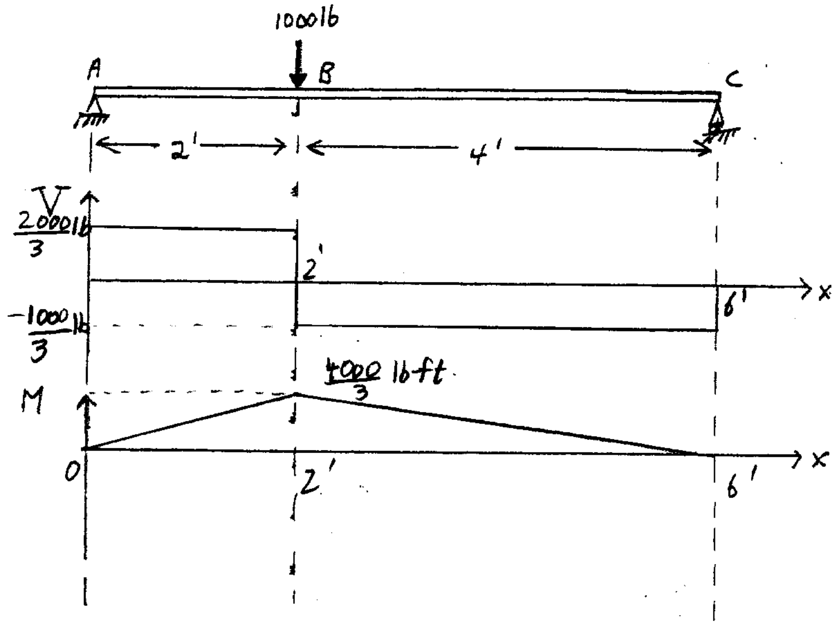
$$\sigma^2_{max} = \frac{6Pl}{b(2h)^2} = \frac{3Pl}{2bh^2}$$

$$\Rightarrow \boxed{\frac{\sigma^1_{max}}{\sigma^2_{max}} = 8}$$

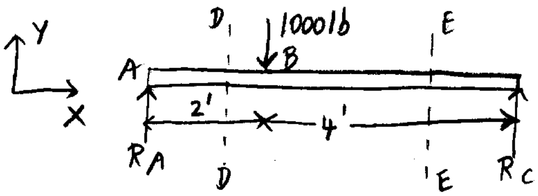
a) $\delta_1/\delta_2 =$ 64

b) $\sigma_1/\sigma_2 =$ 8

8) (20 pts) Draw shear force and bending moment diagrams for the following beam. Clearly label the values (with units) at the ends and at any discontinuities or local maxima or minima.



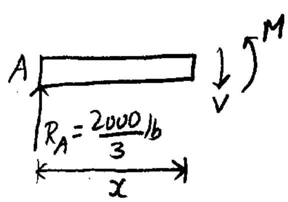
FBD of entire beam:



$$\Sigma M_{/A} = 0 = R_c(6') - (1000\,lb)(2') \Rightarrow R_c = \frac{1000}{3}\,lb$$

$$\Sigma F_y = 0 = R_A + R_c - 1000\,lb \Rightarrow R_A = \frac{2000}{3}\,lb$$

Section D-D: $0 < x < 2'$

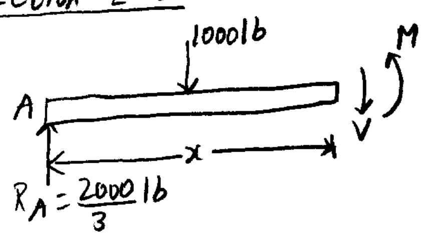


$$\Sigma F_y = 0 = \frac{2000}{3}\,lb - V \Rightarrow V = \frac{2000}{3}\,lb$$

$$\Sigma M_A = 0 = \frac{2000}{3}\,lb \cdot x - M \Rightarrow M = \frac{2000}{3}\,lb \cdot x$$

$$M(x=2') = \frac{4000}{3}\,lb\text{-}ft$$

Section E-E: $2' < x < 6'$



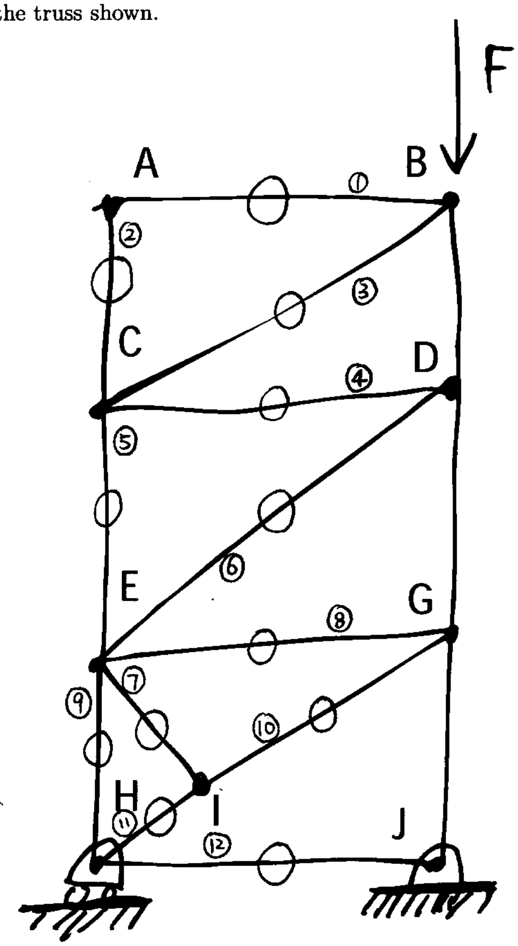
$$\Sigma F_y = 0 = \frac{2000}{3}\,lb - 1000 - V \Rightarrow V = \frac{-1000}{3}\,lb$$

$$\Sigma M_A = 0 = (1000\,lb)(2') - M - \left(\frac{1000}{3}\,lb\right)x = 0$$

$$\Rightarrow M = 2000\,lb\text{-}ft - \frac{1000x}{3}$$

$$M(x=2') = \frac{4000}{3}\,lb\text{-}ft$$

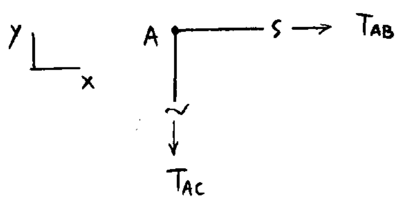
9) **(10 pts)** Mark with a zero on the bar all the zero-force members in the truss shown.



① See the figure on the right.
Small numbers show the order of work

② Example of calculations:

- FBD of joint A:

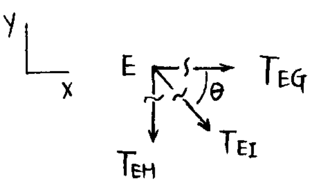


$\Sigma F_x = 0 = T_{AB}$

$\Sigma F_y = 0 = -T_{AC}$

- FBD of joint E:

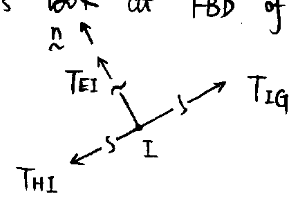
before drawing the FBD of E, we've known $T_{CE} = 0$ from equilibrium of C, and $T_{DE} = 0$ from equilibrium of D. so



$\Sigma F_x = 0 = T_{EG} + T_{EI}\cos\theta$ (1)

$\Sigma F_y = 0 = -T_{EH} - T_{EI}\sin\theta$ (2)

But we have 3 unknowns, so we have to figure out 1 of them beforehand. So let's look at FBD of joint I:



$\Sigma F_n = 0 = T_{EI}$.

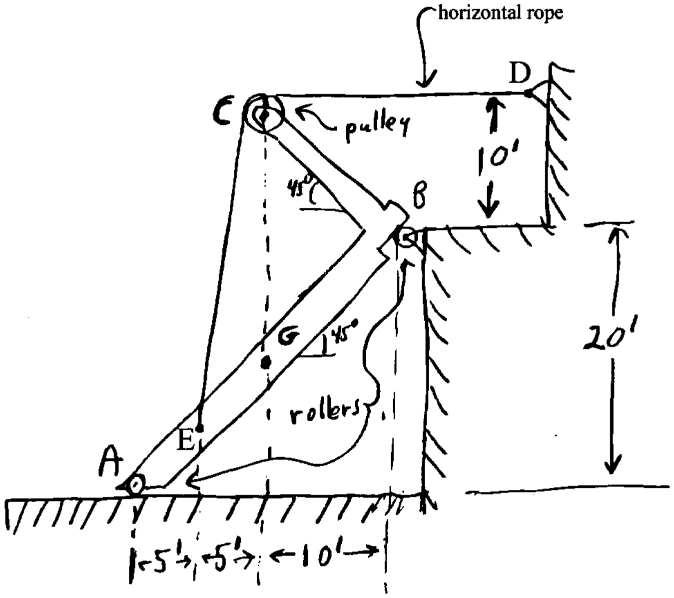
Now we sub $T_{EI} = 0$ into (1) & (2)

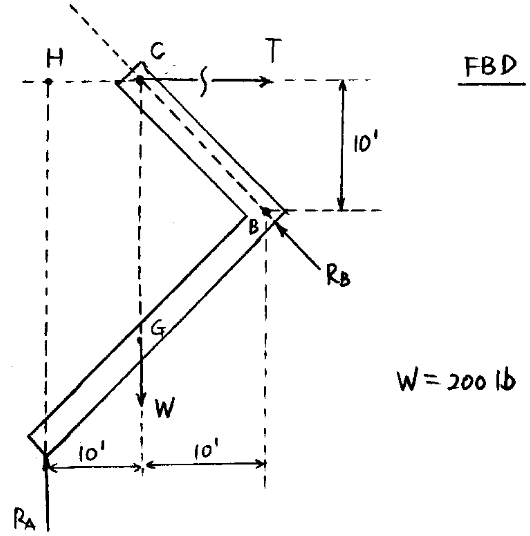
$\Rightarrow$ $T_{EG} = 0$

$T_{EH} = 0$.

Though the load is only carried by the non-zero-force members (BD, DG, GJ) for this loading, the stability of the structure depends on them, as does the ability to withstand load even slightly different than that shown

10) (20 pts) The center of mass of 200 pound structure AEGBC is at G. It is held by rollers at A and B as well as with the rope which starts at E, wraps around the pulley at C, and ends at D. Find the force of the ground on the structure at A and the tension in the rope. Define any base vectors you need.





$\circlearrowleft\oplus \quad \Sigma M_{/C} = 0 = -R_A\,(10') \quad \Rightarrow \quad R_A = 0.$

$\circlearrowleft\oplus \quad \Sigma M_{/B} = 0 = W\,(10') - T\,(10') \quad \Rightarrow \quad T = W = 200 \text{ lb}$

$\underline{\mathbf{F}}_A =$	$\underline{0}$
$T =$	200 lb