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ENGRD 202 Quiz 6 "SOLUTIONS"

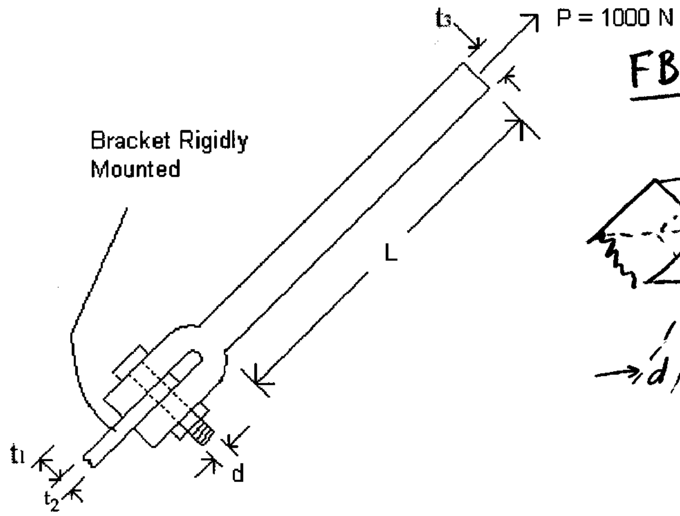
Section day & time: __________

April 18, 2003

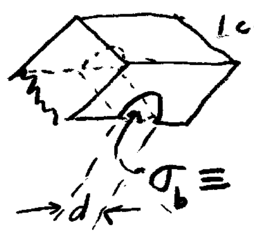
TA name & section #: __________

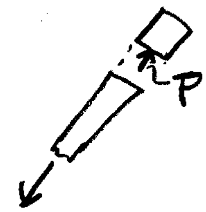
11) (7 pts) A member with a rectangular cross section has a tensile force P applied at its end. It is anchored to a rigid bracket, made of the same material, via a bolt. The bracket slides easily in the gap. Find

a) The bearing stress σ_b on the bracket from the bolt.
b) The average shear stress τ in the bolt on one side of the bracket.
c) Neglecting any stress concentrations, find the elongation δ in the member (length L) due to P.



FBD of bracket





$$\sigma_b \equiv \frac{\text{Load}}{\text{Projected area}}$$

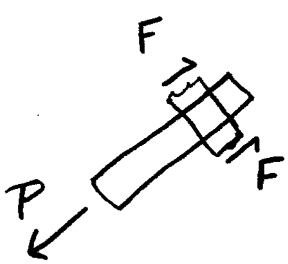
$$\sigma_b = \frac{P}{t_2 \cdot d} = \frac{1000\,N}{1\,cm \cdot 1\,cm} = \frac{1000\,N}{cm^2}$$

$$\sigma_b = 10^7\,N/m^2 = 5 \cdot 10^6\,Pa$$

$L = 1$ m, $d = 1$ cm, $t_1 = 2$ cm, $t_2 =$ bracket thickness$= 1$ cm, $t_3 = 3$ cm.
The depth into page is 2 cm for the bracket and the other part. $= w$
$E_{bracket} = E_{member} = 2.0 \times 10^6$ Pa; $E_{bolt} = 1.0 \times 10^6$ Pa

very compliant material!

FBD of bracket & cut bolt



Symmetry $\Rightarrow$

$$F = P/2$$

$$\tau = \frac{F}{A} = \frac{P}{2A} = \frac{P}{2\,\pi d^2/4}$$

$$\tau = \frac{2 \cdot 1000\,N}{\pi\,(10^{-2}\,m)^2}$$

$$\tau = \frac{2}{\pi}\,10^7\,\frac{N}{m^2} = \frac{2}{\pi}\,10^7\,Pa$$

$$\delta = \frac{PL}{AE}$$

$$= \frac{(1000\,N)(1\,m)}{(2\,cm)(3\,cm) \cdot 2 \cdot 10^6\,N/m^2}$$

$$= \frac{10^7}{12 \cdot 10^6}\,m$$

$$\boxed{\delta = \frac{5}{6}\,m}$$

[Violates small strain theory because of wildly compliant material]

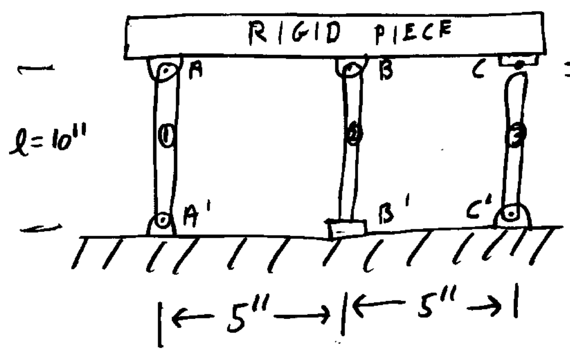
$\sigma_b = 10^7$ Pa

$\tau = \frac{2}{\pi}\,10^7$ Pa

$\delta = \frac{5}{6}$ m huge!

12) (10 pts) Three identical parallel posts are equally spaced. Two are connected to an initially-horizontal massless rigid beam. The weld at B' keeps the structure from collapsing. The third bar CC' misses being connected by the gap $\delta_g = 0.005$ inches which closes when that bar (and only that bar) is sufficiently heated. The rigid beam is then lifted by the expanding CC', and also tilts. After bar CC' is heated, what surface (what angle cut) in which vertical bar has the maximum shear stress? What is that shear stress τ?
[You get full credit for a correct answer that has all letters or, alternatively (your choice) for one with all numbers and units. Make the usual small-strain, small-slope, linear elastic assumptions (no need to state them), and don't account for stress concentrations at the tops and bottoms of the bars.]

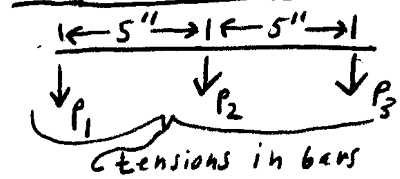
[Eqs. ①-⑤ are the keys. The rest is algebra/arithmetic.]



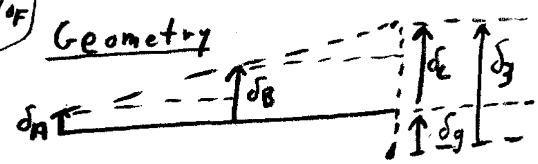
before ΔT: $\delta_g = 0.005''$

$E = 3\times 10^7\ lb/in^2$
$G = 1.2\times 10^7\ lb/in^2$
$\nu = .3,\ \alpha = 5\cdot 10^{-6}/°F$
$A = 2\ in^2$
$l = 10\ in$
$\Delta T = 200°F$

FBD of rigid piece



Geometry



Similar Δs $\Rightarrow$ $\delta_B - \delta_A \overset{①}{=} \delta_c - \delta_B$; $\delta_B = \delta_2$, $\delta_A = \delta_1$, ② $\delta_c = \delta_3 - \delta_g$, $\delta_B = \delta_2$ } Geometry

Mechanics ③: $\Sigma M_{/B} = 0$, $\Sigma F = 0$ } $\Rightarrow P_1 = P_3 = -\frac{P_2}{2}$

$\Rightarrow\ 2\delta_2 - \delta_1 - \delta_3 = -\delta_g$

Mat. Props $\Rightarrow$ ④: $\delta_2 = \frac{P_2 l}{AE}$, $\delta_1 = P_1 l/AE$ with $P_1 = -P_2/2$, $\delta_3 = \frac{P_3 l}{AE} + \alpha(\Delta T)l$ with $P_3 = -P_2/2$

$$\Rightarrow\ \frac{P_2 l}{AE}\left[2 - (-1/2) - (-1/2)\right] = \alpha l(\Delta T) - \delta_g$$

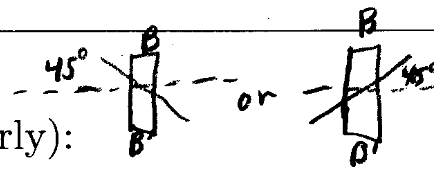
$$\Rightarrow\ \boxed{P_2 = \frac{AE}{3}\left(\alpha(\Delta T) - \delta_g/l\right)} \Rightarrow P_1 = P_3 = \frac{AE}{6}\left[-\alpha(\Delta T) + \delta_g/l\right] = -5000\,lb$$

$$= \frac{(2\ in^2)(3\cdot 10^7\ lb/in^2)}{3}\left((5\cdot 10^{-6}/°F)(200°F) - \left(\frac{0.005\ in}{10\ in}\right)\right)$$

$$= (2\cdot 10^7)\cdot(5\cdot 10^{-4})\ lb = \boxed{10^4\ lb = P_2}$$

$$\tau_{max} \overset{⑤}{=} \frac{|P_{max}|}{2A} = \frac{P_2}{2A} = \frac{10^4\ lb}{2\cdot 2\ in^2} = 2500\ lb/in^2$$

a) bar and surface (describe or sketch clearly):



b) $\tau_{max} = \frac{E}{6}\left[\alpha(\Delta T) - \delta_g/l\right]$

$= 2500\ lb/in^2$