Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

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Cornell TAM/ENGRD 2030

Prelim 1

March 1, 2011

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifiable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems ***poorly defined***, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem. Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 1: ____ /25

Problem 2: ____ /25

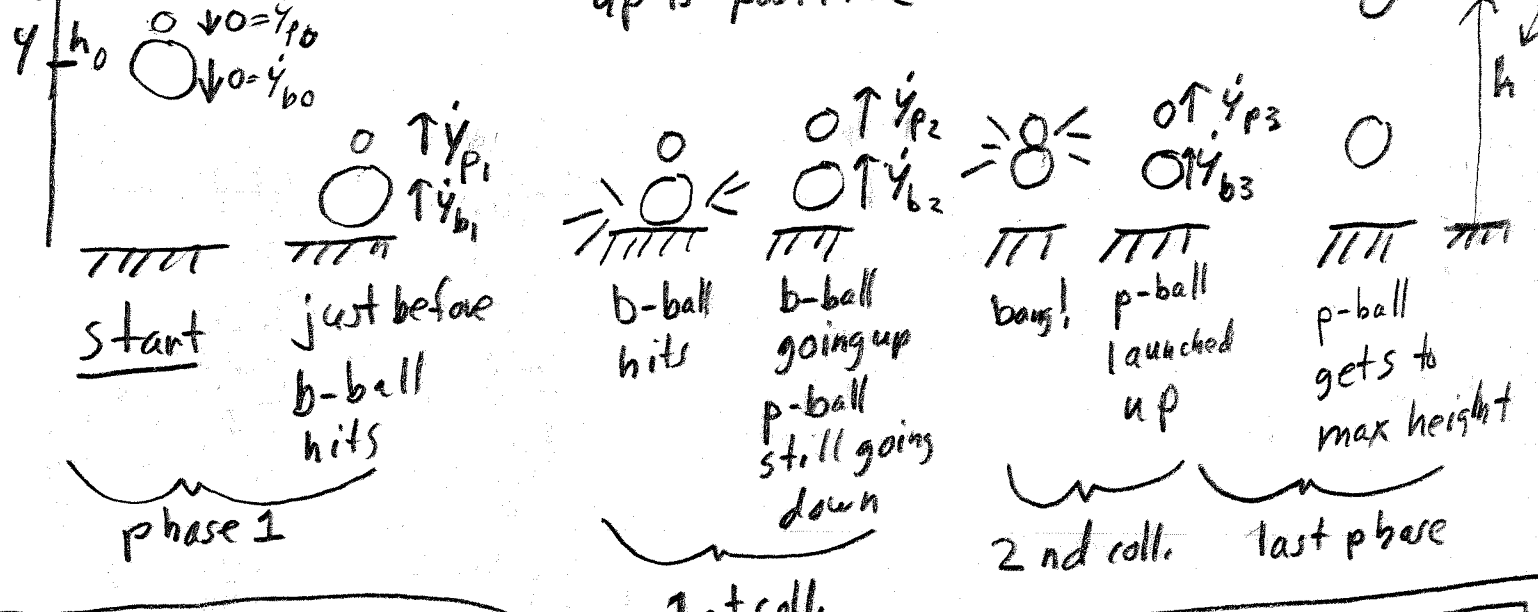
Problem 3: ____ /25

1a) A basketball M is dropped from height h_0 onto a hard surface. Falling just directly above it is a small ping-pong ball m. The Basketball hits the surface, bounces up and immediately hits the ping-pong ball which bounces up off it. Assume $M \gg m$. Assume $e = 1$ for both collisions. Neglect the size of the balls in relation to h ($h \gg$ ball radius). Neglect air friction. How high does the ping-pong ball bounce? of interest

b) Now assume that both collisions have coefficient of restitution e but $e \neq 1$. For what what value of e is the height of the ping-pong ball flight = h_0?

c) The answer to (b) above is a famous number. For one bonus point, do you know anything interesting about that number?

up is positive



Phase 1 FBDs

Mg $\quad mg$

Cons. of En. $\Rightarrow Mgh_0 = \frac{1}{2} M \dot{y}_{b1}^2 \Rightarrow \dot{y}_{b1} = -\sqrt{2gh_0}$

Likewise, $\dot{y}_{p1} = -\sqrt{2gh_0}$ going down

1st Coll $\quad \dot{y}_{b2} = -e\,\dot{y}_{b1}, \quad \dot{y}_{p2} = \dot{y}_{p1}$

2nd Coll. $\quad$ Cons. of mom. $\Rightarrow \cancel{M}\dot{y}_{b2} = \cancel{M}\dot{y}_{b3}$ (because $M \gg m$)

Coll. eqn. $\quad \dot{y}_{b3} - \dot{y}_{p3} = -e(\dot{y}_{b2} - \dot{y}_{p2}) \quad$ 1

$\dot{y}_{p2} = \dot{y}_{p1} = \dot{y}_{b1}$

$\dot{y}_{b2} = -e\,\dot{y}_{b1}$

$\dot{y}_{b3} = \dot{y}_{b2} = -e\,\dot{y}_{b1}$

1a) cont'd

① is one eqn for $\dot{y}_{p3}$, solving

$$\dot{y}_{p3} = -e\dot{y}_{b1} + e(-e\dot{y}_{b1} - \dot{y}_{b1})$$
$$\dot{y}_{p3} = -(e^2+2e)\dot{y}_{b1} \quad ②$$

Take $e=1$, $\dot{y}_{p3} = -3\dot{y}_{b1} = -3(-\sqrt{2gh_0}) = 3\sqrt{2gh_0}$

Last phase

Cons. of energy $\cancel{m}gh = \frac{1}{2}\cancel{m}V_{p3}^2$

$$gh = \frac{1}{2}\left(3\sqrt{2gh_0}\right)^2$$
$$= 9gh_0 \Rightarrow \boxed{h = 9h_0} \text{ (a)}$$

b) $h = h_0 \Rightarrow |\dot{y}_{p3}| = |\dot{y}_{p1}| \Rightarrow e^2+2e = 1$ ②

$e^2+2e-1=0 \Rightarrow e = \dfrac{-2 \pm \sqrt{8}}{2} \Rightarrow e = -1+\sqrt{2}$ only sensible value

$\boxed{e \approx 0.414}$ (b)

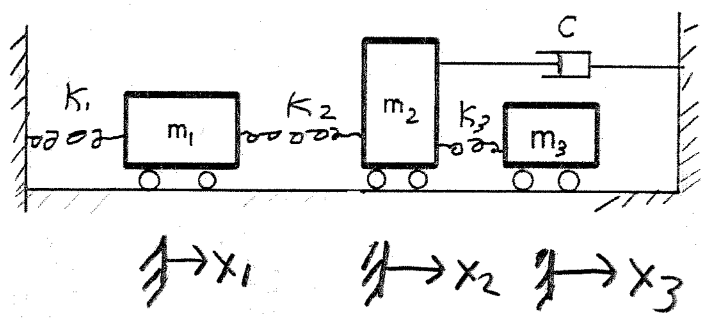
c) Actually, no!

2) As for missing information, please read the front cover.

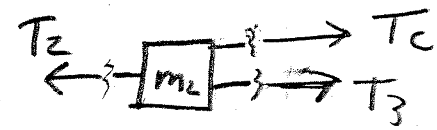
a) Given the positions and velocities of the three masses find the acceleration of mass 2.

b) Assuming $k_2 = 0$ and $k_3 = 0$ and that mass 2 has an initial speed v, find the position of mass 2 as a function of time.

call it v_0



FBD of mass 2



Assume all springs relaxed at $x_1 = x_2 = x_3 = 0$

$$T_2 = k_2(x_2 - x_1)$$

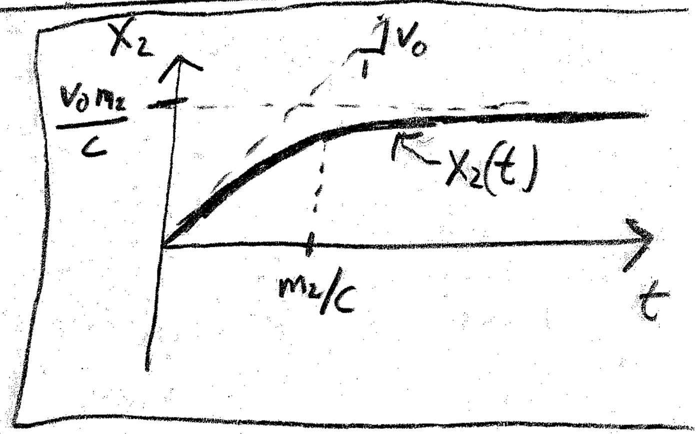
$$T_3 = k_3(x_3 - x_2)$$

$$T_c = c\dot{l} = -c\dot{x}_2$$

LMB $\quad \sum F = ma$

$$-T_2 + T_3 + T_c = m_2 a_2$$

$$\boxed{a_2 = \frac{1}{m_2}\left[-k_2(x_2 - x_1) + k_3(x_3 - x_2) - c\dot{x}_2\right]} \quad (a)$$



b) $k_2 = k_3 = 0 \Rightarrow \quad a_2 = \frac{-c}{m_2}\dot{x}_2$

devine $v_2 = \dot{x}_2 \quad \Rightarrow \ddot{x}_2 = \frac{-c}{m_2}\dot{x}_2$

$$\Rightarrow \dot{v}_2 = \frac{-c}{m_2} v_2$$

$$\Rightarrow v_2 = v_0 e^{-(c/m_2)t}$$

$$\Rightarrow x_2(t) = \int_0^t v_2(t')\,dt' = \int_0^t v_0 e^{-(c/m_2)t'}\,dt' = \frac{-v_0 m_2}{c} e^{-(c/m_2)t'}\Big|_0^t$$

(assume $x_2(0) = 0$)

$$\Rightarrow \boxed{x_2(t) = \frac{v_0 m_2}{c}\left[1 - e^{-(c/m_2)t}\right]} \quad (b)$$

3) The Matlab text below is in one file. It is known to run without error. It is bad code because it has no comments and does not use suggestive variable names.

a) As accurately as possible show the output from running this file. Please annotate (write comments about and explain) the output with any key features or numbers.

b) Same, but changing `p = 0;` to `p = .1;`.
Do not do any long calculations (e.g., arithmetic with square roots) in in detail. Just show clearly how the solution is different than (a) above.

$Z = \begin{bmatrix} X \\ Y \end{bmatrix}$ } just names for z_1 & z_2

$t_0 = 0$, $t_f \approx 2\pi$ (a bit less)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function prellq3()
 tspan=[0 6.2];      n = 10000;
 z0    = [1 0]';     p = 0;
 [tarray zarray] = eulmeth(tspan,z0,n,p);
 u = zarray(:,1);
 w = zarray(:,2);
 plot(u,w,'r', tarray, u,'b');
 axis('equal')
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

$\Rightarrow \boxed{X_0 = 1,\ Y_0 = 0}$ initial conditions

$\underbrace{Y \text{ vs } X}_{\text{red}}$ $\underbrace{X \text{ vs } t}_{\text{blue}}$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [tmat zmat] = eulmeth(tspan,z0,n,p)
tmat    = linspace(tspan(1),tspan(2),n+1);
h       = tmat(2)-tmat(1);
zmat    = zeros(n+1,length(z0));
zmat(1,:) = z0';
 for i=1:n;
     z            = zmat(i,:)';
     t            = tmat(i);
     znew         = z + h*rhs(t,z,p);
     zmat(i+1,:)  = znew';
 end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Solves ODEs numerically. [Exactly from lecture.]

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function  zdot = rhs(t,z,p)
  z1   = z(1);
  z2   = z(2);
  zdot = [ z2;   -z1 - p*z2 ];
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

z1 = X, z2 = Y $\Rightarrow$

$$\boxed{\begin{aligned}\dot{X} &= Y \\ \dot{Y} &= -X - pY\end{aligned}} \quad \text{ODEs} \quad \begin{pmatrix}1a \\ 1b\end{pmatrix}$$

$$1a \Rightarrow \ddot{X} = \dot{Y} \underset{1b}{\Rightarrow} \ddot{X} = -X - p\dot{X} \Rightarrow \boxed{\ddot{X} + p\dot{X} + X = 0} \quad (2)$$

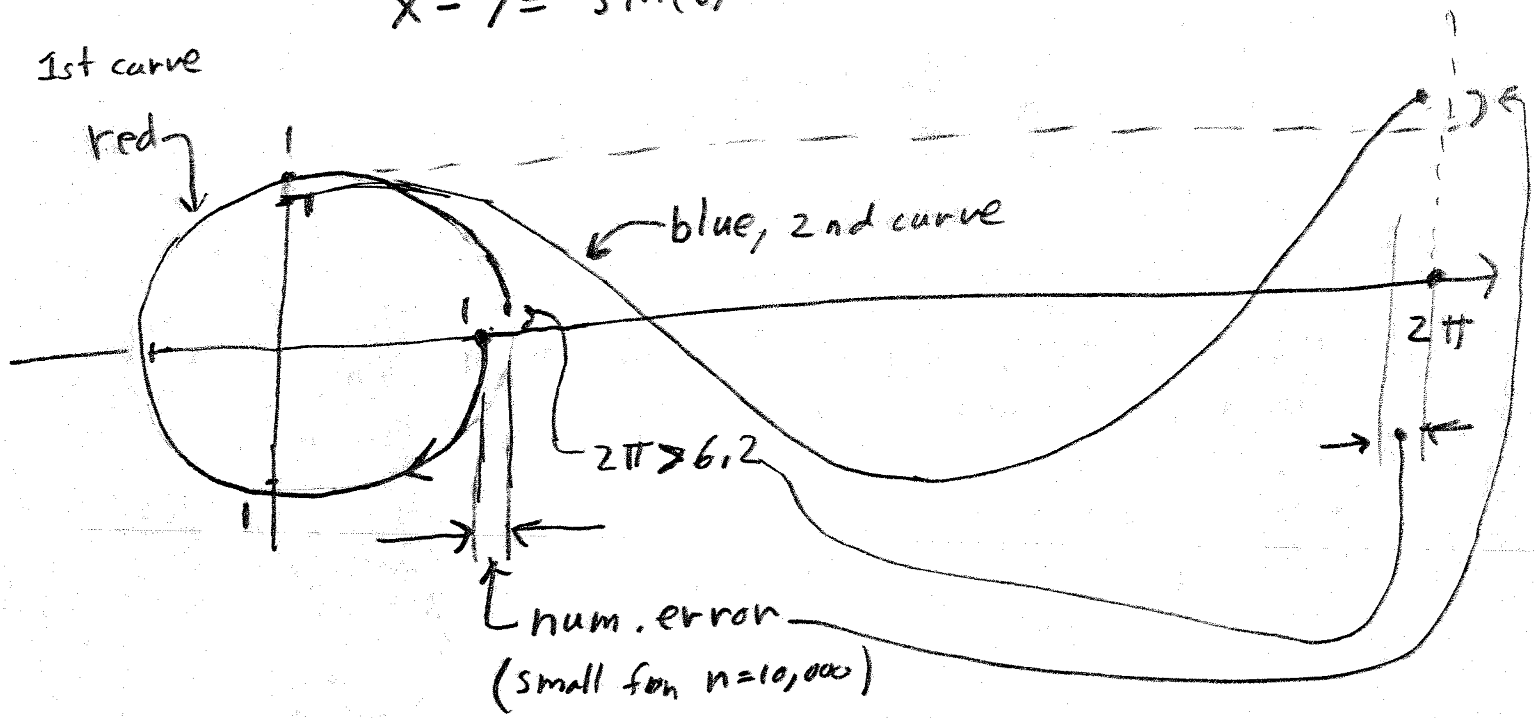
Damped harmonic oscillator!

3a) $p=0 \Rightarrow$ Computer is finding approx. soln. to this problem:

$$\boxed{\begin{array}{ll} \ddot{x}+x=0 & x(0)=1 \\ 0\le t\le 2\pi - \text{a little} & \dot{x}(0)=0 \end{array}}$$

Soln.: $x = \cos(t)$ $\qquad x=u$

$\dot{x} = y = -\sin(t)$ $\qquad y=w$



3b) $p=.1 \Rightarrow$ damping $\Rightarrow$ motions decay $[\ddot{x}+p\dot{x}+x=0]$