Your TA, Section # and Section time:

"SOLUTIONS"

Your name:

RUINA, ANDY

Cornell TAM/ENGRD 2030

Prelim 2

March 29, 2011

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation.**
- A+ Be (I) neat, (II) clear and (III) well organized.
- □ TIDILY REDUCE and box in your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions ($\ell, h, d, \ldots$), coordinates ($x, y, r, \theta \ldots$), variables ($v, m, t, \ldots$), base vectors ($\hat{\imath}, \hat{\jmath}, \hat{e}_r, \hat{e}_\theta, \hat{\lambda}, \hat{n} \ldots$) and signs ($\pm$) with sketches, equations or words.
- →| **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- ≈ Work for **partial credit** (from 60 – 100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

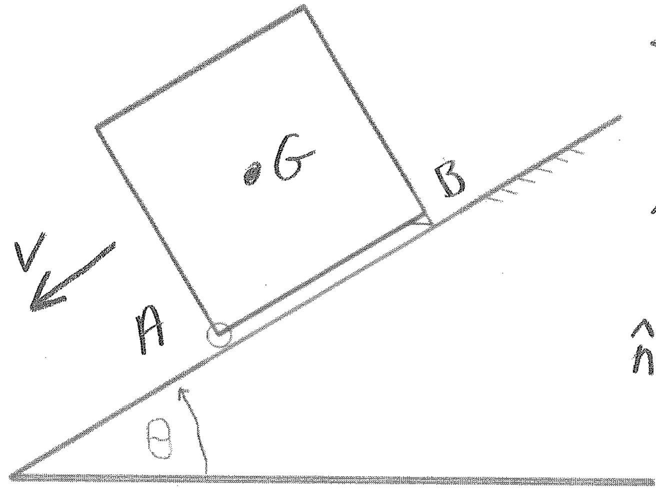
Problem 4: ____ /25

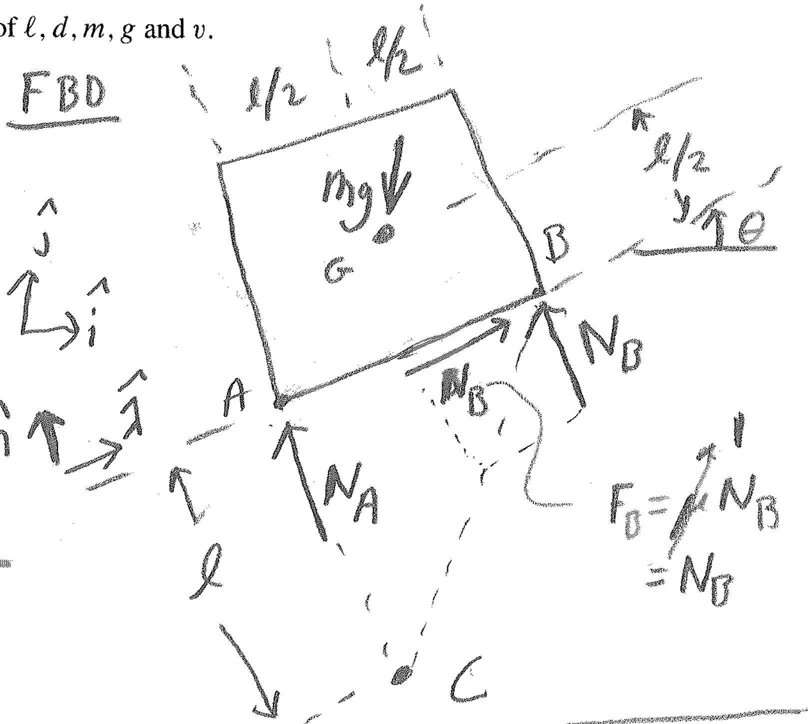
Problem 5: ____ /25

Problem 6: ____ /25

1) At the time of interest a uniform square suitcase with mass m, sides ℓ and thickness d is rolling and sliding down a ramp with speed $v > 0$. The front wheels (downhill) are ideal massless wheels. The other end is sliding on a rubber stub with $\mu = \tan(\phi) = 1$. Do not neglect gravity g.

a) For $\theta = 45°$ find $\dot v$. Answer in terms of some or all of ℓ, d, m, g and v.
b) For what value of θ is $\dot v = 0$?





AMB/C

$$\sum \vec M_{/C} = \dot{\vec H}_{/C}$$

$$\vec r_{G/C} \times (-\cancel{m}g\hat j) = \vec r_{G/C} \times (\cancel{m}\overbrace{\dot v(-\hat\lambda)}^{\vec a})$$

$$\vec r_{G/C} = \frac{3\ell}{2}\hat n + \frac{\ell}{2}\hat\lambda$$

$$\left(\frac{3\ell}{2}\hat n + \ell/2\,\hat\lambda\right) \times (g\hat j) = \left(\frac{3\ell}{2}\hat n + \frac{\ell}{2}\hat\lambda\right) \times (-\dot v\hat\lambda)$$

$\hat n \times \hat j = -\sin\theta\hat k$, $\hat\lambda \times \hat\lambda = \hat 0$, $\hat n \times \hat\lambda = -\hat k$, $\hat\lambda \times \hat j = \cos\theta\hat k$

$$\left\{\left[\frac{3g\cancel{\ell}}{\cancel{2}}\sin\theta - \frac{\cancel{\ell}g}{\cancel{2}}\cos\theta\right]\hat k = \frac{-3\cancel{\ell}}{\cancel{2}}\dot v\hat k\right\}$$

$\{\ \}\cdot\hat k \Rightarrow$
$$\dot v = g\left[\frac{\cos\theta}{3} - \sin\theta\right] \quad (*)$$

$\theta = 45° \Rightarrow$
$$\dot v = g\frac{\sqrt2}{2}(-2/3) = \boxed{-g\sqrt2/3 = \dot v}\ (a)$$

$\dot v = 0$, $(*)$ $\Rightarrow$
$$\frac{\sin\theta}{\cos\theta} = \frac{1}{3} \Rightarrow \boxed{\theta = \tan^{-1}(1/3)}\ (b)$$

Alt. soln. to b.

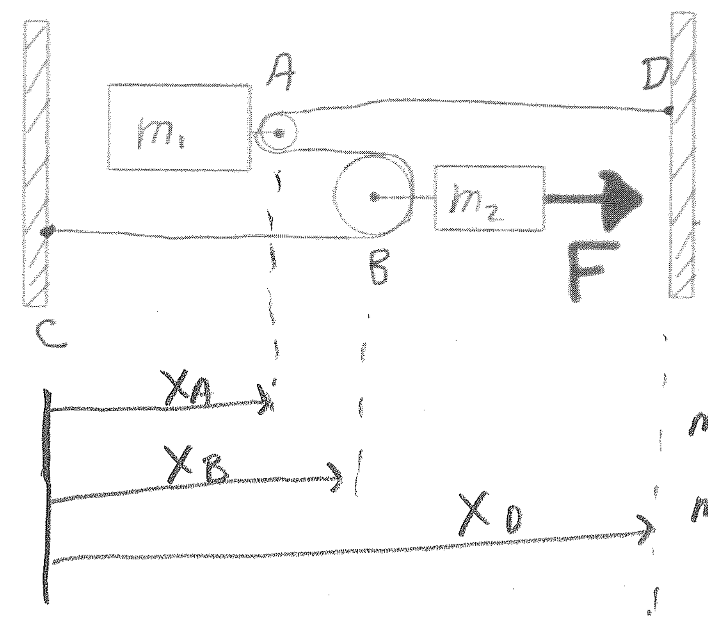
$\dot v = 0 \Rightarrow$ statics
$\Rightarrow$ G above C
$\Rightarrow \tan\theta = 1/3$

Check (*)

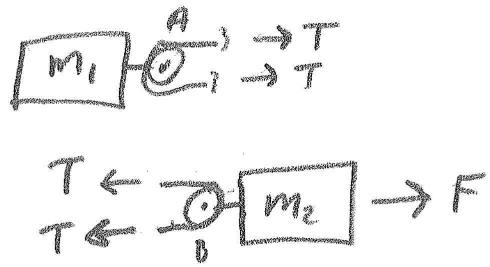
$\theta = 90° \Rightarrow \cos\theta = 0$, $\sin\theta = 1$

$\dot v = -g$ ✓ as expected

2) In terms of m_1, m_2 and F find a_B, the acceleration of B.



FBDs



LMB

m_2: $F - 2T = a_B m_2$ (2)

m_1: $2T = a_A m_1$ (3)

Kinematics

$$\{ \text{const} = l = (x_D - x_A) + (x_B - x_A) + x_B + \pi r_A + \pi r_B \}$$

$$\frac{d^2}{dt^2}\{ \} \Rightarrow 0 = (0 - \ddot{x}_A) + (\ddot{x}_B - \ddot{x}_A) + \ddot{x}_B + 0 + 0$$

$$\Rightarrow \underbrace{\ddot{x}_A}_{a_A} = \underbrace{\ddot{x}_B}_{a_B} \quad (1)$$

[both masses have same motion.]

Apply (1) to (3) and sub into (2)

$$\Rightarrow \quad F - a_B m_1 = a_B m_2 \Rightarrow \boxed{a_B = \frac{F}{m_1 + m_2}}$$

to the right

3) A canonball of mass $m = 2\,\text{kg}$ is launched from the top edge of a tall cliff, with a velocity of $\vec{v}_0 = (300\hat{\imath} + 400\hat{\jmath})\,\text{m/s}$. x and y are the distances from the launch to the right and up, respectively. Gravity points down with $g = 10\,\text{m/s}^2$. There is a quadratic drag force on the ball, opposing the motion, with magnitude

$$F = cv^2 \qquad \text{where } c = 0.1\,\text{N/(m/s)}^2.$$

Goal: find the$x$ position of the ball, in meters, after 100 seconds.

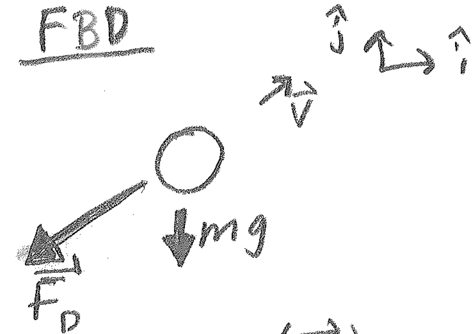
a) Supply the two missing blocks of code (A and B below), so that running `prel2q3.m` below will provide the desired numerical solution to the equations of motion.

Please use reasonably-named intermediate variables to make your code as readable as possible. No other changes or additions to the code below are needed, but you can make other changes if you like.

b) Extra credit (easy): Estimate the velocity at $t = 100\,\text{s}$. The answer should be a single arithmetic formula involving the numbers above. An exact analytic formula is not possible. But it is possible, making reasonable assumptions based on the nature of the motion, to get an accurate estimate.

b) Extra credit (challenge): Estimate the value of x at $t = 100\,\text{s}$. This estimate need not be as good as that above. Again an arithmetic formula is desired.

FBD




```
function prel2q3()
t_end = 100; x0=0; y0=0; vx0=300; vy0 = 400;
m = 2; c = 0.1;  g = 10;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       A. FILL IN MISSING LINES HERE    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[tarray zarray] = rk2(tspan,z0,n,p);  array
% The next line nicely prints out zarray(end,1)
disp(['x(100s)='  num2str(zarray(end,1))  'm'])
end
```

$\vec{F}_D = -cv^2\left(\frac{\vec{v}}{v}\right)$

unit vector in $\vec{v}$ dir.

```
function [tmat zmat] = rk2(tspan,z0,n,p)
%Midpoint integration. No problems here.
tmat    = linspace(tspan(1),tspan(2),n+1);
h       = tmat(2)-tmat(1);
zmat    = zeros(n+1,length(z0));
zmat(1,:) = z0';
 for i=1:n;
    z       = zmat(i,:)';      t     = tmat(i);
    ztemp   = z +(h/2)*rhs( t,       z,    p);
    znew    = z +   h   *rhs((t+h/2),ztemp,p);
    zmat(i+1,:) = znew';
 end
end
```

LMB

$\Sigma\vec{F} = m\vec{a}$

$-mg\hat{\jmath} - cv|\vec{v}| = m\dot{\vec{v}}$

$\Rightarrow$ (1) $\boxed{\begin{aligned}\dot{\vec{r}} &= \vec{v}\\ \dot{\vec{v}} &= \frac{1}{m}\vec{F}_D - mg\hat{\jmath}\end{aligned}}$

$\vec{F}_D = -cv\vec{v}$ or $-cv^2\left(\frac{\vec{v}}{v}\right)$

Eqs. of motion

```
function  zdot = rhs(t,z,p)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       B. FILL IN MISSING LINES HERE    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
```

```
%A. MISSING CODE
p.m = m; p.c = c  ;  p.g = g;
tspan = [0 t_end];  n = 10000;
z0    = [x0 y0 vx0 vy0]';
```

A

```
% B. MISSING CODE
m=p.m; c = p.c;  g = p.g;
r= z(1:2);  vvec = z(3:4); %position and velocity vectors
v = norm(vvec);  uv = vvec/v; % mag of vel, unit vector in dir. of vel.

Fdrag = -c*v^2*uv; % drag force, a 2-comp vector

rdot    = vvec;                        % first two ODEs
vvecdot = (1/m)*(Fdrag -[0 m*g]'); % 2nd   two ODEs (F = ma)

zdot = [rdot; vvecdot];  % All 4 comps of rhs.
```

1

a) Assume terminal velocity

$\Rightarrow \dot{V} = 0$

$\uparrow F_D$

$\downarrow mg$

$F_D = mg$

$cv^2 = mg$

$V = \sqrt{mg/c}$

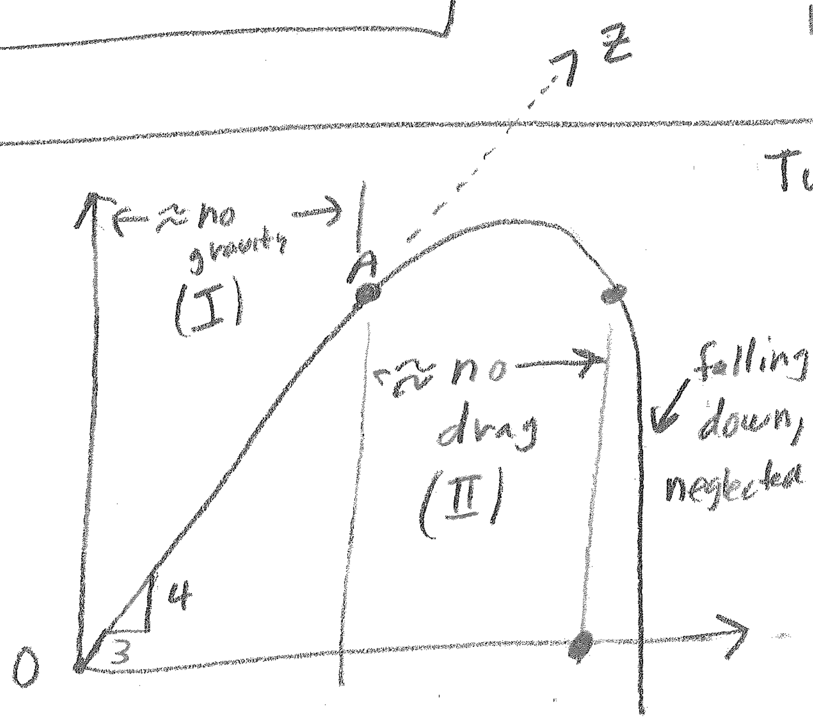
$v = \sqrt{\frac{(2\ kg)(10\ m/s^2)}{0.1\ N/(m/s)^2}}$

$v = \sqrt{200}$ m/s

$\Leftarrow$

a $\boxed{\vec{V} = -\sqrt{200}\ m/s\ \hat{j} = -10\sqrt{2}\ m/s\ \hat{j}}$

b)



Two regimes:

I: slowing (neglect gravity)

II. top of trajectory (neglect drag)

3b) (cont'd)

Regime I ;

$$m\frac{dV}{dt} = -cV^2 \Rightarrow \frac{dV}{dz}V = -\frac{c}{m}V^2$$

$$\Rightarrow \frac{dV}{V} = -\frac{c}{m}dx \Rightarrow \ln V_A - \ln V_0 = -\frac{c}{m}z$$

$$\Rightarrow z_I = \frac{m}{c}\ln\frac{V_0}{V_A}$$

use V_A = term vel. for transition

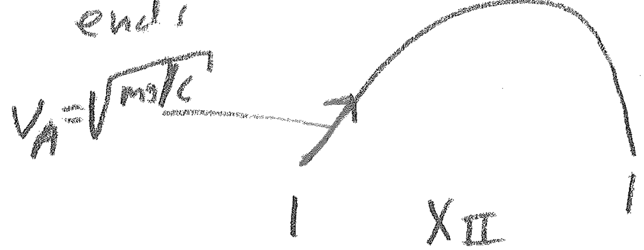
$$\Rightarrow z_I = \frac{m}{c}\ln\left(\frac{V_0}{\sqrt{mg/c}}\right)$$

$$X_I = \frac{3}{5}z_I$$ (project on to z axis)

Regime II

parabolic flight starts where regim I ends

t = time of flight $= 2\frac{4}{5}V_A/g$



$$X_{II} = \overbrace{\frac{3}{5}V_A}^{X\,vel}\cdot t = 2\cdot\frac{3}{5}\cdot\frac{4}{5}V_A^2/g$$

$$\text{Estimate} = \boxed{X_{TOT} = X_I + X_{II} = \frac{3}{5}\frac{m}{c}\ln\left(\frac{\sqrt{mg/c}}{V_0}\right) + \frac{24}{25}mg/cg}$$

Using #s from problem $\Rightarrow$

$$X_{TOT} = \left[\frac{3}{5}20\ln\left(\frac{500}{\sqrt{200}}\right) + \frac{24}{25}\frac{20}{1}\right]m \quad *$$

POSTPLAY MATLAB check:

ODE Soln $\Rightarrow X_{TOT} = 62.4m$

* above $\Rightarrow X_{TOT} = 61.98m$

Pretty close! (Better than it deserves)