Your TA, Section # and Section time:

Your name:

Cornell TAM/ENGRD 2030

No calculators, books or notes allowed.

3 Problems, 90 minutes (+ up to 90 minutes overtime)

Prelim 3

April 19, 2011

How to get the highest score?

Please do these things:

- Draw **Free body diagrams** whenever force, moment, linear momentum, or angular momentum balance are used.
- Use correct **vector notation**.
- A+ Be (I) neat, (II) clear and (III) well organized.
- TIDILY REDUCE and $\boxed{\text{box in}}$ your answers (Don't leave simplifyable algebraic expressions).
- >> Make appropriate `Matlab code` clear and correct.
  You can use shortcut notation like "$T_7 = 18$" instead of, say, "`T(7) = 18`".
  Small syntax errors will have small penalties.
- Clearly **define** any needed dimensions $(\ell, h, d, \dots)$, coordinates $(x, y, r, \theta \dots)$, variables $(v, m, t, \dots)$, base vectors $(\hat{\boldsymbol{\imath}}, \hat{\boldsymbol{\jmath}}, \hat{\boldsymbol{e}}_r, \hat{\boldsymbol{e}}_\theta, \hat{\boldsymbol{\lambda}}, \hat{\boldsymbol{n}} \dots)$ and signs $(\pm)$ with sketches, equations or words.
- **Justify** your results so a grader can distinguish an informed answer from a guess.
- If a problem seems *poorly defined*, clearly state any reasonable assumptions (that do not oversimplify the problem).
- $\approx$ Work for **partial credit** (from 60–100%, depending on the problem)
  - Put your answer is in terms of well defined variables even if you have not substituted in the numerical values.
  - Reduce the problem to a clearly defined set of equations to solve.
  - Provide Matlab code which would generate the desired answer (and explain the nature of the output).
- **Extra sheets.** Put your name on each extra sheet, fold it in, and refer to it at the relevant problem.
  Note the last page is **blank** for your use. Ask for more extra paper if you need it.

Problem 7: ____/25

Problem 8: ____/25

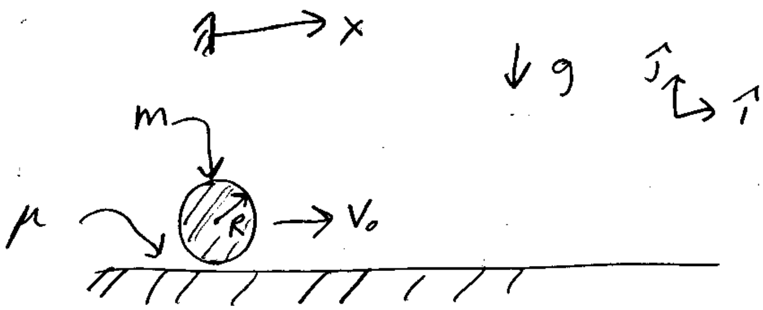
Problem 9: ____/25

1) A uniform cylinder (mass m, radius R) is initially moving horizontally (velocity of its center of mass is $\vec{\boldsymbol{v}}(0) = v_0\hat{\boldsymbol{\imath}}$, with $v_0 > 0$) and not rotating ($\vec{\boldsymbol{\omega}}_0 = \vec{\mathbf{0}}$) when placed on a horizontal flat smooth frictional surface with friction coefficient μ. It slides for a while and then rolls. Answer in terms of some or all of $v_0, m, R, g, \mu, \hat{\boldsymbol{\imath}}$ and $\hat{\boldsymbol{\jmath}}$.

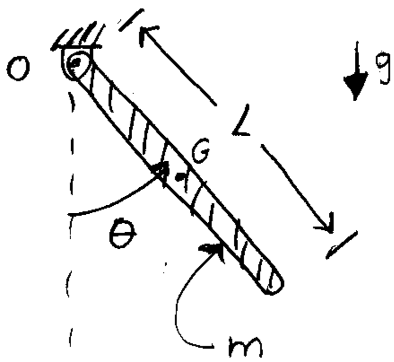
a) When the cylinder eventually rolls what is the velocity of the center of mass?

b) When it eventually rolls what is its angular velocity?

c) How far does it slide before it starts rolling?



2) A uniform rigid stick (length L, mass m) hangs from a hinge with negligible friction at one end (point O). Immediately after it is released from rest with initial angle $\theta = \theta_0$ what is the force (a vector) of the hinge on the stick? Answer in terms of some or all of m, g, L, θ_0, $\hat{\imath}$ and $\hat{\jmath}$. Define $\hat{\imath}$ and $\hat{\jmath}$ any way you like with a clear sketch.



3) A two-dimensional object $\mathcal{B}$ moves in the plane. At the instant of interest its center of mass has position $\vec{r}_\mathrm{G} = \vec{r}_{\mathrm{G/O}}$, velocity $\vec{\boldsymbol{v}}$. and counter-clockwise angular velocity $\omega \neq 0$.

> *Interesting fact:*
> *So long as* $\omega \neq 0$ *a point C, called the 'instantaneous center of rotation' (COR), always exists such that*
> - *point C is instantaneously stationary:* $\vec{\boldsymbol{v}}_\mathrm{C} = \vec{\boldsymbol{0}}$, *and*
> - *the velocities of all points D on the object are calculated by treating the object as rotating about C:* $\vec{\boldsymbol{v}}_\mathrm{D} = \vec{\boldsymbol{\omega}} \times \vec{\boldsymbol{r}}_{\mathrm{D/C}}$.
>
> *Point C is not necessarily literally on the object, but rather is somewhere on an infinite rigid extension of the object (that is, C is on a large imagined rigid piece of graph paper glued to the object).*

a) Find $\vec{r}_\mathrm{C} = \vec{r}_{\mathrm{C/O}}$ in terms of some or all of $\vec{r}_\mathrm{G}$, $\vec{\boldsymbol{v}}$, ω, $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$. That is, write a formula that answers the question: $\vec{r}_\mathrm{C} =$? If you happen to have memorized this formula, you must show how to obtain it.

b) For the special case that

$$\begin{aligned} \vec{r}_\mathrm{G} &= 2\,\mathrm{m}\hat{\boldsymbol{\imath}}, \\ \vec{\boldsymbol{v}} &= 3\,\mathrm{m/s}\hat{\boldsymbol{\imath}} + 4\,\mathrm{m/s}\hat{\boldsymbol{\jmath}} \qquad \text{and} \\ \omega &= 1\,\mathrm{s}^{-1} \end{aligned}$$

find x_C and y_C. A neat sketch may help your work and may help you better communicate your understanding.