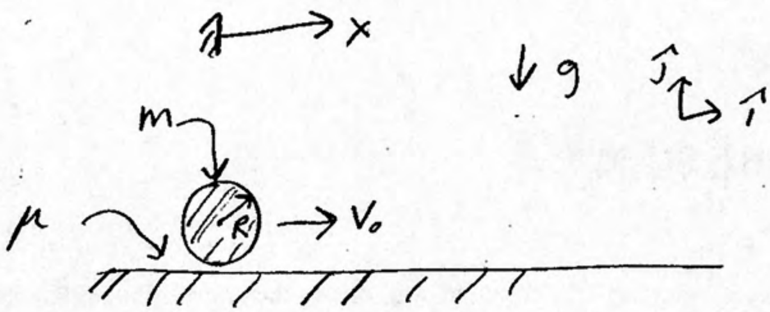
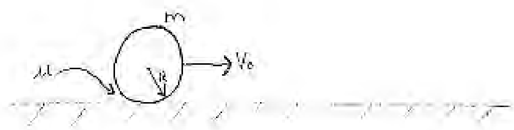
1) A uniform cylinder (mass m, radius R) is initially moving horizontally (velocity of its center of mass is $\vec{v}(0) = v_0\hat{\imath}$, with $v_0 > 0$) and not rotating ($\vec{\omega}_0 = \vec{0}$) when placed on a horizontal flat smooth frictional surface with friction coefficient μ. It slides for a while and then rolls. Answer in terms of some or all of $v_0, m, R, g, \mu, \hat{\imath}$ and $\hat{\jmath}$.

a) When the cylinder eventually rolls what is the velocity of the center of mass?

b) When it eventually rolls what is its angular velocity?

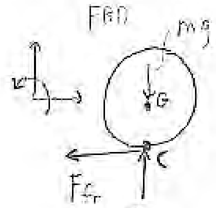
c) How far does it slide before it starts rolling?





Cylinder is initially moving horizontally at $V = V_0\hat{\imath}$ without rotation $\vec{\omega}_0 = \vec{0}$ is on a frictional surface and eventually starts rolling after sliding

a) what is the cylinder velocity once it starts rolling



AMB/c

$$\sum \vec{M}_{/c} = \sum \dot{\vec{H}}_{/c}$$

$$\int_0^t 0 = \int_0^t I^G \dot{\omega} + \vec{r}_{G/c} \times m\vec{a}_G \, dt$$

$(\vec{a}_G = a\hat{\imath}$ due to horizontal surface

$\vec{r}_{G/c} = R\hat{\jmath}$)

$I^G = \frac{mR^2}{2}$

$$0 = I^G(\vec{\omega} - \vec{\omega}_0) + \vec{r}_{G/c} \times m(\vec{V} - \vec{V}(0))$$

$\vec{V} = V\hat{\imath}$

$$I^G \cancel{\vec{\omega}_0}^{\,0} + m\vec{r}_{G/c} \times \vec{V}(0) = I^G\vec{\omega} + \vec{r}_{G/c} \times m\vec{V}$$

let $\vec{\omega} = (\omega)(-\hat{k})$

$$\left\{ mR\hat{\jmath} \times V_0\hat{\imath} = \frac{mR^2}{2}\omega\hat{k} + R\hat{\jmath} \times mV\hat{\imath} \right\} \cdot \hat{k}$$

$$(1) \quad -mRV_0 = -\frac{mR^2}{2}\omega - mRV$$

from rolling condition, $\vec{\omega} \times R_{C/G} + \vec{V} = 0 \implies \omega(-\hat{k}) \times R(-\hat{\jmath}) + V\hat{\imath} = 0$

$$-\omega R\hat{\imath} + V\hat{\imath} = 0$$

$$V = \omega R \text{ or } \omega = V/R \quad (2)$$

subs (2) into (1) and simplifying somewhat

$$V_0 = \frac{1}{2}V + V \implies \boxed{V = \frac{2}{3}V_0}$$

substituting V into (2), $\boxed{\omega = \frac{2}{3}\frac{V_0}{R}}$

c) using energy, translational only

$W = \Delta KE$

$-F_{fr}\, d = \frac{1}{2} m V^2 - \frac{1}{2} m V_o^2$

$$d = \frac{(\frac{2}{3} V_o)^2 - V_o^2}{-2\mu g}$$

$$d = \frac{\frac{4}{9} V_o^2 - V_o^2}{-2\mu g}$$

$$\boxed{d = \frac{5}{18} \frac{V_o^2}{\mu g}}$$

LMB

$mg(-\hat{j}) + R\hat{j} - F_{fr}(\hat{i}) = m\vec{a}$

$\{\}\cdot\hat{j} \Rightarrow -mg + R = 0$, $R = mg$

from friction, $F_{fr} = \mu R$

alternately;

from $\{\}\cdot\hat{i}$, $a = -\mu g$

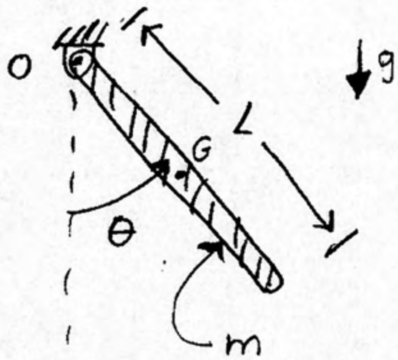
$V = -\mu g t + V_o$

$X = \frac{-\mu g}{2} t^2 + V_o t$

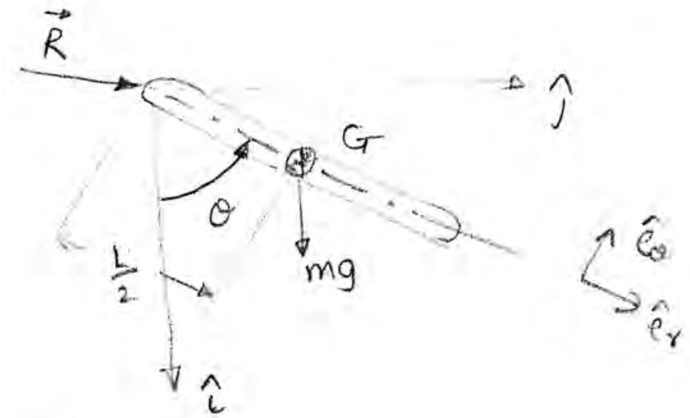
find t for $V = \frac{2}{3} V_o$ $\Rightarrow$ $\frac{2}{3} V_o = -\mu g t + V_o$ $\Rightarrow$ $t = \frac{V_o}{3\mu g}$

$$X = \frac{-\mu g}{2}\left(\frac{V_o}{3\mu g}\right)^2 + \frac{V_o^2}{3\mu g} = \left(\frac{-1}{18} + \frac{1}{3}\right)\frac{V_o^2}{\mu g} = \boxed{\frac{5}{18}\frac{V_o^2}{\mu g}}$$

2) A uniform rigid stick (length L, mass m) hangs from a hinge with negligible friction at one end (point O). Immediately after it is released from rest with initial angle $\theta = \theta_0$ what is the force (a vector) of the hinge on the stick? Answer in terms of some or all of $m, g, L, \theta_0, \hat{\imath}$ and $\hat{\jmath}$. Define $\hat{\imath}$ and $\hat{\jmath}$ any way you like with a clear sketch



② FBD



Find $\vec R$.

AMB$_{/O}$ $\quad \dot{\vec H}_{/O} = \vec M_{/O}$

$$\Rightarrow \vec r_{G/O} \times m\vec a_G + I_G\ddot\theta\,\hat k = \vec r_{G/O} \times mg\,\hat\imath$$

$$\Rightarrow m\frac{L}{2}\hat e_r \times \left\{-\frac{L}{2}\dot\theta^2\hat e_r + \frac{L}{2}\ddot\theta\,\hat e_\theta\right\} + I_G\ddot\theta\,\hat k = mg\frac{L}{2}\overbrace{\hat e_r \times \hat\imath}^{-\sin\theta\,\hat k}$$

$$\Rightarrow \left(\frac{mL^2}{4} + I_G\right)\ddot\theta\,\hat k = -mg\frac{L}{2}\sin\theta\,\hat k$$

$\{\ \}\cdot\hat k$ and put $I_G = \frac{mL^2}{12}$

$$\ddot\theta = -\frac{3}{2}\frac{g}{L}\sin\theta$$

LMB $\quad \vec F = m\vec a_G$

$$\vec R + mg\,\hat\imath = m\left\{-\frac{L}{2}\cancelto{0}{\dot\theta^2}\hat e_r + \frac{L}{2}\ddot\theta\,\hat e_\theta\right\}$$

$$\vec R + mg\,\hat\imath = \frac{mL}{2}\left\{-\frac{3}{2}\frac{g}{L}\sin\theta\left(-\sin\theta\,\hat\imath + \cos\theta\,\hat\jmath\right)\right\}$$

Solving for $\vec R$ and putting $\theta = \theta_0$ gives

$$\boxed{\vec R = mg\left\{\left(\frac{3}{4}\sin^2\theta_0 - 1\right)\hat\imath - \frac{3}{4}\sin\theta_0\cos\theta_0\,\hat\jmath\right\}}$$

3) A two-dimensional object $\mathcal{B}$ moves in the plane. At the instant of interest its center of mass has position $\vec{r}_G = \vec{r}_{G/O}$, velocity $\vec{v}$. and counter-clockwise angular velocity $\omega \neq 0$.

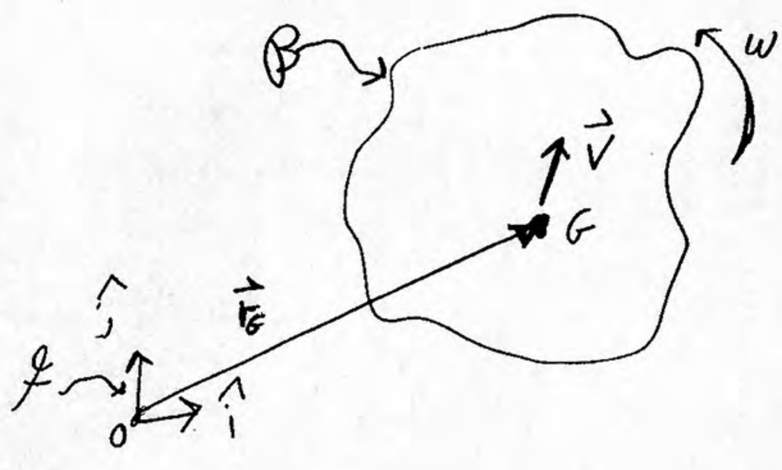
> *Interesting fact:*
> *So long as $\omega \neq 0$ a point C, called the 'instantaneous center of rotation' (COR), always exists such that*
> - *point C is instantaneously stationary: $\vec{v}_C = \vec{0}$, and*
> - *the velocities of all points D on the object are calculated by treating the object as rotating about C: $\vec{v}_D = \vec{\omega} \times \vec{r}_{D/C}$.*
>
> *Point C is not necessarily literally on the object, but rather is somewhere on an infinite rigid extension of the object (that is, C is on a large imagined rigid piece of graph paper glued to the object).*

a) Find $\vec{r}_C = \vec{r}_{C/O}$ in terms of some or all of $\vec{r}_G, \vec{v}, \omega, \hat{\imath}, \hat{\jmath}$ and $\hat{k}$. That is, write a formula that answers the question: $\vec{r}_C = ?$ If you happen to have memorized this formula, you must show how to obtain it.

b) For the special case that

$$
\begin{aligned}
\vec{r}_G &= 2\,\mathrm{m}\hat{\imath}, \\
\vec{v} &= 3\,\mathrm{m/s}\hat{\imath} + 4\,\mathrm{m/s}\hat{\jmath} \qquad \text{and} \\
\omega &= 1\,\mathrm{s}^{-1}
\end{aligned}
$$

find x_C and y_C. A neat sketch may help your work and may help you better communicate your understanding.



(3) Using the definition of instantaneous center

a)

$$\vec{V}_G = \vec{\omega} \times \vec{r}_{G/C}$$

$$\vec{V} = \vec{\omega} \times (\vec{r}_G - \vec{r}_C)$$

Cross both sides with $\vec{\omega}$

$$\vec{\omega} \times \vec{V} = \vec{\omega} \times \{ \vec{\omega} \times (\vec{r}_G - \vec{r}_C) \}$$

$$= -|\vec{\omega}|^2 (\vec{r}_G - \vec{r}_C)$$

Solving for $\vec{r}_C$ gives

$$\boxed{\vec{r}_C = \vec{r}_G + \frac{\vec{\omega} \times \vec{V}}{|\vec{\omega}|^2}}$$

b) Put $\vec{r}_G = 2\hat{i}$ $\quad \vec{\omega} = 1\hat{k}$ $\quad \vec{V} = 3\hat{i} + 4\hat{j}$

$$\vec{r}_C = 2\hat{i} + \frac{\hat{k} \times \{3\hat{i} + 4\hat{j}\}}{1^2}$$

$$= 2\hat{i} + 3\hat{j} - 4\hat{i}$$

$$\boxed{\vec{r}_C = -2\,\text{m}\,\hat{i} + 3\,\text{m}\,\hat{j}}$$