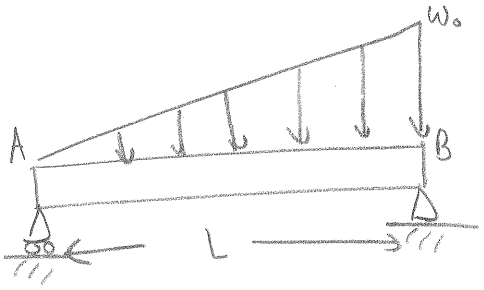
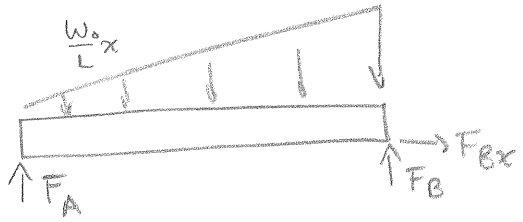
15.12)



a) Find max deflection

b) Find max deflection if beam is W18x50 shape and $w_0 = 4.5 \frac{kip}{ft}$, $L = 18$ Ft, $E = 29 \cdot 10^6$ psi

a)



$F_{Bx} = 0$

$$\sum M_A = -\int_0^L \left(\frac{w_0}{L}x\right) x\,dx + F_B L = 0$$

$$-\frac{w_0 L^2}{3} + F_B L = 0 \Rightarrow F_B = \frac{1}{3} w_0 L$$

$$\sum F_y = 0 = F_A + F_B - \int_0^L \frac{w_0}{L} x\,dx = F_A + \frac{1}{3} w_0 L - \frac{1}{2} w_0 L \Rightarrow F_A = \frac{1}{6} w_0 L$$

$$w = \frac{w_0}{L} x$$

$$V = -\int w\,dx = \frac{1}{6} w_0 L - \frac{w_0}{2L} x^2$$

$$M = \int V\,dx = \frac{1}{6} w_0 L x - \frac{1}{6} \frac{w_0}{L} x^3 = EIu''$$

$$EIu' = \int M\,dx = \frac{1}{12} w_0 L x^2 - \frac{1}{24} \frac{w_0}{L} x^4 + C_1$$

$$EIu = \int EIu'\,dx = \frac{1}{36} w_0 L x^3 - \frac{1}{120} \frac{w_0}{L} x^5 + C_1 x + C_2$$

$$EIu(0) = 0 \Rightarrow C_2 = 0$$

$$EIu(L) = \frac{1}{36} w_0 L^4 - \frac{1}{120} w_0 L^4 + C_1 L = 0 \Rightarrow C_1 = \left(\frac{1}{120} - \frac{1}{36}\right) w_0 L^3 = -\frac{7}{360} w_0 L^3$$

$$EIu' = 0 = \frac{1}{12} w_0 L x^2 - \frac{1}{24} w_0 \frac{x^4}{L} - \frac{7}{360} w_0 L^3 = 0 \quad \text{at maximum deflection}$$

$$15x^4 - 30L^2x^2 + 7L^4 = 0, \quad x^2 = \left(\pm\frac{2\sqrt{30}}{15} + 1\right)L^2$$

max deflection @ $\boxed{x = 0.519L}$

$$u_{max} = \frac{w_0 L^4}{EI}\left[\frac{1}{36}(0.519)^3 - \frac{1}{120}(0.519)^5 - \frac{7}{360}(0.519)\right]$$

$$\boxed{u_{max} = -6.52 \cdot 10^{-3} \frac{w_0 L^4}{EI} \quad @ \quad x = 0.519L}$$

b) for W18x50 shape, $E = 29 \cdot 10^6$ psi, $w_0 = 4.5 \frac{kip}{ft}$, $L = 18$ ft

$I = 800 \text{ in}^4$ for W18x50 shape beams,

$$u_{max} = (-6.52 \cdot 10^{-3}) \frac{(4.5 \cdot 10^3 \frac{lb}{ft})(18 \text{ ft})^4 (\frac{12 \text{ in}}{1 \text{ ft}})^3}{(29 \cdot 10^6 \frac{lb}{in^2})(800 \text{ in}^4)}$$

$$\boxed{u_{max} = -0.229 \text{ in}}$$