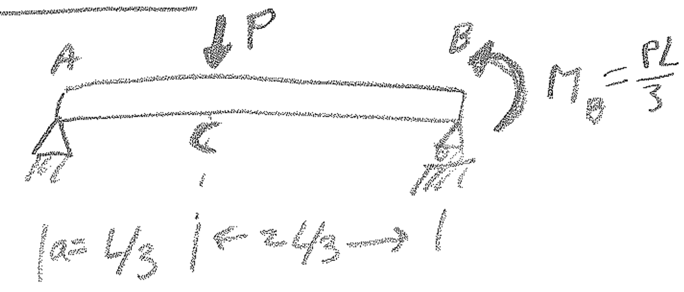
BJ 15.27

(1)




a) $u_C = ?$

b) $u'_A = ?$

Given

$EI, L, F, M_B$

Ans. in back of book

a) $u_C = -8PL^3/243EI$

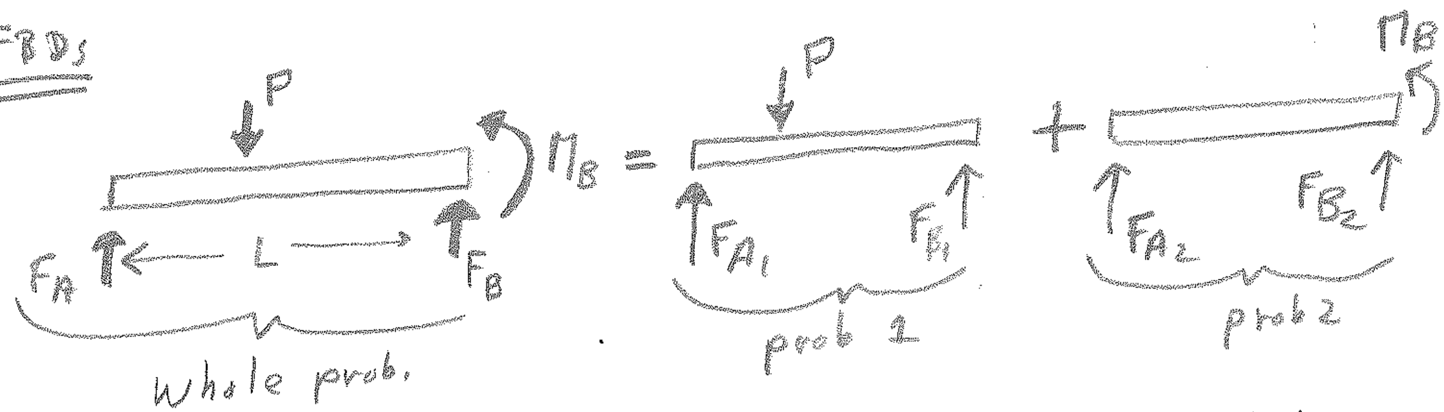
b) $u'_A = -19PL^2/162EI$

(Looks like a mess!)

Solution Method

1. Solve w/ F & No $M_B$
2. " " $M_B$ " " F
3. Add Solutions

FBDs




$\Sigma F_y = 0, \Sigma M_B = 0 \Rightarrow$

$\Rightarrow \quad F_A = \left(1 - \frac{a}{L}\right)P + M_B/L \qquad \Leftarrow \qquad F_{A1} = \left(1 - \frac{a}{L}\right)P \qquad F_{A2} = M_B/L$

$F_B = \frac{a}{L}P - M_B/L \qquad F_{B1} = \frac{a}{L}P \qquad F_{B2} = -M_B/L$

1. Solve prob 1 (no need to include $F_{B1}$)

$W(x) = -F_{A1}\langle x\rangle^{-1} + P\langle x-a\rangle^{-1}$

$V(x) = F_{A1}\langle x\rangle^0 - P\langle x-a\rangle^0$

$M(x) = F_{A1}\langle x\rangle^1 - P\langle x-a\rangle^1 = EIu''$

$EIu' = F_{A1}\langle x\rangle^2/2 - P\langle x-a\rangle^2/2 + d_1$

$EIu = F_{A1}\langle x\rangle^3/6 - P\langle x-a\rangle^3/6 + d_1 x + \cancel{d_2}$ $\quad 0 \quad (u(0)=0)$

$u(L)=0 \Rightarrow \quad F_{A1} L^3/6 - P(L-a)^3/6 + d_1 L = 0$

$$\Rightarrow d_1 = \underbrace{-F_{A1}}_{\frac{2P}{3}} L^2/6 + P(L-\underbrace{a}_{L/3})^3/6L$$

$$= -PL^2\left[2/18 - (2/3)^3/6\right] = -PL^2\left[1/9 - \frac{8}{27\cdot 6}\right]$$

$$= -PL^2\left[\frac{18-8}{27\cdot 6}\right] = -PL^2\frac{10}{27\cdot 6} = -PL^2\frac{5}{3\cdot 27} = -PL^2\frac{5}{81}$$

$$\Rightarrow u'_{A_1} = u_1'(0) = c_1/EI = \boxed{\frac{-PL^2}{EI}\frac{5}{81}} = u'_{A1}$$

$$u_{C_1} = u_1(a) = \left[\underbrace{F_{A1}}_{2P/3}\underbrace{a}_{L/3}^3/6 + P\cdot 0 + d_1\underbrace{a}_{L/3}\right]/EI$$

$$= \frac{PL^3}{EI}\left[\frac{2}{3\cdot 3^3\cdot 6} - \frac{5}{81\cdot 3}\right] = \frac{PL^3}{EI}\left[\frac{1-5}{243}\right]$$

$$= \frac{-4}{3^5}\frac{PL^3}{EI} = \boxed{\frac{-4}{243}\frac{PL^3}{EI}} = u_{C_1}$$

2. Solve Prob. 2 (easier!)

$$\hat{w}(x) = -F_{A2}\langle x\rangle^{-1} = \frac{-M_B}{L}\langle x\rangle^{-1} = \frac{-P}{3}\langle x\rangle^{-1}$$

$$V(x) = \frac{P}{3}\langle x\rangle^0$$

$$EIu'' = M(x) = \frac{P}{3}\langle x\rangle^1$$

$$EIu' = P/3\langle x\rangle^2/2 + c_1$$

$$EIu = P/3\langle x\rangle^3/6 + c_1x + \cancel{c_2}^{\,0} \quad (u(0)=0)$$

$$u(L)=0 \Rightarrow \frac{PL^3}{18} + c_1L = 0 \Rightarrow c_1 = \frac{-PL^2}{18}$$

$$\boxed{u'_{A2} = \frac{-PL^2}{18EI}}$$

$$EI\,u_{C2} = \frac{P}{18}(L/3)^3 + c_1 a = PL^3\left(\frac{1}{18\cdot 27} - \frac{1}{18\cdot 3}\right)$$

$$\boxed{u_{C2} = \frac{PL^3}{EI}\left(\frac{1}{2\cdot 3^5} - \frac{1}{2\cdot 3^3}\right) = \frac{-PL^3}{EI}\frac{4}{3^5}}$$

## 3. Add 2 Solutions

$$u_A' = u_{A1} + u_{A2}$$

$$= \frac{-PL^2}{EI}\left[\frac{5}{81} + \frac{1}{18}\right] = \frac{-PL^2}{EI}\left[\frac{10}{81\cdot 2} + \frac{9}{81\cdot 2}\right]$$

$$\boxed{u_A' = \frac{-19}{162}\frac{PL^2}{EI}} \quad (b)$$

$$u_C = u_{C1} + u_{C2}$$

$$= \frac{-PL^3}{EI}\left[\frac{4}{3^5} + \frac{4}{3^5}\right]$$

$$= \frac{-PL^3}{EI}\frac{8}{3^5}$$

$$\boxed{u_C = \frac{-PL^3}{EI}\frac{8}{243}} \quad (a)$$

(w/ arithmetic help from Jon Kuriloff)