Help for HW-10

Q1, Q3
- are about solving ode's
- look for that in lecture 20 online

Q2

car mass = 1 (m)

gear ratio = anything (G)

peak power = P_{peak} = 10 W (P_{peak})

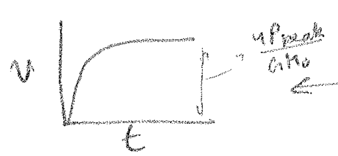
no drag, motor law of straight line

→ the differential equation relating acceleration and speed for car is exact same as that of bike problem in HW-8, page (7)

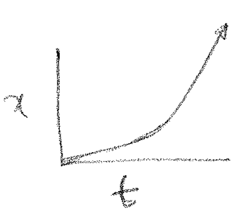
$$\boxed{m\frac{dv}{dt} = GM_0 - \frac{(GM_0)^2}{4P_{peak}}v} \qquad (1)$$

for this we solve as in HW-8

to get



$$V = \frac{4P_{peak}}{GM_0}\left(1 - e^{-\frac{(GM_0)^2}{4mP_{peak}}t}\right) \qquad (2)$$



$$x = \frac{4P_{peak}}{GM_0}\left[t + \frac{\left(e^{-\frac{(GM_0)^2 t}{4mP_{peak}}} - 1\right)}{\frac{(GM_0)^2}{4mP_{peak}}}\right] \qquad (3)$$

ideas

- in here we want to find time to go $x = 15$
- see on last page in the graph of (x, t)
  as t increases x increases
- so you can increase t in steps and find where $x = 15$;
  or use fsolve etc
- ~~the~~

→ but we don't have G

→ we don't know M_0 either!! only m and P_{peak} are given

- but notice in the equation (1) $\boxed{GM_0 \text{ always occur together}}$
  Hence in solutions (2) and (3) also they occur together.
  we can choose any G, that means in effect we can choose any (GM_0).
  all motor with same P_{peak}, but different ~~G~~ M_0 ~~are~~ can be made equivalent by some proper choice of G.

if you are writing differential equation not in terms of ~~G~~ M_0, but say ω_f or C, you'll see that $\frac{G}{\omega_f}$ always occur together. etc.

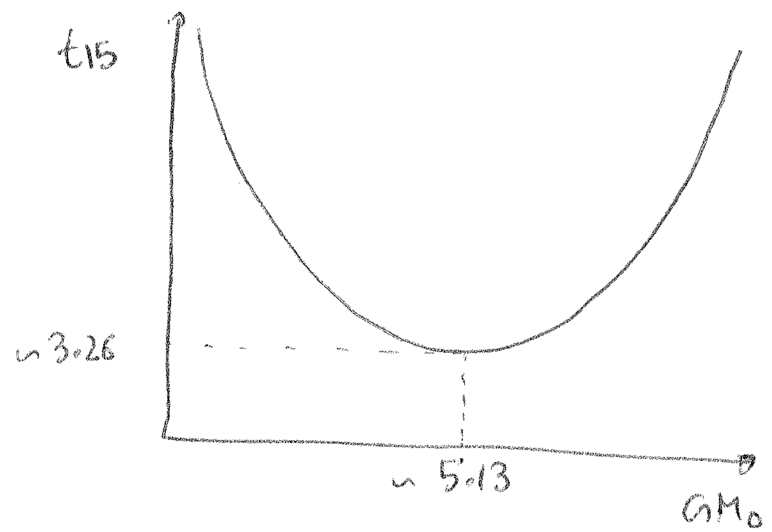
finally

- pick a (GM_0) varying from 0 to say 2000;
- from (3) find t to go $x = 15$, by
  - slowly increasing t in steps until x is just > 15
    (u probably will not get x exactly 15)
  - interpolate to get t for $x = 15$

  → u may neglect this to get approximate ans.
- plot $(t_{\text{for } x=15}$, vs $GM_0)$
- find GM_0 for minimum $t_{...}$

code $\boxed{\text{very simplistic}}$

```
m = 1;     Ppeak = 10;

GM0 = 0.1: .1 : 300 ;          % GM0 starts at .1 because
                               % otherwise we get divide by 0
                               % warning; x has G in denominator.
t15 = zeros(1, length(GM0))    % initialise a vector t15
                               % to record t15 for each
                               % GM0 from 0 to 300

for  i = 1 : length(GM0)

     t = 0;
     while ( 4*Ppeak / GM0(i) * (t + (exp(-GM0(i)^2*t/(4*m*Ppeak))
                 -1)/(GM0(i)^2/(4*m*Ppeak)))
                 < 15 )

      t = t + .01;
     end

     % this loop find time to go just above 15 for a
     % particular GM0(i) according to for loop.
     % this takes time, better use binary search.

     t15(i) = t;

     % this is approximating t15. its accurate
     % upto .01 because t = t + .01, step size = .01
     % u can devise better root finder to get better t
     % or linearly interpolate like we did in page (13)
     % of HW-8; can use binary search like we did to solve
     % roots for cubic long time back etc.
end

plot(GM0, t15).        % now read from graph the optimal GM0
                       % for minimum t15, by zooming in to
                       % desired decimal place.

% after few tries you can find abt GM0 ad. smaller time step increment
```



approximate shape.

⇒) the Best possible $GM_0 \cong 5.13$

⇒) ∴ if your motor has $P_{peak} = 10$ W and $\omega_f = 14000$ rpm

then best G for minimum t15

$$G_{optimal} = \frac{5.13}{M_0} = \frac{5.13}{\frac{4 \times P_{peak}}{\omega_f}} = \frac{5.13}{40/(14000 \times 2\pi/60)} \cong 188$$

⇒) To code all this in more elegant way using ode45 look up the lecture 20 online.