Think of this as a potential energy problem. The potential energy of the water in the bucket will be some constant multiplied by the distance of the center of gravity below the fixed pivot point. Now, if we find a function for the y-coordinate of the center of mass with respect to some tilt angle theta, we can find points of equilibrium and test them for a possible solution. Finding the center of mass isn't particularly neat, but it's nothing more than a complex geometry problem, so it can be done without much pain. The equations on the next page will work as long as $A_1 \geq 0$ and $A_2 \geq 0$ which happens when $|\tan(\theta)| \leq \frac{2 \cdot d}{w}$. More importantly, it will work for some open interval around $\theta = 0$, which turns out to be the point of interest.

$$\bar{y} = \frac{A_1 \cdot \bar{y}_1 + A_2 \cdot \bar{y}_2}{A_1 + A_2} = \frac{12 \cdot d \cdot (d - 2 \cdot h) \cdot \cos^2(\theta) - w^2 \cdot \sin^2(\theta)}{24 \cdot d \cdot \cos(\theta)}$$

(see last page for calculations)

If $\frac{d\bar{y}}{d\theta} \neq 0$ then the center of mass will want to decrease its energy and move towards the lower value, so at a point where $\frac{d\bar{y}}{d\theta} = 0$ there exists a point of equilibrium. In order to determine when this point is stable, we must find $\frac{d^2\bar{y}}{d\theta^2} > 0$.

$$\frac{d\bar{y}}{d\theta} = 0 = \sin\theta \cdot f(\theta, h, w, d) \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$$

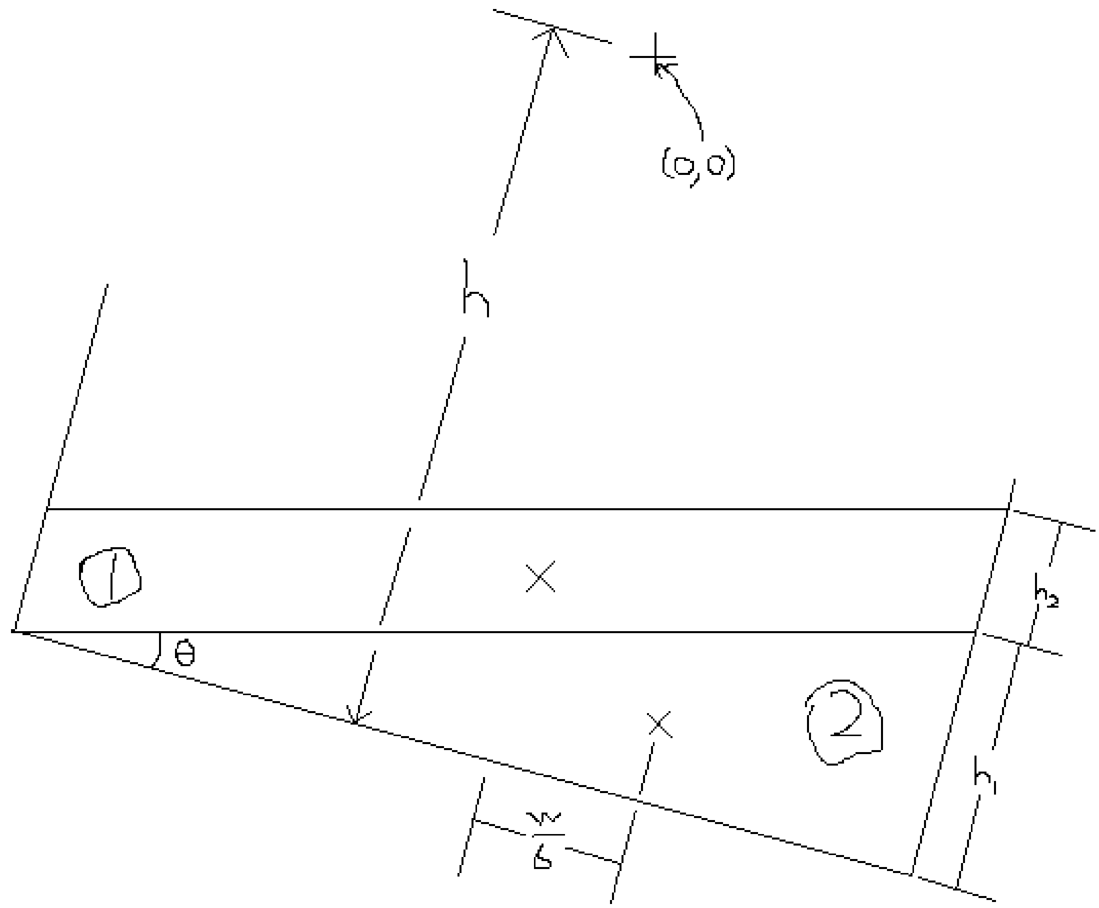
(note $f(\theta, h, w, d)$ is rather messy and unimportant to the problem at hand)

Here we find that $\theta = 0$ is a critical point (as you could have guessed). So now we evaluate $\frac{d^2\bar{y}}{d\theta^2}(\theta)$ for the case when $\theta = 0$.

$$\frac{d^2\bar{y}}{d\theta^2}(0) > 0 \Rightarrow h - \frac{d}{2} - \frac{w^2}{12 \cdot d} > 0$$

$$\boxed{\Rightarrow h > \frac{d}{2} + \frac{w^2}{12 \cdot d}}$$

This is a solution for a suitable value of h for a stable equilibrium at the normal upright position of the bucket ($\theta = 0$).



$$h_1 = w \cdot \tan(\theta)$$

$$2 \cdot h_2 + h_1 = 2 \cdot d \Rightarrow 2 \cdot h_2 = 2 \cdot d - w \cdot \tan(\theta) \Rightarrow h_2 = d - \frac{w}{2} \cdot \tan(\theta)$$

$$A_2 = \frac{1}{2} \cdot w \cdot w \cdot \tan(\theta)$$

$$A_1 = d \cdot w - A_2 = w \cdot \left(d - \frac{w}{2} \cdot \tan(\theta) \right)$$

$$\bar{y}_2 = \left(\frac{1}{3} \cdot w \cdot \tan(\theta) - h \right) \cdot \cos(\theta) - \frac{w}{6} \cdot \sin(\theta)$$

$$\bar{y}_1 = \left(-h + \frac{1}{2} \cdot w \cdot \tan(\theta) + \frac{d}{2} - \frac{1}{4} \cdot w \cdot \tan(\theta) \right) \cdot \cos(\theta)$$