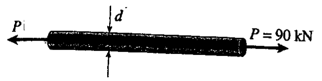
T&AM 202 HW 11

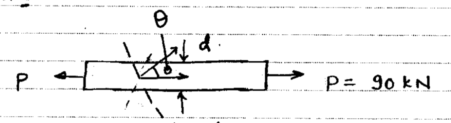
Due Nov 14, 02 Prepared by: Pankaj

Q.1

2.6-2 A circular steel rod of diameter d is subjected to a tensile force $P = 90$ kN (see figure). The allowable stresses in tension and shear are 110 MPa and 50 MPa, respectively. What is the minimum permissible diameter d_{min} of the rod?



PROB. 2.6-2



A = Cross-sectional area

Max Normal stress $\sigma_x = \frac{P}{A}$

Max. Shear stress $\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$ $(\theta = \pm 45°)$.

$\sigma_{allow} = 110$ MPa $\tau_{allow} = 50$ MPa

Because τ_{allow} is less than one half σ_{allow} $\left(\frac{\sigma_{max}}{\tau_{max}} = 2\right)$ the shear stress governs

so

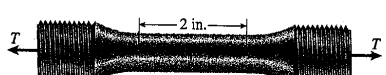
$\tau_{max} = \frac{P}{2A} \Rightarrow 50\,\text{MPa} = \frac{90\,\text{kN}}{2\left(\frac{\pi d^2}{4}\right)}$

$\Rightarrow$ $\boxed{d_{min} = 33.9\text{ mm}}$

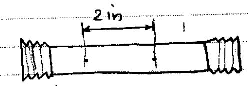
Q.2

2.6-7 During a tension test of a mild-steel specimen (see figure), the extensometer shows an elongation of 0.00140 in. with a gage length of 2 in. Assume that the steel is stressed below the proportional limit and that the modulus of elasticity $E = 30 \times 10^6$ psi.

(a) What is the maximum normal stress σ_{max} in the specimen? (b) What is the maximum shear stress τ_{max}? (c) Draw a stress element oriented at an angle of 45° to the axis of the bar and show all stresses acting on the faces of this element.



Tension test



Elongation = .00140 in & $E = 30 \times 10^6$ psi

strain $\epsilon = \frac{.00140\text{ in}}{2\text{ in}} = .0007$

Using Hooke's Law (steel is stressed below the proportional limit)

$\sigma_x = E\epsilon = 30\times10^6\text{ psi} \times .0007$

$\boxed{\sigma_x = 21,000\text{ psi}}$

a) max. normal stress

σ_x is the max normal stress

so $\boxed{\sigma_{max} = 21,000\text{ psi}}$

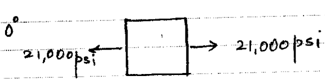
b) Max. Shear Stress

max shear stress is on 45° plane and

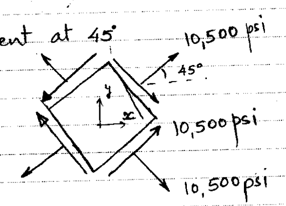
$\boxed{\tau_{max} = \frac{\sigma_x}{2} = \frac{21,000\text{ psi}}{2} = 10,500\text{ psi}}$

(c) stress Element at $\theta = 45°$

element at 0°



stress element at 45°



$\tau_{45°} = \tau_{max} = 10,500$ psi

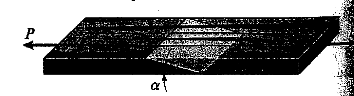
$\sigma_{45°} = \sigma_x \cos^2 45° = \frac{\sigma_x}{2} = 10,500$ psi

$\boxed{\sigma_{45°} = 10,500\text{ psi}}$

Q.3.

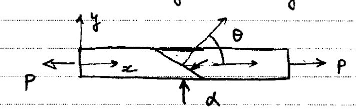
2.6-14 Two boards are joined by gluing along a scarf joint, as shown in the figure. For purposes of cutting and gluing, the angle α between the plane of the joint and the faces of the boards must be between 10° and 40°. Under a tensile load P, the normal stress in the boards is 4.9 MPa.

(a) What are the normal and shear stresses acting on the glued joint if $\alpha = 20°$? (b) If the allowable shear stress on the joint is 2.25 MPa, what is the largest permissible value of the angle α? (c) For what angle α will the shear stress on the glued joint be numerically equal to twice the normal stress on the joint?



PROB. 2.6-14

Two boards joined by a scarf joint



$10° \le \alpha \le 40°$

Due to load P $\sigma_x = 4.9$ MPa

a) Stresses on the joint when $\alpha = 20°$

$\theta = 90 - \alpha = 70°$

$\sigma_\theta = \sigma_x \cos^2\theta$

$= 4.9 \cos^2 70$

$\boxed{\sigma_\theta = 0.57\text{ MPa}}$

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= -4.9\text{ MPa} \sin(70°)\cos(70°)$

$\boxed{\tau_\theta = -1.58\text{ MPa}}$

b) Largest angle α if $\tau_{allow} = 2.25$ MPa

$\tau_{allow} = -\sigma_x \sin\theta \cos\theta$

The shear stress has -ve sign. Its numerical value can not exceed $\tau_{allow} = 2.25$ MPa

Therefore

$$-2.25\,MPa = -\frac{(4.9\,MPa)}{2}\sin 2\theta$$

$\Rightarrow \theta = 33.34^\circ$ or 56.66°

$\alpha = 90-\theta$

$\Rightarrow \alpha = 56.66$ or 33.34

α must be between 10° and 40° so

$\boxed{\alpha = 33.34^\circ}$

Note: if $10 \le \alpha \le 33.34^\circ$

$|\tau_0| \le 2.25$ MPa

$33.34^\circ \le \alpha \le 40^\circ$

$|\tau_0| \ge 2.25$ MPa

(c) what is α if $\tau_\theta = 2\sigma_\theta$

take numerical values

$\Rightarrow |\tau_\theta| = \sigma_x \sin\theta \cos\theta$

$|\sigma_\theta| = \sigma_x \cos^2\theta$

and $\left|\frac{\tau_\theta}{\sigma_\theta}\right| = 2$

$$\Rightarrow \frac{\sigma_x \sin\theta\cos\theta}{\sigma_x \cos^2\theta} = 2$$

$\Rightarrow \tan\theta = 2$

$\theta = 63.43^\circ$

$\Rightarrow \alpha = 90-\theta = 26.6^\circ$

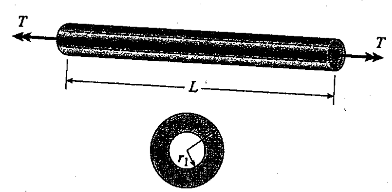
so $\boxed{\alpha = 26.6^\circ}$

Q.4.

(4) 3.2-4 A circular steel tube of length $L = 1.25$ m is loaded in torsion by torques T (see figure).

(a) If the inner radius of the tube is $r_1 = 38$ mm and the measured angle of twist between the ends is 0.6°, what is the shear strain γ_1 (in radians) at the inner surface?

(b) If the maximum allowable shear strain is 0.0004 rad and the angle of twist is to be kept at 0.6° by adjusting the torque T, what is the maximum permissible outer radius $(r_2)_{max}$?



Circular steel tube

$L = 1.25$ m, $r_1 = 38$ mm, $\phi = 0.6\left(\frac{\pi}{180}\right)$ rad $= 0.01047$ rad

$\gamma_{max} = 0.0004$ rad.

a) shear strain at the inner surface

(from eqn 3.5b)

$$\gamma_{min} = \gamma_1 = r_1\frac{\phi}{L} = \frac{(38\,mm)(.01047\,rad)}{(1250\,mm)}$$

$\Rightarrow \boxed{\gamma_1 = 318\times10^{-6}\ rad}$

b) max outer radius

(from eq 3.5a)

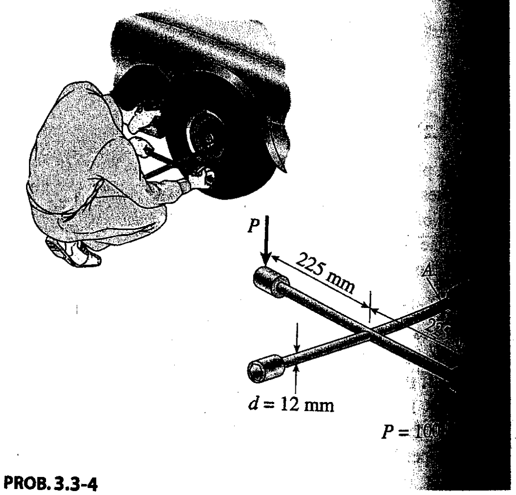
$$\gamma_{max} = \gamma_2 = r_2\frac{\phi}{L} \Rightarrow r_2 = \frac{\gamma_{max}L}{\phi}$$

$$\text{so}\quad (r_2)_{max} = \frac{(.0004\,rad)(1250\,mm)}{.01047\,rad}$$

$\boxed{r_2 = 47.8\ mm}$

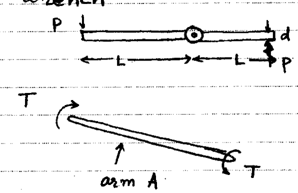
Q.5 3.3-4 While removing a wheel to change a tire, a[...] applies forces $P = 100$ N at the ends of the arms [...] wrench (see figure). The wrench is made of steel wi[...] modulus of elasticity $G = 78$ GPa. Each arm of the [...] is 225 mm long and has a solid circular cross sec[...] diameter $d = 12$ mm.

(a) Determine the maximum shear stress in the [...] that is turning the lug nut (arm A). (b) Determine the [...] of twist (in degrees) of this same arm.



PROB. 3.3-4

Lug wrench



$P = 100$ N

$L = 225$ mm

$d = 12$ mm

$G = 78$ GPa

$T = 2PL$

$= 2(100\,N)(.225\,m)$

$T = 45$ N.m

a) Max Shear Stress

(From eqn 3.12)

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(45\,N.m)}{\pi(.012\,m)^3}$$

$\boxed{\tau_{max} = 132.63\ MPa}$

(b) Angle of twist

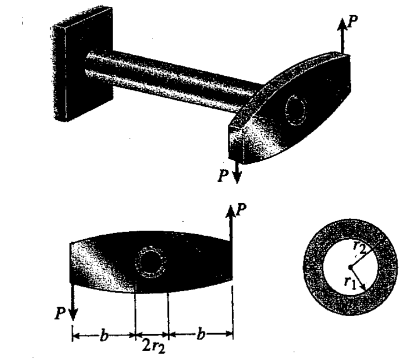
(From eq^n 3.15)

$$\phi = \frac{TL}{GI_p} = \frac{(45\ N\cdot m)(.225\ m)}{(78\ GPa)\left(\frac{\pi}{32}\right)(.012\ m)^4}$$

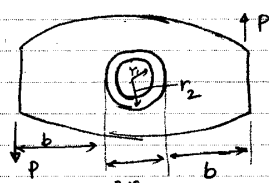
$$\Rightarrow \boxed{\phi = 0.06376\ rad = 3.65^\circ}$$

Q.6.

... of inner radius r_1 and outer radius ... torque produced by forces $P = 900$ lb ... forces have their lines of action at a dis- ... the outside of the tube.
... shear stress in the tube is 6300 psi and ... 1.2 in., what is the minimum permis-



PROB. 3.3-17



$P = 900\ lb$

$b = 5.5\ in$

$\tau_{allow} = 6300\ psi$

$r_1 = 1.2\ in$

$r_2 = ?$

Torsion formula

$$T = 2P(b + r_2)$$

$$I_p = \frac{\pi}{2}(r_2^4 - r_1^4)$$

$$\tau_{max} = \frac{Tr_2}{I_p} = \frac{4P(b+r_2)r_2}{\pi(r_2^4 - r_1^4)}$$

r_2 is the only unknown in above eq^n

$$\Rightarrow (6300\ psi) = \frac{4(900\ lb)(5.5\ in + r_2)r_2}{\pi\left[r_2^4 - (1.2\ in)^4\right]}$$

$$\text{or}\quad \frac{r_2^4 - 2.07360}{r_2(r_2 + 5.5)} - 0.181891 = 0$$

$$\text{or}\quad r_2^4 - 0.181891\,r_2^2 - 1.0004\,r_2 - 2.07360 = 0.$$

Solve using matlab.

$$r_2 = 1.3988\ in \simeq 1.4\ in$$

Matlab code for numerical solution.

```
/home/pkp2/f.m
November 6, 2002

% James Gere 3.3-17     Matlab file f.m
% Numerical solution of the equation

function y=f(r)
y=r.^4-0.181891*r.^2-1.0004*r-2.07360

% just write this file f.m in your home
% directory and give command
%  "fzero(@f,2)" it will give you zero
% of the function near 2

% Output of the matlab program


>> fzero(@f,2)
ans =

    1.3988
```