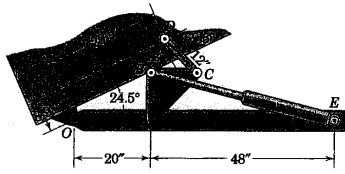
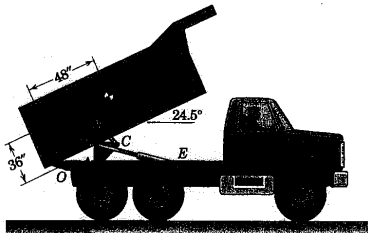


4/115 The design of a hoisting mechanism for the dump truck is shown in the enlarged view. Determine the compression  $P$  in the hydraulic cylinder  $BE$  and the magnitude of the force supported by the pin at  $A$  for the particular position shown, where  $BA$  is perpendicular to  $OAE$  and link  $DC$  is perpendicular to  $AC$ . The dump and its load together weigh 20,000 lb with center of mass at  $G$ . All dimensions for the indicated geometry are given on the figure.  
 Ans.  $P = 26,900$  lb,  $A = 14,600$  lb

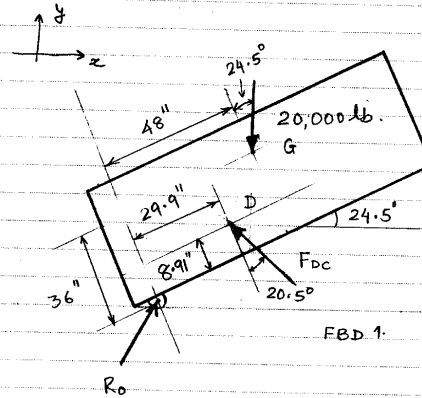


Detail of hoisting mechanism

Problem 4/115

- Note: 1. Pin at point B connects hydraulic cylinders and triangular plate, not the truck box.  
 2. members DC and the hydraulic cylinder BE are two force members so the direction of force in these members will be along them.

Considers FBD of "Dump"



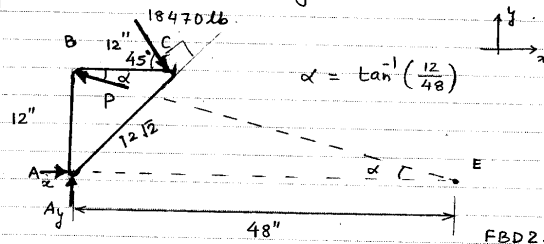
FBD 1

$$\sum M_O = 0$$

$$\Rightarrow -20,000 \text{ lb} (\cos 24.5) 48 + 20,000 \text{ lb} (\sin 24.5) \times 36 + F_{DC} (\cos 20.5) \times 29.9 + F_{DC} (\sin 20.5) \times 8.91 = 0$$

$$\Rightarrow F_{DC} = 18,470 \text{ lb}$$

Considers FBD of "Triangular Plate"



FBD 2

$$\sum M_A = 0$$

$$\Rightarrow P \cos 12 - 18,470 \text{ lb} 12\sqrt{2} = 0$$

$$\Rightarrow P = 26,900 \text{ lb}$$

$$\sum F_x = 0 \Rightarrow A_x - 26,900 \text{ lb} \cos 12 + 18,470 \text{ lb} \cos 45 = 0$$

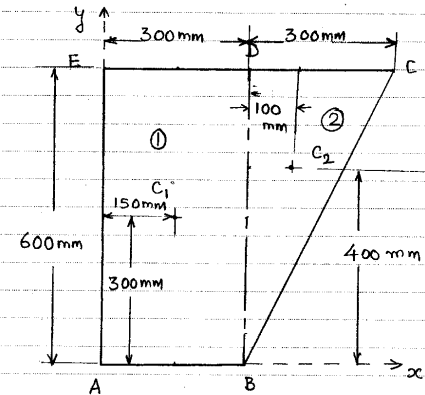
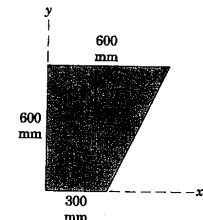
$$\Rightarrow A_x = 13,060 \text{ lb}$$

$$\sum F_y = 0 \Rightarrow A_y + 26,900 \text{ lb} \sin 12 - 18,470 \text{ lb} \sin 45 = 0$$

$$\Rightarrow A_y = 6,530 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(13,060)^2 + (6,530)^2} = 14,600 \text{ lb}$$

5/43 Determine the coordinates of the centroid of the trapezoidal area shown.  
 Ans.  $\bar{X} = 233$  mm,  $\bar{Y} = 333$  mm



We divide the plate in two parts. ① & ②

For Plate ① i.e. ABDE  
 Area:  $A_1 = 600 \times 300 \text{ mm}^2 = 18 \times 10^4 \text{ mm}^2$

Coordinates of centroid  $C_1$

$$x_1 = 150 \text{ mm} \quad y_1 = 300 \text{ mm}$$

For Plate ② i.e. BCD

$$\text{Area: } A_2 = \frac{1}{2} (600 \times 300) = 9 \times 10^4 \text{ mm}^2$$

Coordinates of centroid  $C_2$

$$x_2 = (300 + 100) \text{ mm} = 400 \text{ mm}$$

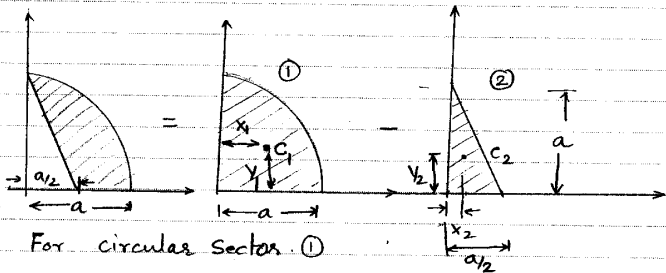
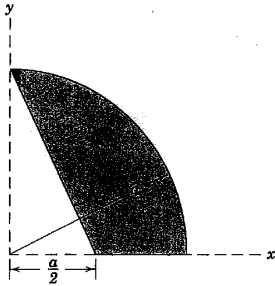
$$y_2 = 400 \text{ mm}$$

$$\bar{X} = \frac{\sum A x}{\sum A} = \frac{(18 \times 10^4 \times 150 + 9 \times 10^4 \times 400)}{18 \times 10^4 + 9 \times 10^4} \text{ mm} = 233 \text{ mm}$$

$$\bar{Y} = \frac{\sum A y}{\sum A} = \frac{(18 \times 10^4 \times 300 + 9 \times 10^4 \times 400)}{18 \times 10^4 + 9 \times 10^4} \text{ mm} = 333 \text{ mm}$$

5/51 By the method of this article, determine the x- and y-coordinates of the centroid of the shaded area of Prob. 5/19, repeated here.

Ans.  $\bar{X} = \frac{7a}{6(\pi - 1)}$ ,  $\bar{Y} = \frac{a}{\pi - 1}$



Area:  $A_1 = \frac{1}{2}(\pi/2 a^2)$  & Centroid  $x_1 = \frac{4a}{3\pi} = y_1$

For Triangle ②

Area:  $A_2 = -\frac{a^2}{4}$  & Centroid  $x_2 = \frac{1}{3}(\frac{a}{2}) = \frac{a}{6}$   
 $y_2 = \frac{1}{3}a = \frac{a}{3}$

For the Given Section

$$X = \frac{\sum Ax}{\sum A} = \frac{\frac{\pi a^2}{4} \cdot \frac{4a}{3\pi} - \frac{a^2}{4} \cdot \frac{a}{6}}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

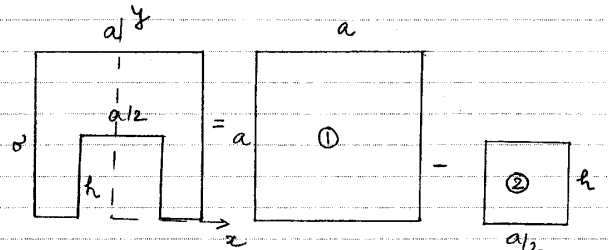
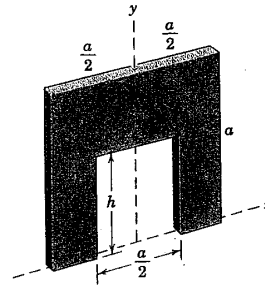
$$\Rightarrow X = \frac{7a}{6(\pi - 1)}$$

$$Y = \frac{\sum AY}{\sum A} = \frac{\frac{\pi a^2}{4} \cdot \frac{4a}{3\pi} - \frac{a^2}{4} \cdot \frac{a}{3}}{\frac{\pi a^2}{4} - \frac{a^2}{4}}$$

$$Y = \frac{a}{\pi - 1}$$

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5/67 Determine the dimension h of the rectangular opening in the square plate which will result in the mass center of the remaining plate being as close to the upper edge as possible. Ans.  $h = 0.586a$



For Plate ①

Area:  $A_1 = a^2$ , centroid  $x_1 = 0$ ,  $y_1 = a/2$

For Plate ②

Area:  $A_2 = -\frac{ah}{2}$ , centroid  $x_2 = 0$ ,  $y_2 = h/2$

For Given plane

$$x = 0 \text{ \& \; } y = \frac{a^2 \cdot \frac{a}{2} - \frac{ah}{2} \cdot \frac{h}{2}}{(a^2 - \frac{ah}{2})} = \frac{1}{2} \frac{(a^2 - h^2/2)}{(a - h/2)}$$

we want to maximize y so

$$\frac{dy}{dh} = 0 \Rightarrow \frac{1}{2} \frac{(a - h/2)(-h) - (a^2 - h^2/2)(-1/2)}{(a - h/2)^2} = 0$$

$$\Rightarrow h^2/4 - ah + a^2/2 = 0$$

$$\Rightarrow h = a(2 \pm \sqrt{2})$$

h has to be less than a so we discard + sign.

$$\Rightarrow h = a(2 - \sqrt{2}) = 0.586a$$