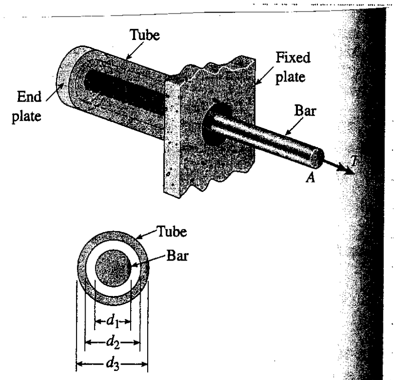
TAM202, HW 12 Solutions Prepared by Vijay Muralidharan
Due on Nov. 19, 2002

3.4-3 A circular tube of outer diameter $d_3 = 2.75$ in. and inner diameter $d_2 = 2.35$ in. is welded at the right-hand end to a fixed plate and at the left-hand end to a rigid end plate (see figure). A solid circular bar of diameter $d_1 = 1.60$ in. is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the end plate.

The bar is 40 in. long and the tube is half as long as the bar. A torque $T = 10{,}000$ lb-in. acts at end A of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity $G = 3.9 \times 10^6$ psi.

(a) Determine the maximum shear stresses in both the bar and tube.

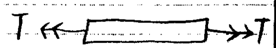
(b) Determine the angle of twist (in degrees) at end A of the bar.

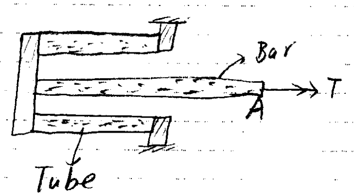


PROB. 3.4-3

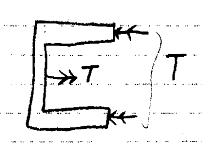
Solution:

Bar





Tube



Tube

$d_3 = 2.75\,in,\ d_2 = 2.35\,in$

$G = 3.9\times10^6$ psi

$L_{tube} = 20\,in$

$(I_P)_{tube} = \frac{\pi}{32}(d_3^4 - d_2^4)$

$= 2.621\,in^4$

Bar

$d_1 = 1.60\,in,$

$G = 3.9\times10^6$ psi

$L_{bar} = 40\,in$

$(I_P)_{bar} = \frac{\pi}{32} d_1^4$

$= 0.6434\,in^4$

(Continued)

Torque $T = 10{,}000$ lb-in

a) Maximum Shear Stresses

Bar: $T_{bar} = \frac{16T}{\pi d_1^3}$

$= \frac{16\times(10000\ lb\text{-}in)}{\pi\,(1.6\,in)^3} = 12{,}430$ psi

Tube: $T_{tube} = \frac{T(d_3/2)}{(I_P)_{tube}} = \frac{(10000\ lb\text{-}in)(\frac{2.75}{2}\,in)}{2.621\,in^4}$

$= 5{,}250$ psi

b) $\boxed{\phi_A = \phi_{bar} + \phi_{tube}}$

Bar: $\phi_{bar} = \frac{T L_{bar}}{G(I_P)_{bar}} = \frac{(10000\,lb\text{-}in)(40\,in)}{(3.9\times10^6\,psi)(0.6434\,in^4)}$

$= 0.1594$ rad

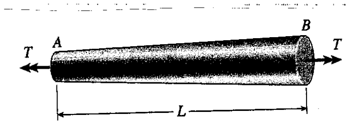
Tube: $\phi_{tube} = \frac{T L_{tube}}{G(I_P)_{tube}} = \frac{(10{,}000\,lb\text{-}in)(20\,in)}{(3.9\times10^6\,psi)(2.621\,in^4)}$

$= 0.0196$ rad

$\phi_A = \phi_{bar} + \phi_{tube} = 0.1790\,rad = 10.3^\circ$

$\therefore \boxed{\phi_A = 10.3^\circ = 0.179\,rad}$

3.4-10



PROBS. 3.4-9 and 3.4-10

3.4-10 The bar shown in the figure is tapered linearly from end A to end B and has a solid circular cross section. The diameter at the smaller end of the bar is $d_A = 25$ mm. The bar is made of steel with shear modulus of elasticity $G = 82$ GPa.

If the torque $T = 90$ N·m and the allowable rate of twist is 0.5°/m, what is the minimum allowable diameter d_B at the larger end of the bar? (*Hint:* Use the results of Example 3-5.)

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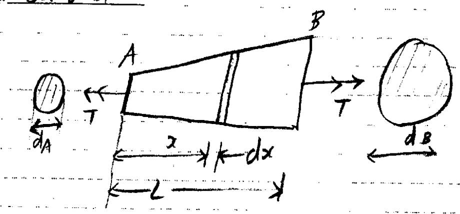

To determine the general expression for Angle of twist:

Diameter d at a distance x from end A,

$$d = d_A + \frac{d_B - d_A}{L}x \longrightarrow (1)$$

Polar moment of Inertia:

$$I_p(x) = \frac{\pi d^4}{32} = \frac{\pi}{32}\left[d_A + \frac{d_B - d_A}{L}x\right]^4$$

$$\therefore \phi = \int_0^L \frac{T\,dx}{G\,I_p(x)} \quad [\because \text{Torque } T \text{ is constant}]$$

$$\phi = \frac{32T}{\pi G}\int_0^L \frac{dx}{\left(d_A + \frac{d_B - d_A}{L}x\right)^4} \longrightarrow (2)$$

The above integral is of the form

$$\int \frac{dx}{(a+bx)^4} \quad \text{where } a = d_A,\ b = \frac{d_B - d_A}{L}$$

Now, $\int \frac{dx}{(a+bx)^4} = -\frac{1}{3b(a+bx)^3} \longrightarrow (3)$

$\therefore$ Using (3), (2) gets simplified to

$$\phi = \frac{32\,TL}{3\pi G(d_B - d_A)}\left(\frac{1}{d_A^3} - \frac{1}{d_B^3}\right) \longrightarrow (4)$$

A convenient form to (4) is

$$\phi = \frac{TL}{G(I_P)_A}\left(\frac{\beta^2 + \beta + 1}{3\beta^3}\right) \longrightarrow (5)$$

where $\beta = \frac{d_B}{d_A}$, $(I_P)_A = \frac{\pi d_A^4}{32}$

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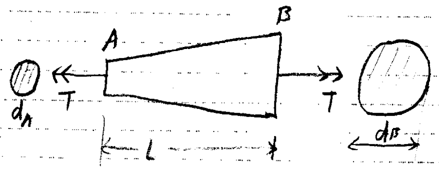
Tapered Bar:



$d_A = 25mm$ $G = 82\,GPa$, $T = 90\ N.m$

$\theta_{allow} = 0.5°/m$

Using ⑤, we get

$$\theta = \frac{T}{G(I_P)_A}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right)$$

where, $\theta = \frac{\phi}{L}$, $\beta = \frac{d_B}{d_A}$, $(I_P)_A = \frac{\pi}{32} d_A^4$

$$\therefore (0.5°/m)\times\left(\frac{\pi}{180}\frac{rad}{deg}\right) = \frac{90\,N.m}{(82GPa)\left(\frac{\pi}{32}\right)(25mm)^4}\left(\frac{\beta^2+\beta+1}{3\beta^3}\right)$$

$$\Rightarrow 0.3049 = \frac{\beta^2+\beta+1}{3\beta^3}$$

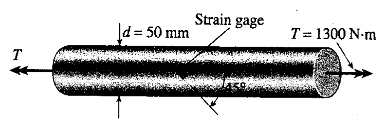
Solving, $\beta = 1.9445$

$\therefore d_B = \beta d_A \Rightarrow \boxed{d_B = 48.6mm}$

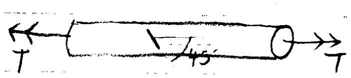
3.5-4

3.5-4 A solid circular bar of diameter d = 50 mm (see figure) is twisted in a testing machine until the applied torque reaches the value T = 1300 N·m. At this value of torque, a strain gage oriented at 45° to the axis of the bar gives a reading $\epsilon = 331 \times 10^{-6}$.

Determine the shear modulus G of the material.



PROB. 3.5-4



Strain gage at 45°: $\epsilon_{max} = 331\times10^{-6}$

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$d = 50mm$, $T = 1300\ N\text{-}m$

From Eq. 3-33 (Gere),

$\gamma_{max} = 2\,\epsilon_{max} = 662\times10^{-6}$

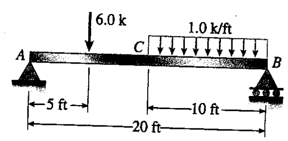
$$\tau_{max} = G\gamma_{max} = \frac{16\,T}{\pi d^3}$$

$$\Rightarrow G = \frac{16\,T}{\pi d^3 \gamma_{max}} = \frac{16\,(1300\,N\text{-}m)}{\pi\,(0.05m)^3(662\times10^{-6})}$$

$\boxed{G = 80.0\,GPa}$

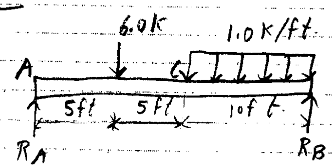
4.3-1

4.3-1 Determine the shear force V and bending moment M at the midpoint C of the simple beam AB shown in the figure.



PROB. 4.3-1

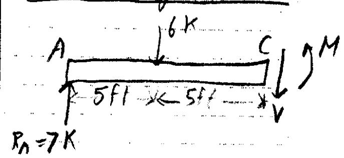
Solution:



$\Sigma M_B = 0$: $(6K\times15ft)+(10K\times5ft) - R_A\times20ft = 0$

$\Rightarrow R_A = 7.0K$

FBD of segment AC



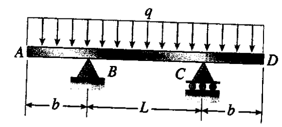
$\Sigma F_y = 0$: $V = 7K - 6K = 1.0K$

$\Sigma M_C = 0$: $(6K\times5ft) + M - (7K\times10ft) = 0$

$\Rightarrow \boxed{M = 40K\text{-}ft}$

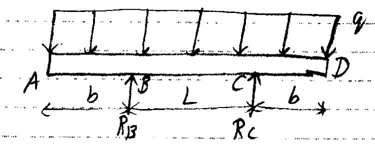
4.3-4

4.3-4 The beam $ABCD$ shown in the figure has overhangs at each end and carries a uniform load of intensity q. For what ratio b/L will the bending moment at the midpoint of the beam be zero?



PROB. 4.3-4

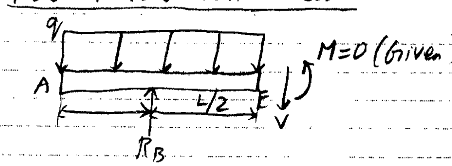
Solution:



From symmetry and equilibrium in vertical direction,

$$R_B = R_C = \frac{q}{2}(L+2b)$$

FBD of left half of beam:



$\Sigma M_E = 0$ (+↺):

$$-R_B\cdot\frac{L}{2} + q\cdot\frac{1}{2}\left(\frac{L}{2}+b\right)^2 = 0$$

$$\Rightarrow \frac{q}{2}\left(\frac{L}{2}+b\right)^2 = \frac{qL}{2}\left(\frac{L}{2}+b\right)$$

$$\Rightarrow \frac{L}{2}+b = L \Rightarrow b = L/2$$

or $\boxed{\frac{b}{L} = \frac{1}{2}}$