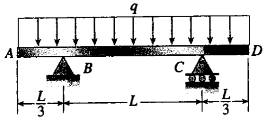
TAM 202, HW 13 SOLUTIONS, HW DUE ON 26th Nov, 2002, (prepared by Peeyush Bhargava)

4.5-9 Beam *ABCD* is simply supported at *B* and *C* and has overhangs at each end (see figure). The span length is *L* and each overhang has length *L*/3. A uniform load of intensity *q* acts along the entire length of the beam.

Draw the shear-force and bending-moment diagrams for this beam.

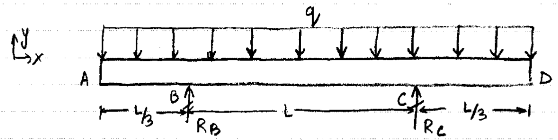


PROB. 4.5-9

Solution

Span length = distance between B and C = L.
length of each overhang = L/3

FBD for beam ABCD.



By symmetry $R_B = R_C = (5qL/6)$
In gory detail:

$$\Sigma F_y = 0 = -q\left(L + \frac{L}{3} + \frac{L}{3}\right) + R_B + R_C$$

$$\text{or} \quad R_B + R_C = \frac{5L}{3}q$$

To find other equation to solve for reactions, do moment balance about B.

(Anticlockwise moments (+))

$\Sigma M_B = 0$

$$+q\left(\frac{L}{3}\right)\left(\frac{L}{6}\right) + R_C(L) - q\left(\frac{L}{3} + L\right)\left(\frac{L}{3} + L\right)\frac{1}{2} = 0$$

$$\text{or} \quad R_C \cdot L = q\left(\frac{15}{18}L^2\right)$$

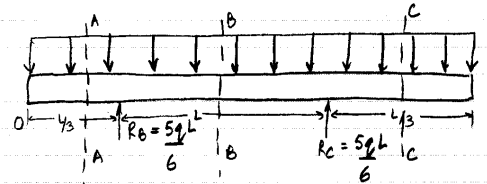
$$\text{or} \quad \boxed{R_C = \frac{5}{6}qL}$$

As you could do by inspection

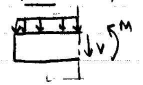
$$\boxed{R_B = \frac{5L}{3}q - \frac{5L}{6}q = \frac{5L}{6}q}$$

Note: the moment due to a distributed load with constant magnitude is $\frac{qx^2}{2}$ where q is the magnitude & x is the distance from the point about which moment is taken

To draw shear force diagram, start from either end and take cuts [i.e. Method 1: FBD's]



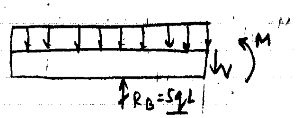
Cut A-A



$\Sigma F_y = 0 \Rightarrow V = -qx$ (Linear function of x with negative slope) −(1)

$M = -\frac{qx^2}{2}$ (quadratic function of x with (−) slope). −(2)

Cut B-B



$$\Sigma F_y = 0 \quad V = R_B - qx = \frac{5}{6}qL - qx$$

$$= q\left(\frac{5}{6}L - x\right) \quad -(3)$$

See that q = 0 when $x = \frac{5}{6}L$
the distance of this point from B
$= \frac{5}{6}L - \frac{L}{3} = \frac{L}{2}$

$\Sigma M = 0$

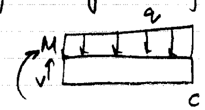
$$\Rightarrow M = \frac{5}{6}qL\left(x - \frac{L}{3}\right) - \frac{qx^2}{2}$$

$$\boxed{M = -\frac{qx^2}{2} + \frac{5}{6}qLx - \frac{5}{18}qL^2} \quad -(4)$$

M = 0 will have 2 roots.

cut C-C can continue the same way by taking all the forces and moments

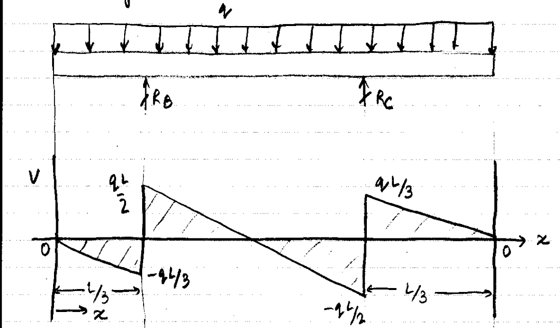
Or by starting over from the other side.

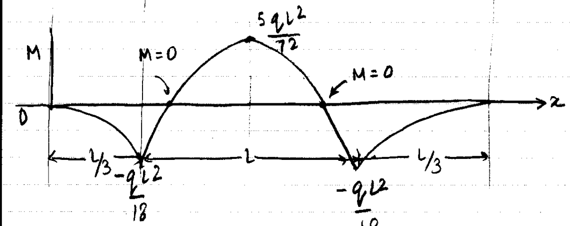


$M = -qx^2/2$ −(5)

$V = +qx.$ −(6)

Using (1), (3), (6), Draw shear force diagram.
Note: Diagram Not to Scale.





Use (2), (4), (5) to draw Bending moment diagram. Diagram NOT to Scale.

Method 2:

Use the equations $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = V$, integrate, find out V and M in terms of x and V in terms of x.

In this problem $q = q_0$ (a constant).

$$\frac{dV}{dx} = -q \qquad 0 \le x \le L/3$$

[x measured from Left hand side].

Integrate,

$$V = V(0) - \int_0^x q\,dx = V(0) - qx$$

To calculate $V(0)$, use $V=0$ at $x=0$, so $V_0 = 0$.

$$\text{so } V = -qx \qquad 0 \le x \le L/3 \quad -(1)$$

Similarly

$$\frac{dM}{dx} = V \Rightarrow M = \int_0^x V\,dx + M_0.$$

use $V = -qx \Rightarrow M = M(0) + \int_0^x (-qx)\,dx$

or $\boxed{M = M(0) - \frac{qx^2}{2}}$

To calculate $M(0)$ use $M=0$ at $x=0$.

$\Rightarrow M(0) = 0$.

$$\text{So } \boxed{M = -\frac{qx^2}{2}} \qquad 0 \le x \le L/3 \quad -(2)$$

See that the equation we got here is the same as when we took the cut A-A & calculated V & M.

Now take $\frac{L}{3} < x < \frac{4L}{3}$. [have to take x in the region where there is no discontinuity in load or moment (no concentrated load or moment]

So using the same equation $\left[\frac{dV}{dx} = -q,\ \frac{dM}{dx} = V\right]$ get

$$\left.\begin{aligned}\frac{dV}{dx} &= -q \\ \frac{dM}{dx} &= +V\end{aligned}\right\} \quad \frac{L}{3} < x < \frac{4L}{3}$$

Integrate for V

$$V = V\left(\tfrac{L}{3}\right)^+ - \int_{L/3}^{x} q\,dx$$

$$\text{or } V = V\left(\tfrac{L}{3}\right)^+ - q\left(x - L/3\right)$$

$$\text{Now } V\left(\tfrac{L}{3}\right)^+ = \underbrace{-\frac{qL}{3}}_{V\left(\frac{L}{3}\right)^-} + \underbrace{\frac{5qL}{6}}_{\text{jump in V due to reaction at B } (R_B)}$$

$$V\left(\tfrac{L}{3}\right)^+ = \frac{qL}{2}.$$

$$\text{so } V = V\left(\tfrac{L}{3}\right)^+ - qx + qL/3$$

$$V = \frac{qL}{2} + \frac{qL}{3} - qx$$

$$\text{or } \boxed{V = \frac{5qL}{6} - qx} \quad -(3).$$

Integrate for M

$$M = M\left(\tfrac{L}{3}\right)^+ + \int_{L/3}^{x}\left(\frac{5qL}{6} - qx\right)dx$$

$$\text{or } M = M\left(\tfrac{L}{3}\right)^+ + \frac{5qL}{6}\left(x - \frac{L}{3}\right) - \frac{q}{2}\left(x - \frac{L}{3}\right)^2$$

$$M\left(\tfrac{L}{3}\right)^+ = -\frac{qL^2}{18}.$$

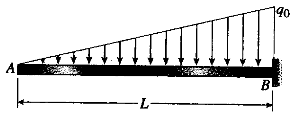
$$\text{so } M = -\frac{qL^2}{18} + \frac{5qL}{6}\left(x - L/3\right) - \frac{q}{2}\left(x - L/3\right)^2$$

$$\text{or } \boxed{M = -\frac{qx^2}{2} + \frac{5qLx}{6} - \frac{5}{18}qL^2}$$

Similarly V & M can be determined for $\frac{4L}{3} < x < \frac{5L}{3}$, or can start over from the other side as done for method 1. Plot the bending moment & the shear force diagrams using equations found out. [of course, would be the same].

4-5-10

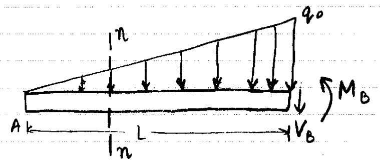
4.5-10 Draw the shear-force and bending-moment diagrams for a cantilever beam AB supporting a linearly varying load of maximum intensity q_0 (see figure).



PROB. 4.5-10

Solution

FBD for the cantilever beam



$\Sigma F_y = 0 \Rightarrow -V_B - (q_0 * L)\frac{1}{2} = 0$

$\frac{1}{2}(q_0 L)$ = Area of triangular region = magnitude of force.

$\Rightarrow \boxed{V_B = -\frac{q_0 L}{2}}$ — ①.

$\Sigma M_B = 0$

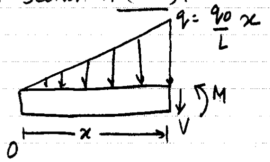
$\Rightarrow M_B - \underbrace{\left(\frac{q_0 L}{2}\right)\left(\frac{L}{3}\right)}_{\text{Magnitude of moment due to distributed load}} = 0$

$\Rightarrow \boxed{M_B = \frac{q_0 L^2}{6}}$ — ②.

Now to draw shear force & bending moment diagram take a cut (n-n) & look at V & M.

Method 1

FBD for Section A-(N-N).



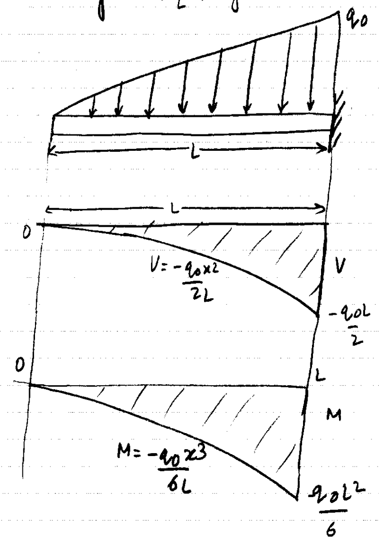
$\Sigma F_y = 0 \Rightarrow V + \frac{1}{2}\left(\frac{q_0 x}{L}\right)(x) = 0$

or $\boxed{V = -\frac{q_0 x^2}{2L}}$ -③

$\Sigma M = 0 \Rightarrow M + \frac{1}{2}\left(\frac{q_0 x}{L}\right)(x)\left(\frac{x}{3}\right) = 0$

$\Rightarrow \boxed{M = -\frac{q_0 x^3}{6L}}$ -④

Using ③ & ④, Draw shear force & Bending moment diagram. [Diagrams not to scale]



Method 2.

Use $\frac{dV}{dx} = -q$ & $\frac{dM}{dx} = +V$.

Here q is a function of x.

$q = \frac{q_0 x}{L}$.

$\frac{dV}{dx} = -\frac{q_0 x}{L}$.

$V = V(0) - \int_0^x \frac{q_0 x}{L}\, dx$

or $V = V(0) - \frac{q_0 x^2}{2L}$

$V(0) = 0 \quad \because V = 0$ at $x = 0$.

So $\boxed{V = -\frac{q_0 x^2}{2L}}$. -①

$M = M(0) + \int_0^x V\, dx$

So $M = M(0) - \int_0^x \frac{q_0 x^2}{2L}\, dx$

or $M = -\frac{q_0 x^3}{6L} + M(0)$

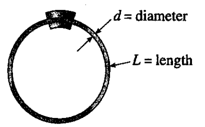
$M(0) = 0 \quad \because M = 0$ at $x = 0$.

$\boxed{M = -\frac{q_0 x^3}{6L}}$. -②

Use ① & ② to plot Bending moment & Shear force diagrams

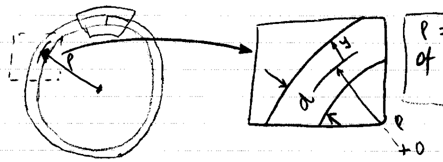
5-4-2

5.4-2 A copper wire having diameter $d = 3$ mm mm is bent into a circle and held with the ends just touching (see figure). If the maximum permissible strain in the copper is $\epsilon_{max} = 0.004$, what is the shortest length L of wire that can be used?



PROB. 5.4-2

Solution: $d = 3\ mm = 3 * 10^{-3}\ m$.
$\epsilon_{max} = 0.004$



$\boxed{\rho = \text{radius of curvature} = L/2\pi}$

Now

$$\epsilon_{max} = \frac{y}{\rho} = \frac{d/2}{L/2\pi} = \frac{\pi d}{L}$$

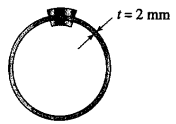
$$\Rightarrow L_{min} = \frac{\pi d}{\epsilon_{max}} = \frac{\pi(3*10^{-3} m)}{0.004}$$

or $\boxed{L_{min} = 2.36\ m}$

5-5-2

5.5-2 A thin strip of hard copper ($E = 113$ GPa) having length $L = 2$ m and thickness $t = 2$ mm is bent into a circle and held with the ends just touching (see figure).

(a) Calculate the maximum bending stress σ_{max} in the strip. (b) Does the stress increase or decrease if the thickness of the strip is increased?



PROB. 5.5-2

Solution: $E = 113\ GPa$.
$L = 2\ m$.
$t = 2\ mm = 2 \times 10^{-3}\ m$.

a) Calculate σ_{max}.

$$\epsilon = \frac{y}{\rho} \Rightarrow \sigma = E\epsilon = \frac{Ey}{\rho}.$$

ρ = radius of curvature $= \frac{L}{2\pi}$ and $y = t/2$

$$\Rightarrow \sigma_{max} = \frac{E(t/2)}{(L/2\pi)} = \frac{\pi E t}{L}$$

or $\sigma_{max} = \frac{\pi(113e9\ GPa)(2e(-3)\ m)}{2\ m}$

or $\sigma_{max} = 355\ MPa$.

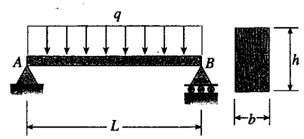
b) $\sigma_{max} \propto t$.

If t increases, σ_{max} increases.

5-5-4

5.5-4 A simply supported wood beam AB with span length $L = 3.75$ m carries a uniform load of intensity $q = 6.4$ kN/m (see figure).

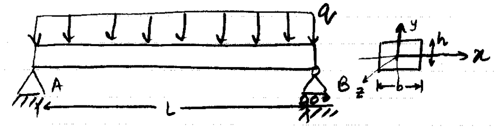
Calculate the maximum bending stress σ_{max} due to the load q if the beam has a rectangular cross section with width $b = 150$ mm and height $h = 300$ mm.



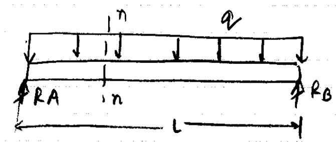
PROB. 5.5-4

Solution: The maximum bending stress occurs where the bending Moment is maximum.

So $\sigma_{max} = \frac{M_{max}\, y_{max}}{I}$



FBD for the beam.

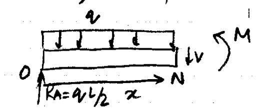


$\Sigma F_y = 0 \quad R_A + R_B = qL$

$\Sigma M_B = 0 \quad \Rightarrow -R_A \cdot L + \frac{qL^2}{2} = 0$

or $\boxed{R_A = \frac{qL}{2} \quad \& \quad R_B = \frac{qL}{2}}$

take a cut, FBD for AN



$\Sigma F_y = 0 \Rightarrow V = \frac{qL}{2} - qx$

$\Sigma M = 0 \Rightarrow M = \frac{qLx}{2} - \frac{qx^2}{2}$

M_{max} occurs at $\frac{dM}{dx} = 0$ or $V = 0$

when $V = 0$ $x = (L/2)$.

$M_{max} = M(@\,x = L/2) = \frac{qL^2}{8}$.

$I = I_x = \frac{1}{12} bh^3$ [From Appendix, D, Pg 877]

$y_{max} = h/2$.

$\Rightarrow \sigma_{max} = \frac{M_{max} y_{max}}{I_x} = \frac{\frac{qL^2}{8} * (h/2)}{\frac{1}{12}(bh^3)} = \frac{3qL^2}{4bh^2}$

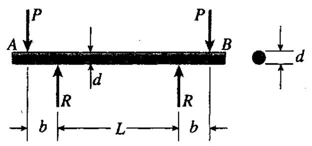
or $\sigma_{max} = \frac{3(6.4\ kN/m)(3.75\ m)^2}{4(150 * 10^{-3}\ m)(300 * 10^{-3}\ m)^2}$

or $\boxed{\sigma_{max} = 5\ MPa}$

(5-5-6)

5.5-6 A freight-car axle *AB* is loaded as shown in the figure, with the forces *P* representing the car loads (transmitted through the axle boxes) and the forces *R* representing the rail loads (transmitted through the wheels). The diameter of the axle is $d = 80$ mm, the wheel gauge is $L = 1.45$ m, and the distance between the forces *P* and *R* is $b = 200$ mm.

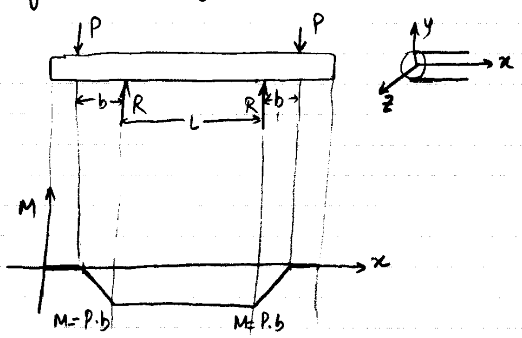
Calculate the maximum bending stress σ_{max} in the axle if $P = 46.5$ kN.



PROB. 5.5-6

Solution: $R = P = 46.5\ kN$.

The Bending moment diagram looks like this: -



$M_{max} = P.b$.

$y_{max} = d/2$

$I = I_x = \frac{\pi d^4}{64}$ [From Appendix D, 9 Pg 879]

So $\sigma_{max} = \frac{M_{max} y_{max}}{I_x} = \frac{P.b.d/2}{(\pi d^4/64)} = \frac{32Pb}{\pi d^3}$.

or $\boxed{\sigma_{max} = \frac{32(46.5 kN)(200 * 10^{-3}\ m)}{\pi(80 * 10^{-3}\ m)^3} = 185.0\ MPa}$