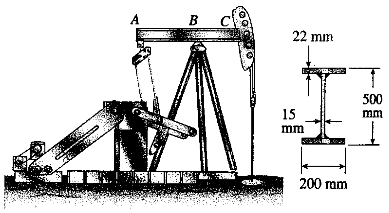
TAM 202 HW 14 Solution prepared by Zhongping Bao & Tian Tang.

5.5-8, 5.5-12, 5.6-16 (due Dec. 3rd, 2002)

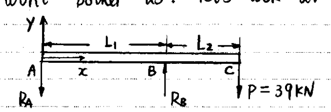
5.5-8 The horizontal beam *ABC* of an oil-well pump has the cross section shown in the figure. If the vertical pumping force acting at end *C* is 39 kN, and if the distance from the line of action of that force to point *B* is 4.5 m, what is the maximum bending stress in the beam due to the pumping force?



PROB. 5.5-8

① Find M_{max}

In this problem, we are not given the length AB, but this won't bother us. let's look at FBD of ABC:

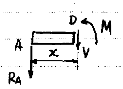


L_1 is not given
$L_2 = 4.5m$

$\Sigma M_{/A} = 0 = R_B L_1 - P(L_1+L_2) \Rightarrow R_B = P(L_1+L_2)/L_1$

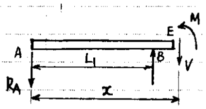
$\Sigma F_y = 0 = R_B - R_A - P \Rightarrow R_A = PL_2/L_1$

Now we can find bending moment distribution in ABC.



$\Sigma M_{/D} = 0 = M + R_A x$

$\Rightarrow M = -Px L_2/L_1 \quad (0 \le x \le L_1)$

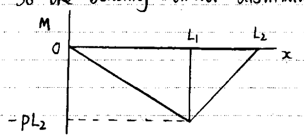


$\Sigma M_{/E} = 0 = M + R_A x - R_B(x - L_1)$

$\Rightarrow M = P(x - L_1 - L_2)$

$(L_1 \le x \le L_2)$

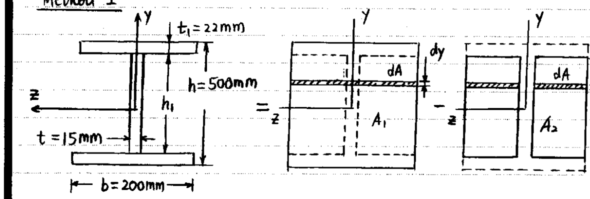
So the bending moment distribution are simply straight lines



$\Rightarrow |M_{max}| = PL_2$ occurs at $x = L_1$, and L_2 is given in this problem.

② Find I for an I-beam

Method I



$$I = I_1 - I_2$$

where

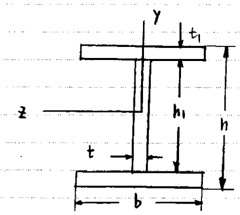
$$I_1 = \int_{A_1} y^2 dA = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \cdot b dy = \frac{bh^3}{12}$$

$$I_2 = \int_{A_2} y^2 dA = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} y^2 \cdot (b-t) dy = \frac{(b-t)h_1^3}{12}$$

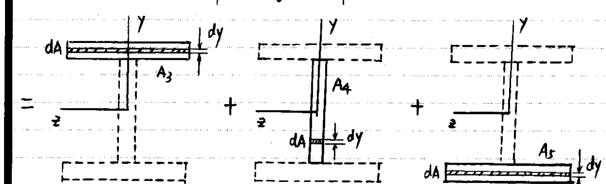
$$= \frac{(b-t)(h-2t_1)^3}{12}$$

$$\Rightarrow I = I_1 - I_2 = \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

Method II



I



$$= I_3 + I_4 + I_5$$

where

$$I_3 = \int_{A_3} y^2 dA = \int_{\frac{h_1}{2}}^{\frac{h}{2}} y^2 \cdot b dy = \frac{b}{24}[h^3 - h_1^3]$$

$$I_4 = \int_{A_4} y^2 dA = \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} y^2 \cdot t dy = \frac{th_1^3}{12}$$

$$I_5 = \int_{A_5} y^2 dA = \int_{\frac{h}{2}}^{\frac{h_1}{2}} y^2 \cdot b dy = \frac{b}{24}[h^3 - h_1^3]$$

(Continued)

$$\Rightarrow I = I_3 + I_4 + I_5$$

$$= \frac{b}{24}[h^3 - h_1^3] + \frac{th_1^3}{12} + \frac{b}{24}[h^3 - h_1^3]$$

$$= \frac{bh^3}{12} - \frac{(b-t)h_1^3}{12}$$

$$= \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

Substitute numbers into I

$$\Rightarrow I = \frac{bh^3}{12} - \frac{(b-t)(h-2t_1)^3}{12}$$

$$= \frac{1}{12}[(200mm)(500mm)^3 - (200mm - 15mm)(500mm - 2\times 22mm)^3]$$

$$= 6.21 \times 10^8 \ mm^4$$

③ Find σ_{max}

$$|\sigma_{max}| = \left|\frac{M_{max}\, y_{max}}{I}\right|$$

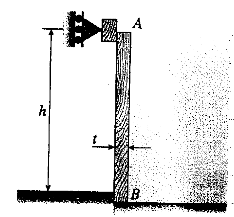
$$= \frac{PL_2(\frac{h}{2})}{I}$$

$$= \frac{(39kN)(4.5m)(\frac{500mm}{2})}{6.21\times 10^8 \ mm^4}$$

$$= \boxed{70.6 \ MPa}$$

5.5-12

5.5-12 A small dam of height $h = 2.4$ m is constructed of vertical wood beams *AB* of thickness $t = 150$ mm, as shown in the figure. Consider the beams to be simply supported at the top and bottom. Determine the maximum bending stress σ_{max} in the beams, assuming that the weight density of water is $\gamma = 9.81$ kN/m^3.



PROB. 5.5-12

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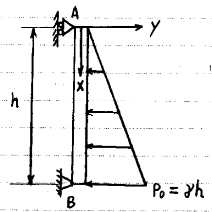
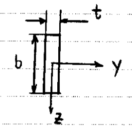
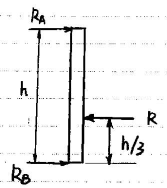
① water pressure on the beam





② Find R_A or R_B (reaction forces).

FBD:



where $R = \bar{P}\,(bh) = \frac{P_0}{2}(bh)$

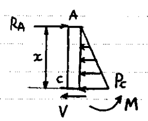
$= \frac{\gamma b h^2}{2}$

$\Sigma M_{/B} = 0 = R\left(\frac{h}{3}\right) - R_A\,(h)$

$\Rightarrow R_A = \gamma b h^2/6$

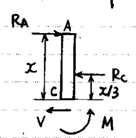
③ Find M_{max}

FBD:



$P_c = \frac{P_0 x}{h} = \gamma x$

This FBD is equivalent to the following one:



$R_c = \frac{P_c}{2}(bx)$

$= \frac{\gamma b}{2} x^2$

$\Sigma M_{/C} = 0 = M + R_c\left(\frac{x}{3}\right) - R_A \cdot x$

$\Rightarrow M = R_A x - R_c x/3 = \gamma b h\left[\frac{hx}{6} - \frac{x^3}{6h}\right]$

To find M_{max}, we set $\frac{dM}{dx} = 0$.

$\frac{dM}{dx} = 0 = \gamma b h\left[\frac{h}{6} - \frac{x^2}{2h}\right] \Rightarrow x = h/\sqrt{3}$

$\Rightarrow M_{max} = M\left(x = \frac{h}{\sqrt{3}}\right) = \gamma b h\left[\frac{h}{6}\frac{h}{\sqrt{3}} - \frac{1}{6h}\left(\frac{h}{\sqrt{3}}\right)^3\right] = \frac{\gamma b h^3}{9\sqrt{3}}$

(Continued)

④ Find σ_{max}

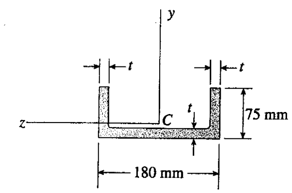
$$|\sigma_{max}| = \left|\frac{M_{max}\, y_{max}}{I}\right| = \frac{\gamma b h^3}{9\sqrt{3}} \cdot \frac{\frac{t}{2}}{\frac{bt^3}{12}} = \frac{2\gamma h^3}{3\sqrt{3}\,t^2}$$

$$= \frac{2\,(9.81\ \mathrm{kN/m^3})\,(2.4\,\mathrm{m})^3}{3\sqrt{3}\ (150\,\mathrm{mm})^2}$$

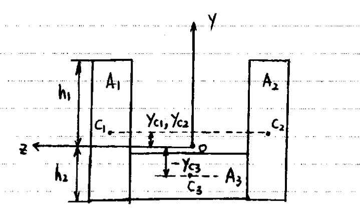
$$= \boxed{2.32\ \mathrm{MPa}}$$

5.6-16

5.6-16 A beam having a cross section in the form of a channel (see figure) is subjected to a bending moment acting about the *z* axis. Calculate the thickness *t* of the channel in order that the bending stresses at the top and bottom of the beam will be in the ratio 7:3, respectively.



PROB. 5.6-16



σ_1: stress at the top

σ_2: stress at the bottom

① First, we can use the condition $\left|\frac{\sigma_1}{\sigma_2}\right| = \frac{7}{3}$ to find the position of neutral axis:

$\sigma = -\frac{My}{I} \Rightarrow \left|\frac{\sigma_1}{\sigma_2}\right| = \frac{h_1}{h_2} = \frac{7}{3}$ (1)

total height $h_1 + h_2 = 75\,\mathrm{mm}$ (2)

(1) & (2) $\Rightarrow$ $h_1 = 52.5\,\mathrm{mm}$ $h_2 = 22.5\,\mathrm{mm}$

② Now, the areas A_i $(i = 1, 2, 3)$ & the position of their centroid y_{ci} depend on the thickness t, but the centroid of the whole section is point O, i.e.

$\sum_{i=1}^{3} y_{ci} A_i = y_0 = 0$ (3)

where c_i is the centroid of the area A_i $(i = 1, 2, 3)$

(Continued)

So we can use (3) to find t. In (3)

$A_1 = A_2 = t\,(75\,\mathrm{mm})$

$A_3 = t\,(180\,\mathrm{mm} - 2t)$

$y_{c1} = y_{c2} = \frac{75}{2}\,\mathrm{mm} - h_2 = 15\,\mathrm{mm}$

$y_{c3} = \frac{t}{2} - h_2 = \frac{t}{2} - 22.5\,\mathrm{mm}$

Sub into (3)

$\Rightarrow 2t\,(75\,\mathrm{mm})(15\,\mathrm{mm}) + t\,(180\,\mathrm{mm} - 2t)\left(\frac{t}{2} - 22.5\,\mathrm{mm}\right) = 0$

$\Rightarrow t^2 - (135\,\mathrm{mm})\,t + 1800\,\mathrm{mm}^2 = 0$

$\Rightarrow t = \boxed{15\,\mathrm{mm}}$