Statics and Strength of Materials Formula Sheet

(12/12/94, revised 5/10/01 — A. Ruina)

Not given here are the conditions under which the formulae are accurate or useful.

Basic Statics

Free Body Diagram

A $\vec{\mathbf{F}}$FBD is a picture of any system for which you would like to apply mechanics equations and of all the external forces and torques which act on the system.

Action & Reaction

If $\mathcal{A}$ feels force $\vec{\mathbf{F}}$ and couple $\vec{\mathbf{M}}$ from $\mathcal{B}$.
then $\mathcal{B}$ feels force $-\vec{\mathbf{F}}$ and couple $-\vec{\mathbf{M}}$ from $\mathcal{A}$.
(With $\vec{\mathbf{F}}$ and $-\vec{\mathbf{F}}$ acting on the same line of action.)

Force and Moment Balance

These equations apply to every system in equilibrium:

$$\overbrace{\sum_{\text{All external forces}} \vec{\mathbf{F}} = \vec{\mathbf{0}}}^{\textit{Force Balance}} \qquad \overbrace{\sum_{\text{All external torques}} \vec{\mathbf{M}}_{/C} = \vec{\mathbf{0}}}^{\textit{Moment Balance about pt C}}$$

- The torque $\vec{\mathbf{M}}_{/C}$ of a force depends on the reference point C. But, for a body in equilibrium, and for any point C, the sum of all the torques relative to point C must add to zero).
- Dotting the force balance equation with a unit vector gives a scalar equation, e.g. $\left\{\sum \vec{\mathbf{F}}\right\} \cdot \hat{\imath} = 0 \quad \Rightarrow \quad \sum F_x = 0.$
- Dotting the moment balance equation with a unit vector gives a scalar equation, e.g. $\left\{\sum \vec{\mathbf{M}}_{/C}\right\} \cdot \hat{\lambda} = 0 \quad \Rightarrow$ net moment about axis in direction $\hat{\lambda}$ through $C = 0$.

Some Statics Facts and Definitions

- The moment of a force is unchanged if the force is slid along its line of action.
- For many purposes the words 'moment', 'torque', and 'couple' have the same meaning.
- **Two-force body.** If a body in equilibrium has only two forces acting on it then the two forces must be equal and opposite and have a common line of action.
- **Three-force body.** If a body in equilibrium has only three forces acting on it then the three forces must be coplanar and have lines of action that intersect at one point.
- **truss:** A collection of weightless two-force bodies connected with hinges (2D) or ball and socket joints (3D).
- **Method of joints.** Draw free body diagrams of each of the joints in a truss.
- **Method of sections.** Draw free body diagrams of various regions of a truss. Try to make the FBD cuts for the sections go through only three bars with unknown forces (2D).
- **Caution:** Machine and frame components are often **not** two-force bodies.
- **Hydrostatics:** $p = \rho g h, \quad F = \int p\, dA$

Stress, strain, and Hooke's Law

	Stress	Strain	Hooke's Law
Normal:	$\sigma = P_{\perp}/A$	$\epsilon = \delta/L_0 = \frac{L - L_0}{L_0}$	$\sigma = E\epsilon$ $[\,\epsilon = \sigma/E + \alpha\Delta T\,]$ $\epsilon_{tran} = -\nu\epsilon_{long}$
Shear:	$\tau = P_{\parallel}/A$	$\gamma =$ change of formerly right angle	$\tau = G\gamma$ $2G = \frac{E}{1+\nu}$

Stress and deformation of some things

	Equilibrium	Geometry	Results
Tension	$P = \sigma A$	$\epsilon = \delta/L$	$\delta = \frac{PL}{AE}$ $[\delta = \frac{PL}{AE} + \alpha L\Delta T]$
Torsion	$T = \int \rho\tau\, dA$	$\gamma = \rho\phi/L$	$\phi = \frac{TL}{JG}$ $\tau = \frac{T\rho}{J}$
Bending and Shear in Beams	$M = -\int y\sigma\, dA$ $\frac{dM}{dx} = V, \quad \frac{dV}{dx} = -w$ $V = \int \tau\, dA$ $\tau t\Delta x = \Delta MQ/I$	$\epsilon = -y/\rho = -y\kappa$ $u'' = \frac{d^2}{dx^2}u = \frac{1}{\rho} = \kappa$	$u'' = \frac{M}{EI}$ $\sigma = \frac{-My}{I}$ $\tau = \frac{VQ}{It}$
Pressure Vessels	$pA_{gas} = \sigma A_{solid}$		$\sigma = \frac{pr}{2t}$ (sphere) $\sigma_l = \frac{pr}{2t}$ (cylinder) $\sigma_c = \frac{pr}{t}$ (cylinder)

Buckling

Critical buckling load $= P_{crit} = \frac{\pi^2 EI}{L_{eff}^2}$.

pinned-pinned	clamped-free	clamped-clamped	clamped-pinned
$L_{eff} = L$	$L_{eff} = 2L$	$L_{eff} = L/2$	$L_{eff} = .7L$

Cross Section Geometry

	Definition	Composite	annulus (circle: $c_1 = 0$)	thin-wall annulus (approx)	rectangle
$A =$	$\int dA$	$\sum A_i$	$\pi(c_2^2 - c_1^2)$	$2\pi ct$	bh
$J =$	$\int \rho^2\, dA$		$\frac{\pi}{2}(c_2^4 - c_1^4)$	$2\pi c^3 t$	
$I =$	$\int y^2\, dA$	$\sum(I_i + d_i^2 A_i)$	$\frac{\pi}{4}(c_2^4 - c_1^4)$	$\pi c^3 t$	$bh^3/12$
$\bar{y} =$	$\frac{\int y\, dA}{\int dA}$	$\frac{\sum y_i A_i}{\sum A_i}$	center	center	center
$Q =$	$\int y\, dA = A'\bar{y}'$	$\sum A_i'\bar{y}_i'$			$\frac{b(\frac{h^2}{4} - y^2)}{2}$

Mohr's Circle

Rotating the surface of interest an angle θ in physical space corresponds to a rotation of 2θ on the Mohr's circle in the same direction.

$$C = \frac{\sigma_1 + \sigma_2}{2} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{(\sigma_x - C)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta = \frac{\tau}{\sigma - C} = \frac{2\tau}{\sigma_x - \sigma_y}$$

Miscellaneous

- **Power in a shaft:** $P = T\omega$.
- **Saint Venant's Principle:** Far from the region of loading, the stresses in a structure would only change slightly if a load system were replaced with any other load system having the same net force and moment.