Your Name: ____________________

Section day and time: ______________

T&AM 202 Final Exam

Wed May 16, 2001

Draft May 14, 2001

5 problems, 100 points, and 150 minutes, no overtime.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A formula sheet is on the last page. You can rip it off and use the back for scratch work. Ask for extra scratch paper if you want it.

b) Full credit if

- →free body diagrams← are drawn when ever force or moment balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;

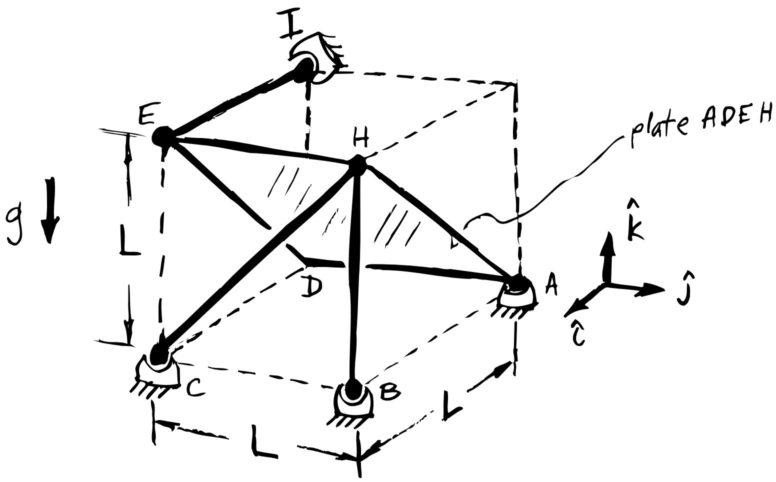
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are boxed in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ `= 18`" instead of, say, "`theta7dot = 18`".

c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	/ 20
Problem	2:	/ 20
Problem	3:	/ 20
Problem	4:	/ 20
Problem	5:	/ 20
TOTAL:		**/100**

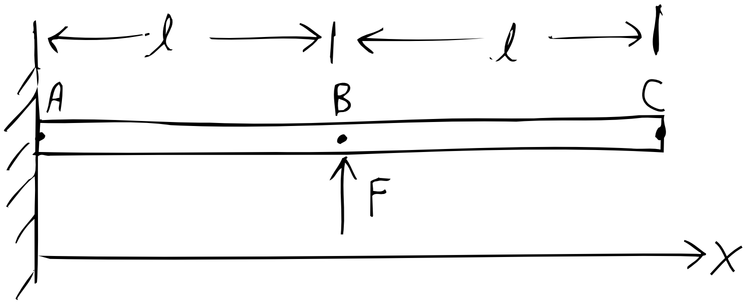
1) (20pt) Uniform plate ADEH with mass m is connected to the ground with a ball and socket joint at A. It is also held by three massless bars (IE, CH and BH) that have ball and socket joints at each end, one end at the rigid ground (at I, C and B) and one end on the plate (at E and H).

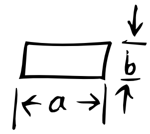
In terms of some or all of m, g, and L find the reaction at A (the force of the ground on the plate) and the three bar tensions T_{IE}, T_{CH} and T_{BH}.



2) (20 pt) The uniform solid linear elastic beam ABC has a rectangular cross section with sides a and b. It has moduli E, G, and ν. A vertical load F is applied halfway along its total length of 2ℓ. Answer the questions below in terms of some or all of a, b, E, G, ν, F and ℓ.

a) What is the maximum tensile stress and where does it occur?
b) What is the deflection at C?





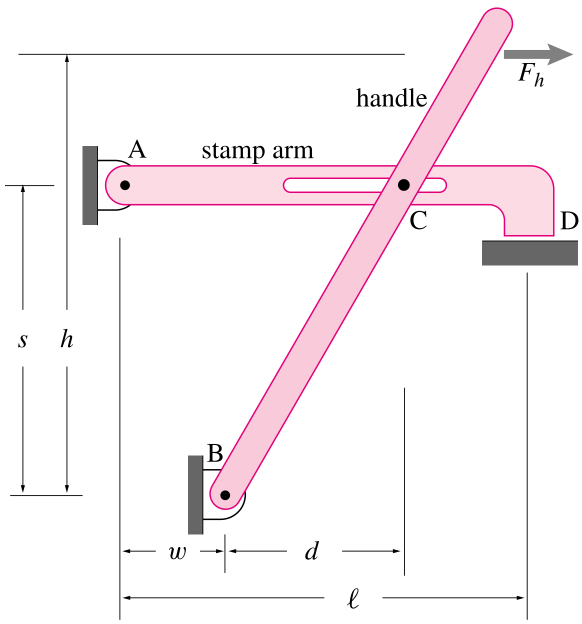
Partial credit if you

i) Find the reactions at A.
ii) Find $V(x)$ and draw a shear force diagram.
iii) Find $M(x)$ and draw a bending moment diagram.
iv) Find $u'(x)$.
v) Find $u(x)$ and neatly draw the deflected shape.

Though you don't need to do these if have got the correct answers to (a) and (b) above by reasoning that does not depend on them.

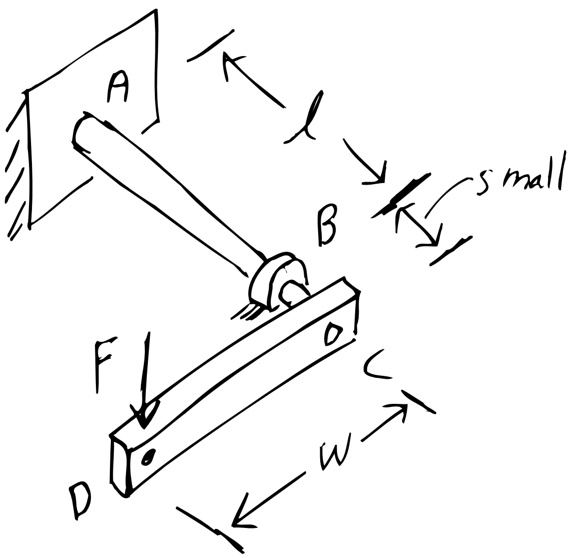
3) (20 pt) Stamp machine: Pulling on the handle causes the stamp arm to press down at D. Neglect gravity and assume that the hinges at A and B, as well as the roller at C, are frictionless.

Find the force N that the stamp machine causes on the support at D in terms of some or all of F_h, w, d, ℓ, h, and s.



4) (20 pt) Torsion bar. The uniform solid round linear elastic bar AB has radius r and length ℓ. It has elastic moduli E, G, and ν. Right next to the bearing at B the rod is attached to a very stiff (modeled as rigid) bar CD with length w. You can neglect the distance BC. You can assume that the total rotation of the shaft end at C is small ($\ll 1$).

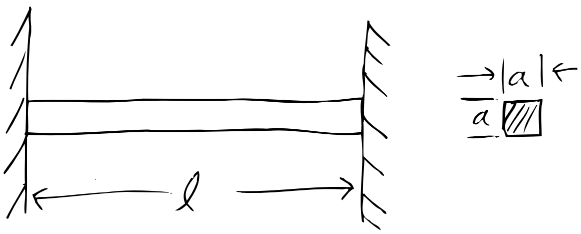
In terms of some or all of ℓ, r, w, E, G, ν, and F find the deflection of the point D.



5) (20 pt) A uniform linear elastic bar with a solid square cross section and length ℓ is welded between two rigid walls at room temperature in a stress-free state. The bar has thermal expansion coefficient α.

a) What is the compression in the bar when the temperature is raised ΔT? Answer in terms of some or all of $E, G, \nu, \alpha, \Delta T, \ell$, and a.

b) What temperature rise ΔT will cause the bar to buckle? Answer in terms of some or all of E, G, ν, α, ℓ, and a.



Statics and Strength of Materials Formula Sheet

(12/12/94, revised 5/10/01 — A. Ruina)

Not given here are the conditions under which the formulae are accurate or useful.

Basic Statics

Free Body Diagram

A **FBD** is a picture of any system for which you would like to apply mechanics equations and of all the external forces and torques which act on the system.

Action & Reaction

If $\mathcal{A}$ feels force $\vec{\mathbf{F}}$ and couple $\vec{\mathbf{M}}$ from $\mathcal{B}$.
then $\mathcal{B}$ feels force $-\vec{\mathbf{F}}$ and couple $-\vec{\mathbf{M}}$ from $\mathcal{A}$.
(With $\vec{\mathbf{F}}$ and $-\vec{\mathbf{F}}$ acting on the same line of action.)

Force and Moment Balance

These equations apply to every system in equilibrium:

$$\overbrace{\sum_{\text{All external forces}} \vec{\mathbf{F}} = \vec{\mathbf{0}}}^{\textit{Force Balance}} \qquad \overbrace{\sum_{\text{All external torques}} \vec{\mathbf{M}}_{/C} = \vec{\mathbf{0}}}^{\textit{Moment Balance about pt C}}$$

- The torque $\vec{\mathbf{M}}_{/C}$ of a force depends on the reference point C. But, for a body in equilibrium, and for any point C, the sum of all the torques relative to point C must add to zero).
- Dotting the force balance equation with a unit vector gives a scalar equation, e.g. $\left\{\sum \vec{\mathbf{F}}\right\} \cdot \hat{\imath} = 0 \quad \Rightarrow \quad \sum F_x = 0.$
- Dotting the moment balance equation with a unit vector gives a scalar equation, e.g. $\left\{\sum \vec{\mathbf{M}}_{/C}\right\} \cdot \hat{\lambda} = 0 \quad \Rightarrow$ net moment about axis in direction $\hat{\lambda}$ through $C = 0$.

Some Statics Facts and Definitions

- The moment of a force is unchanged if the force is slid along its line of action.
- For many purposes the words 'moment', 'torque', and 'couple' have the same meaning.
- **Two-force body.** If a body in equilibrium has only two forces acting on it then the two forces must be equal and opposite and have a common line of action.
- **Three-force body.** If a body in equilibrium has only three forces acting on it then the three forces must be coplanar and have lines of action that intersect at one point.
- **truss:** A collection of weightless two-force bodies connected with hinges (2D) or ball and socket joints (3D).
- **Method of joints.** Draw free body diagrams of each of the joints in a truss.
- **Method of sections.** Draw free body diagrams of various regions of a truss. Try to make the FBD cuts for the sections go through only three bars with unknown forces (2D).
- **Caution:** Machine and frame components are often **not** two-force bodies.
- **Hydrostatics:** $p = \rho g h, \quad F = \int p\, dA$

Cross Section Geometry

	Definition	Composite	annulus (circle: $c_1 = 0$)	thin-wall annulus (approx)	rectangle
$A =$	$\int dA$	$\sum A_i$	$\pi(c_2^2 - c_1^2)$	$2\pi ct$	bh
$J =$	$\int \rho^2\, dA$		$\frac{\pi}{2}(c_2^4 - c_1^4)$	$2\pi c^3 t$	
$I =$	$\int y^2\, dA$	$\sum (I_i + d_i^2 A_i)$	$\frac{\pi}{4}(c_2^4 - c_1^4)$	$\pi c^3 t$	$bh^3/12$
$\bar{y} =$	$\frac{\int y\, dA}{\int dA}$	$\frac{\sum y_i A_i}{\sum A_i}$	center	center	center
$Q =$	$\int y\, dA = A'\bar{y}'$	$\sum A_i' \bar{y}_i'$			$\frac{b(\frac{h^2}{4} - y^2)}{2}$

Stress, strain, and Hooke's Law

	Stress	Strain	Hooke's Law
Normal:	$\sigma = P_\perp / A$	$\epsilon = \delta / L_0 = \frac{L - L_0}{L_0}$	$\sigma = E\epsilon$ $[\,\epsilon = \sigma/E + \alpha \Delta T\,]$ $\epsilon_{tran} = -\nu \epsilon_{long}$
Shear:	$\tau = P_\parallel / A$	$\gamma =$ change of formerly right angle	$\tau = G\gamma$ $2G = \frac{E}{1+\nu}$

Stress and deformation of some things

	Equilibrium	Geometry	Results
Tension	$P = \sigma A$	$\epsilon = \delta / L$	$\delta = \frac{PL}{AE}$ $[\delta = \frac{PL}{AE} + \alpha L \Delta T]$
Torsion	$T = \int \rho \tau\, dA$	$\gamma = \rho \phi / L$	$\phi = \frac{TL}{JG}$ $\tau = \frac{T\rho}{J}$
Bending and Shear in Beams	$M = -\int y \sigma\, dA$ $\frac{dM}{dx} = V\,, \quad \frac{dV}{dx} = -w$ $V = \int \tau\, dA$ $\tau t \Delta x = \Delta M Q / I$	$\epsilon = -y/\rho = -y\kappa$ $u'' = \frac{d^2}{dx^2} u = \frac{1}{\rho} = \kappa$	$u'' = \frac{M}{EI}$ $\sigma = \frac{-My}{I}$ $\tau = \frac{VQ}{It}$
Pressure Vessels	$pA_{gas} = \sigma A_{solid}$		$\sigma = \frac{pr}{2t}$ (sphere) $\sigma_l = \frac{pr}{2t}$ (cylinder) $\sigma_c = \frac{pr}{t}$ (cylinder)

Buckling

Critical buckling load $= P_{crit} = \frac{\pi^2 EI}{L_{eff}^2}$.

pinned-pinned	clamped-free	clamped-clamped	clamped-pinned
$L_{eff} = L$	$L_{eff} = 2L$	$L_{eff} = L/2$	$L_{eff} = .7L$

Mohr's Circle

Rotating the surface of interest an angle θ in physical space corresponds to a rotation of 2θ on the Mohr's circle in the same direction.

$$C = \frac{\sigma_1 + \sigma_2}{2} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{(\sigma_x - C)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta = \frac{\tau}{\sigma - C} = \frac{2\tau}{\sigma_x - \sigma_y}$$

Miscellaneous

- **Power in a shaft:** $P = T\omega$.
- **Saint Venant's Principle:** Far from the region of loading, the stresses in a structure would only change slightly if a load system were replaced with any other load system having the same net force and moment.