Your Name: ANDY RUINA

"SOLUTIONS"

Section day and time: ______________

T&AM 202 Prelim 2

Tuesday April 3, 2001

Draft April 3, 2001

3 problems, 100 points, and 90^+ minutes.

Please follow these directions to ease grading and to maximize your score.

a) No calculators, books or notes allowed. A blank page for tentative scrap work is provided at the back. Ask for extra scrap paper if you need it.

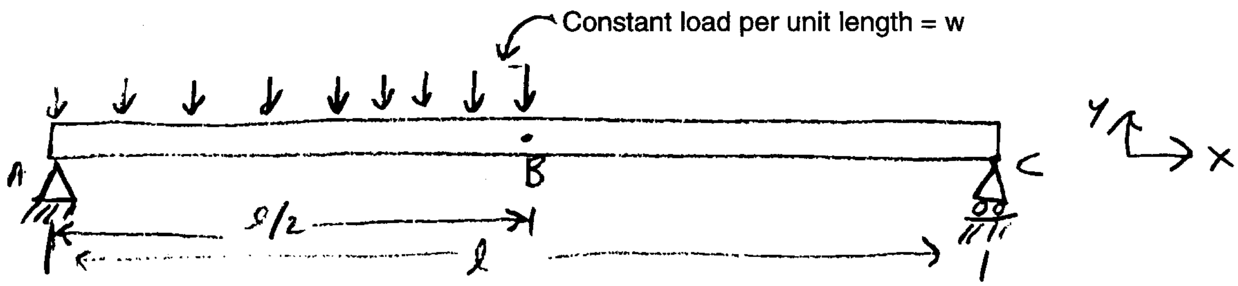
b) Full credit if

- →free body diagrams← are drawn whenever force or moment balance is used;
- correct vector notation is used, when appropriate;
- ↑→ any dimensions, coordinates, variables and base vectors that you add are clearly defined;
- ± all signs and directions are well defined with sketches and/or words;
- →| reasonable justification, enough to distinguish an informed answer from a guess, is given;
- you clearly state any reasonable assumptions if a problem seems *poorly defined*;
- work is I.) neat,
  II.) clear, and
  III.) well organized;
- your answers are TIDILY REDUCED (Don't leave simplifiable algebraic expressions.);
- □ your answers are $\boxed{\text{boxed}}$ in; and
- ≫ unless otherwise stated, you will get full credit for, instead of doing a calculation, presenting Matlab code that would generate the desired answer. To ease grading and save space, your Matlab code can use shortcut notation like "$\dot{\theta}_7$ = 18" instead of, say, "`theta7dot = 18`".

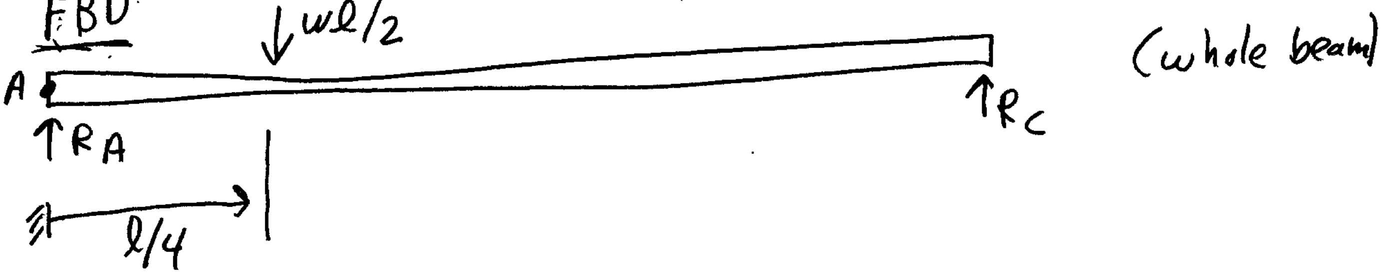
c) Substantial partial credit if your answer is in terms of well defined variables and you have not substituted in the numerical values. Substantial partial credit if you reduce the problem to a clearly defined set of equations to solve.

Problem	1:	___/___
Problem	2:	___/___
Problem	3:	___/___
TOTAL:		**___/100**

1) **(35 pt)** Draw shear and bending moment diagrams of the beam shown. Clearly label the values of the heights of the curves at jumps, kinks and local mazima (if and where they exist).



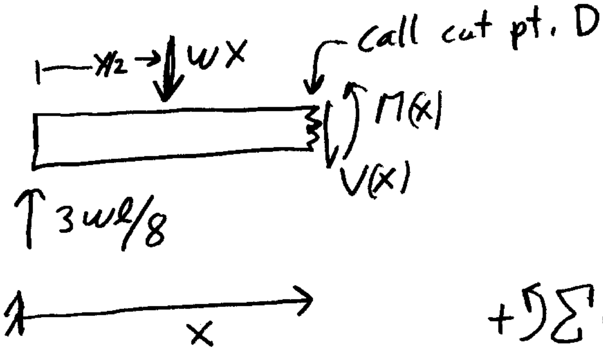
FBD



$$\sum F_y = 0 \Rightarrow R_A + R_C = w\ell/2$$

$$\circlearrowleft \sum M_{/A} = 0 \Rightarrow -\frac{w\ell}{2}\frac{\ell}{4} + R_C\ell = 0 \Rightarrow \boxed{R_C = \frac{w\ell}{8}} \Rightarrow \boxed{R_A = \frac{3w\ell}{8}}$$

FBD for $x < \ell/2$



$$\sum F_y = 0 = \frac{3w\ell}{8} - wx - V$$

$$\Rightarrow \boxed{V(x) = w\left(\frac{3\ell}{8} - x\right)} \quad (1)$$

(for $x \le \ell/2$)

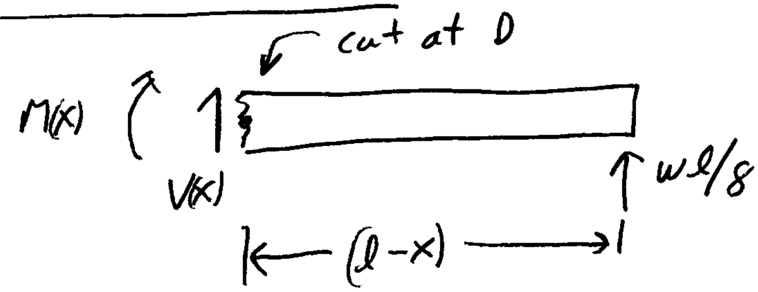
$$+\circlearrowleft \sum M_{/D} = 0 = -\frac{3w\ell}{8}x + \frac{wx^2}{2} + M(x)$$

$$\Rightarrow \boxed{M(x) = w\left[\frac{3\ell}{8}x - \frac{x^2}{2}\right]} \quad (2)$$

(for $x \le \ell/2$)

1) cont'd

FBD for $x \geq l/2$



$\sum F_y = 0 \Rightarrow \boxed{V(x) = -wl/8}$ (for $x \geq l/2$) (3)

$+\circlearrowleft \sum M_{/D} = 0 \Rightarrow \frac{wl}{8}(l-x) - M(x) = 0$

$\Rightarrow \boxed{M(x) = \frac{wl^2}{8} - \frac{wlx}{8}}$ (for $x \geq l/2$) (4)

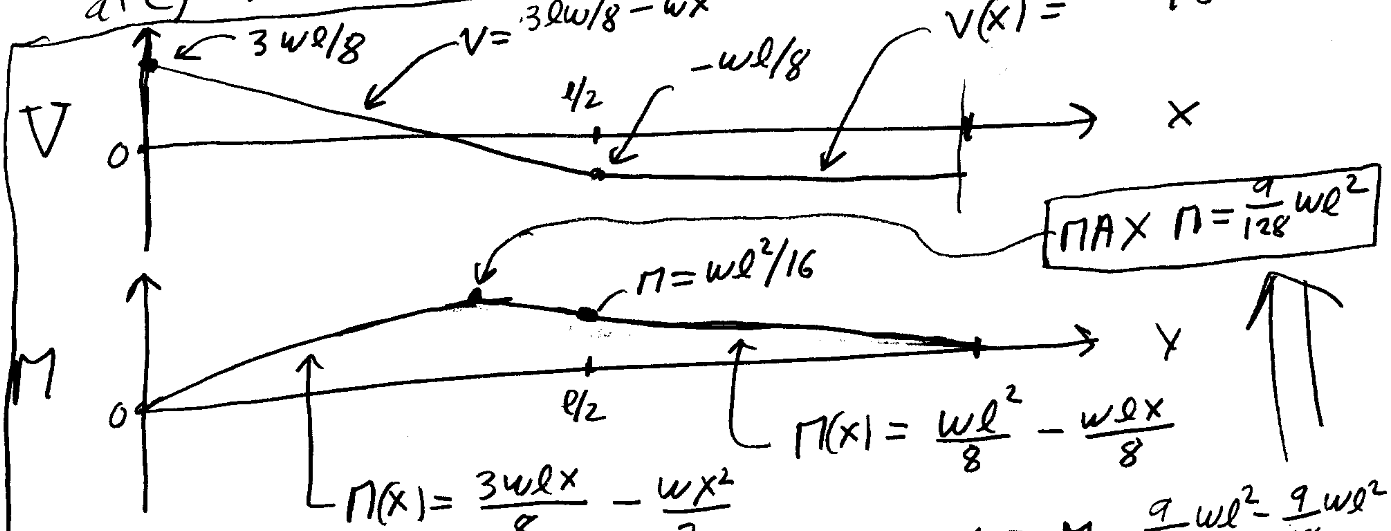
at A: $V(0) = 3wl/8$ $\quad M(0) = 0$

at B) $V(\frac{x}{2}) = \underbrace{w(\frac{3l}{8} - \frac{l}{2})}_{(1)} = \frac{-wl}{8}$ also $V(\frac{x}{2}) = -\frac{wl}{8}$ from (3)

$M(\frac{x}{2}) = \underbrace{w\left[\left(+\frac{3}{8}\right)\frac{l^2}{2} - \frac{l^2}{8}\right]}_{(2)} = \frac{wl^2}{16}$ also $\overbrace{M(\frac{x}{2}) = \frac{wl^2}{8} - \frac{wl^2}{16}}^{(4)}$

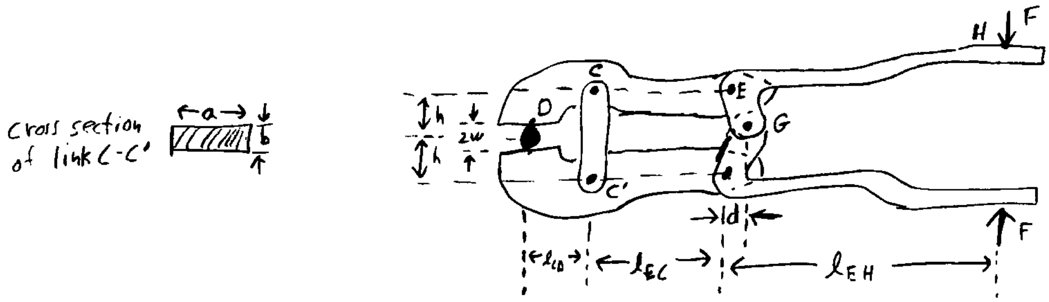
$= wl^2/16$

at C) $V(l) = -wl/8$ $\quad M(l) = 0$



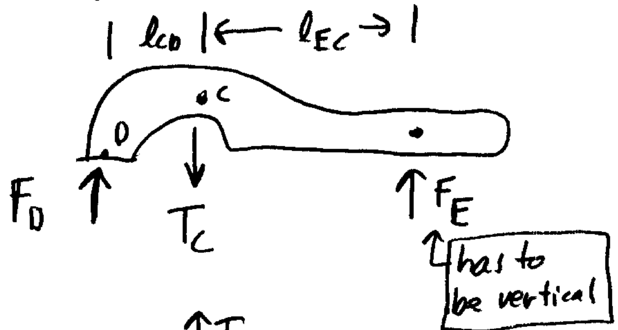
Max M at $dM/dx = 0 \Rightarrow 3wl/8 - wx = 0 \Rightarrow x = 3l/8 \Rightarrow M = \frac{9}{64}wl^2 - \frac{9}{128}wl^2$

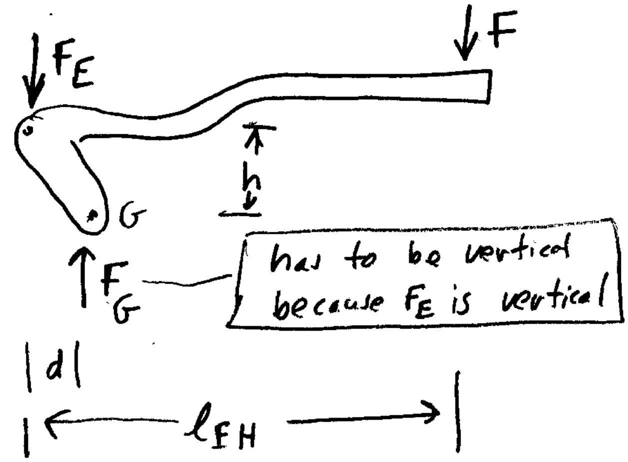
2) (35 pt) The plyers shown are made of five pieces modeled as rigid: HEG and its mirror image, DCE and its mirror image, and link CC'. You may assume that the goemetry is symmetric about a horizontal line (the top is a mirror image of the bottom). The load F and dimensions shown are given.



FBD of whole structure $\Rightarrow F = F$ (big deal)

a) Find the force squeezing the piece at D, and
b) The stress in piece CC'.
c) What happens to the squeezing force if d is made smaller, approaching zero? (full credit for a coherently explained answer even if the work above is wrong). Why can't this work in practice?





$$+\circlearrowleft \sum M_{/G} = 0 = d F_E - (l_{EH} - d) F$$

$$\Rightarrow \boxed{F_E = \left(\frac{l_{EH} - d}{d}\right) F}$$

$$+\circlearrowleft \sum M_{/C} = 0 = F_E l_{EC} - F_D l_{CD}$$

$$\Rightarrow F_D = \frac{l_{EC}}{l_{CD}} F_E$$

(a) $$\boxed{F_D = \frac{l_{EC}}{l_{CD}} \left(\frac{l_{EH} - d}{d}\right) F}$$

2) (cont'd)

b) For piece DCE $\sum F_y = F_D - T_C + F_E$

$$\Rightarrow T_C = F_D + F_E$$

$$= \frac{l_{EC}}{l_{CD}}\left(\frac{l_{EH}-d}{d}\right)F + \left(\frac{l_{EH}-d}{d}\right)F$$

$$= F\left(\frac{l_{EH}-d}{d}\right)\left[\frac{l_{EC}}{l_{CD}}+1\right]$$

(b) $$\boxed{\sigma_{CC'} = \frac{T_C}{A} = \frac{T_C}{ab} = \frac{F}{ab}\left(\frac{l_{EH}-d}{d}\right)\left(\frac{l_{EC}}{l_{CD}}+1\right)}$$

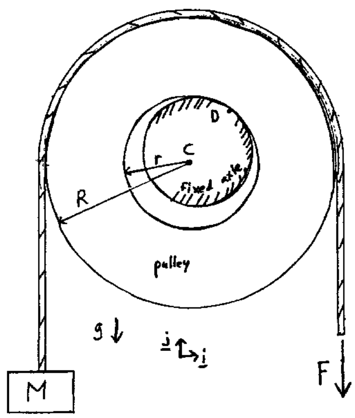
(c) as $d \to 0$ $F_D \to \infty$ [super plyers]

But, for real plyers deformation of the metal would keep this from happening. That is, the rigidity assumption would brake down.

3) **(30 pt)** A weight M is steadily raised by pulling with a force F on a rope going over a neglige-mass pulley on an unlubricated journal bearing (no ball bearings). For an ideal frictionless pulley $F = Mg$. Here we have a friction coefficient between the bearing and its axle which is is $\mu = \tan\phi$. (25 pt if either part below is answered correctly, so you can jump to part (b) if you strongly prefer numbers.) *No Matlab code allowed in this problem.*

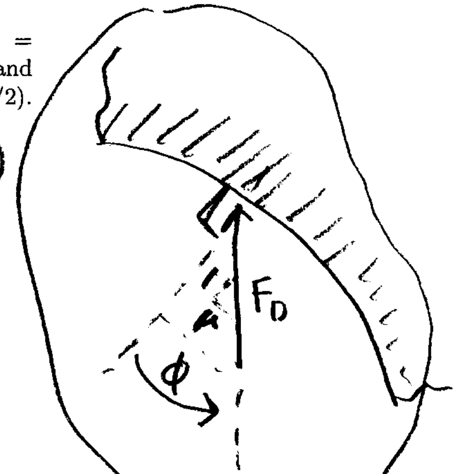
[Hint: Finding the location of the contact point D is probably part of your solution.]

see pgs 120-121 PR for a similar but more difficult problem



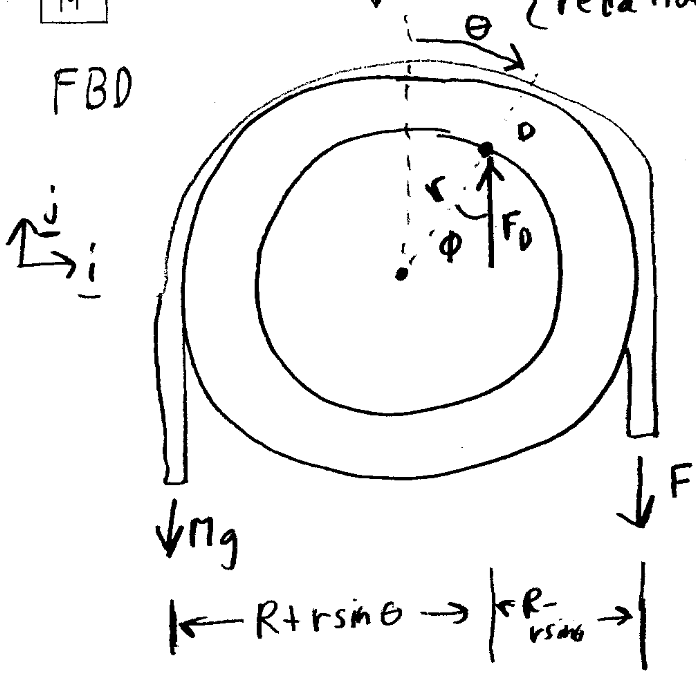
a) Find F in terms of M, g, R, r and μ (or ϕ or $\sin\phi$ or $\cos\phi$ — whichever is most convenient. For example $\cos(\tan^{-1}(\mu))$ is just $\cos\phi$), *and*

b) Evaluate F in the special case that $M = 100\,\text{kg}, g = 10\text{m/s}^2, r = 1\,\text{cm}, R = 2\,\text{cm}$, and $\mu = \sqrt{3}/3$ (so $\phi = \pi/6, \sin\phi = 1/2, \cos\phi = \sqrt{3}/2$).



blow up of pt. D
(friction force opposes relative motion)

FBD



θ defines location of pt. D

the key

$\{\sum \underline{F} = \underline{0}\} \cdot \underline{i}$

$\Rightarrow F_D \sin(\theta - \phi) = 0$

$\Rightarrow \sin(\theta - \phi) = 0$

$\Rightarrow \boxed{\theta = \phi}$, F_D is vertical

$$+\circlearrowleft \sum M_{/D} = Mg\,(R + r\sin\theta) - F(R - r\sin\theta) = 0$$

$$\Rightarrow \boxed{F = Mg\,\frac{R + r\sin\phi}{R - r\sin\phi}} \text{ (a)}$$

$$= (100\,\text{kg})\left(10\,\frac{\text{N}}{\text{kg}}\right)\left(\frac{2\,\text{cm} + (1\,\text{cm})(1/2)}{2\,\text{cm} + (1\,\text{cm})(1/2)}\right)$$

$$\boxed{F = 1000\,\text{N}\,\frac{2.5}{1.5} = \frac{5}{3}\,1000\,\text{N} = 1667\,\text{N}} \text{ (b)}$$