3.2, 3.6b, 3.24, 3.38, 3.50b

3.2. (a) Determine the T which causes $\tau_{max} = 45\,MPa$ in a hollow shaft

(b). Determine τ_{max} caused by T in a solid shaft of the same cross area.

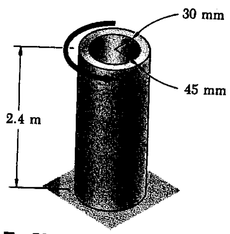


Fig. P3.2

Solution:

(a). From the eqn:

$$\tau_{max} = \frac{TC}{J} \Rightarrow$$

$$T = \frac{\tau_{max} J}{C}, \quad J = \frac{1}{2}\pi(45^4 mm^4 - 30^4 mm^4) = 5.17 \times 10^{-6}\ m^4$$

$$\Rightarrow T = \frac{45\times10^6\,Pa \times 5.17\times10^{-6}\,m^4}{45\times10^{-3}\ m} = 5.17\ kN\cdot m$$

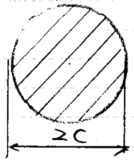
$$\boxed{T = 5.17\ kN\cdot m}$$

(b).

$$\tau_{max} = \frac{TC}{J}$$

$$J = \frac{1}{2}\pi C^4$$

$$\Rightarrow \tau_{max} = \frac{T}{\frac{1}{2}\pi C^3}$$



Solid shaft.

$$\pi C^2 = \pi(45^2 mm - 30^2 mm)$$

$$\Rightarrow C = 33.5\ mm$$

$$\Rightarrow \tau_{max} = \frac{5.17\times10^3\,N}{\frac{1}{2}\pi(33.5mm)^3} = 87.2\ MPa$$

$$\boxed{\tau_{max} = 87.2\ MPa}$$

With the same amount of material, the stress is about half as big w. the hollow tube

3.6 b.

Given: the electric motor exerts a 12 kip.in torque at E, each shaft is solid

Find: (b) the maximum shearing stress in CD.

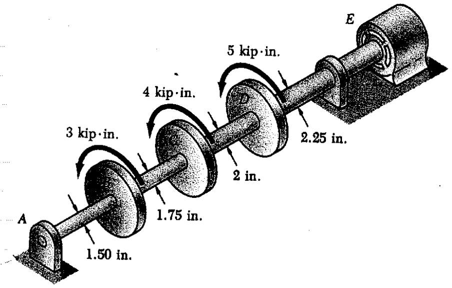


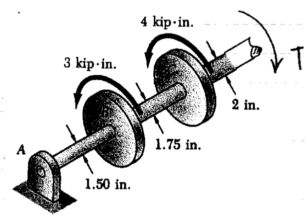
Fig. P3.6

Soln:

(b)

$$\tau_{max} = \frac{T_{CD}\,C}{J}$$

Draw the F.B.D to get T_{CD}:



$$T = 7\ kip\cdot in$$

$$\Rightarrow \tau_{max} = \frac{(7\,kip\cdot in)(1\ in)}{\frac{1}{2}\pi\ (1\,in)^4} = 4.46\ ksi$$

$$\boxed{\tau_{max} = 4.46\ ksi}$$

3.24. (a). Determin ϕ caused by T = 40 kip.in in a solid shaft w/. $G = 3.7\times10^6$ psi

(b). Solve part a for a hollow shaft w/. $d_{out} = 3''$, $d_{in} = 1$ in.

Cont. 3.24.

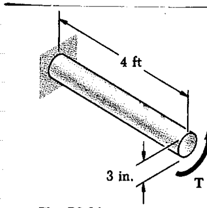


Fig. P3.24

Solution:

(a) Twist of ange ϕ is:

$$\phi = \frac{TL}{JG} = \frac{(40\ kip\cdot in)(4\times12\ in)}{\frac{1}{2}\pi(1.5\,in)^4(3.7\times10^6\,psi)}$$

$$= 65.25\times10^{-3}\ rad$$

$$= \frac{65.25\times10^{-3}}{\pi}\times180^\circ$$

$$= 3.74^\circ$$

$$\boxed{\phi = 3.74^\circ}$$

(b).

$$\phi = \frac{TL}{JG}$$

$$= \frac{(40\ kip\cdot in)(4\times12\,in)}{\frac{1}{2}\pi[(1.5'')^4 - (0.5'')^4](3.7\times10^6\,psi)}$$

$$= 66.1\times10^{-3}\ rad$$

$$= \frac{66.1\times10^{-3}\times180^\circ}{\pi}$$

$$= 3.79^\circ$$

$$\boxed{\phi = 3.79^\circ}$$

moral: the innea inch of material has about no contribution to the stiffness,

3.38.

Given: shafts w/. $G = 11.2\times10^6$ psi, $\tau_{all} = 12$ ksi

$T_A = 2$ kip.in $\phi_A \le 7.5^\circ$

Find: Required diameter

Solution:

Rotation of end A:

$$\phi_A = \phi_{A/B} + \phi_B \qquad (1)$$

ϕ_B is the twist angle at B = rotation of gear B

• Need ϕ_B. can be found from gear system:

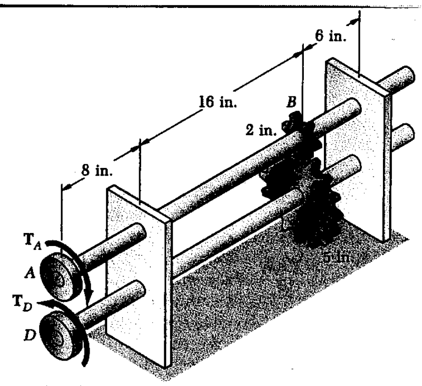


Fig. P3.38

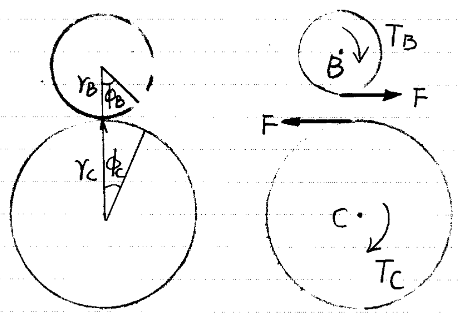


Figure for ϕ. FBD.

From the FBD:

$$\left.\begin{array}{l} +\circlearrowleft \Sigma M_B = 0 \Rightarrow \quad T_B = \gamma_B F \\ +\circlearrowleft \Sigma M_C = 0 \Rightarrow \quad T_C = \gamma_C F \end{array}\right\} \Rightarrow \frac{T_B}{T_C} = \frac{\gamma_B}{\gamma_C} \qquad (2)$$

From the figure for ϕ:

$\gamma_B \phi_B = \gamma_C \phi_C$ (They'll have the same length of route)

$$\Rightarrow \frac{\phi_B}{\phi_C} = \frac{\gamma_C}{\gamma_B} \qquad (3)$$

Using (3):

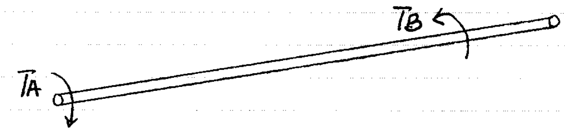
$$\phi_B = \frac{\gamma_C}{\gamma_B}\phi_C = \left(\frac{\gamma_C}{\gamma_B}\right)\frac{T_C L_{CD}}{JG}$$

Using (2):

$$\phi_B = \left(\frac{\gamma_C}{\gamma_B}\right)\left(\frac{T_B \gamma_C}{\gamma_B}\ \frac{L_{CD}}{JG}\right)$$

$$= \left(\frac{\gamma_C}{\gamma_B}\right)^2 \frac{T_B L_{CD}}{JG} = \frac{25}{4}\ \frac{T_B L_{CD}}{JG} \qquad (4)$$

$T_B = T_A$ From the F.B.D. of shaft AB



The same for $T_C = T_D$

$$\Rightarrow \phi_B = \frac{25}{4}\ \frac{T_A(24\,in)}{JG}$$

$$\phi_{A/B} = \frac{T_A L_{AB}}{JG} = \frac{T_A(24\,in)}{JG}$$

$$\Rightarrow \phi_A = \frac{1}{J}\left[\frac{T_A(24\,in)}{G}\left(\frac{25}{4}+1\right)\right]$$

$$\Rightarrow J = \frac{(174\,in)\,T_A}{\phi_A G}$$

$\phi_A \le 7.5°$

$$\Rightarrow J = \frac{1}{2}\pi\left(\frac{d}{2}\right)^4 \ge \frac{(174\,in)(2\,kip\cdot in)}{(7.5°)(11.2\times10^6\,psi)}$$

$$\ge \frac{348\times10^3\ lb\cdot in^2}{\left(7.5°\times\frac{\pi}{180°}\right)(11.2\times10^6\,psi)}$$

$$\ge 0.237\ in^4$$

$$\Rightarrow \left(\frac{d}{2}\right)^4 \ge \frac{2\times0.237\,in^4}{\pi}$$

$$\Rightarrow d \ge 1.24\ in.$$

So, to have $\phi_A \le 7.5°$, $d \ge 1.24$ in.

But we also have $\tau_{all} = 12$ ksi, need to check it out:

• $$\tau_{AB} = \frac{T_A C}{J} = \frac{(2\,kip\cdot in)(0.62\,in)}{\frac{1}{2}\pi(0.62\,in)^4}$$
$$= 5.34\ ksi$$
$$< 12\ ksi$$

So $d = 1.24$ in is fine for AB

• $$\tau_{CD} = \frac{T_C C}{J} = \left(\frac{T_B \gamma_C}{\gamma_B}\right)\left(\frac{C}{J}\right) = \left(\frac{5}{2}\right)\frac{(2\,kip\cdot in)\,C}{J}$$
$$= 13.134\ ksi > \tau_{all}$$

Cont. 3.38.

⇒ $d = 1.24\ in$ doesn't satisfy $\tau_{CD} < \tau_{all}$

For $\tau_{CD} = \left(\frac{5}{2}\right)\frac{(2\ kip\cdot in)}{\frac{1}{2}\pi c^3} < \tau_{all} = 12\ ksi$

⇒ $c \geq 0.6425\ in$
⇒ $d \geq 1.285\ in$

We need to take the larger one to make
$\tau \leq \tau_{all}$
$\phi_A \leq 7.5^\circ$

$$\boxed{d \geq 1.285\ in}$$

3.50. b.
Given: $T = 20\ kip\cdot in$
Find: (b) τ_{max} in sleeve CD.

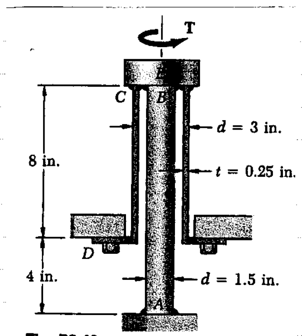


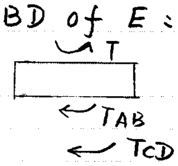
Fig. P3.49

Soln: $G_s = 11.2\times10^6\ psi$, $(\tau_{all})_s = 12\ ksi$
$G_b = 5.6\times10^6\ psi$, $(\tau_{all})_b = 7\ ksi$

$$\tau^{CD}_{max} = \frac{T_{CD}\,c}{J} = \frac{T_{CD}\,(1.5\ in)}{\frac{1}{2}\pi[(1.5\ in)^4 - (1.25\ in)^4]}$$
$$= \frac{0.364\ T_{CD}}{in^3}$$

We need T_{CD}:

① From the FBD of E:



Cont. 3.50 (b)

$T = T_{AB} + T_{CD}$ (1)

② From the geometry condition:

$\phi_E = \phi_{AB} = \phi_{CD}$ (They're all fixed to each-other)

$$\Rightarrow \frac{T_{AB}L_{AB}}{J_{AB}G_{AB}} = \frac{T_{CD}L_{CD}}{J_{CD}G_{CD}}$$

$$\Rightarrow \frac{T_{AB}}{T_{CD}} = \frac{J_s G_s}{J_b G_b}\,\frac{L_{CD}}{L_{AB}}$$

$$= \frac{(0.75\ in)^4\ (11.2\times10^6\ psi)\ (8\ in)}{[(1.5\ in)^4 - (1.25\ in)^4](5.6\times10^6\ psi)\ (12\ in)}$$
$$= 0.161$$

⇒ $T_{AB} = 0.161\ T_{CD}$ (2)

Put (2) → (1) ⇒

$T = 1.161\ T_{CD} = 20\ kip\cdot in$

⇒ $T_{CD} = 17.2\ kip\cdot in$

$$\Rightarrow \tau^{CD}_{max} = \frac{0.364}{in^3}T_{CD}$$
$$= 0.364\times17.2\ \frac{kip}{in^2}$$
$$= 6.26\ ksi$$

$$\boxed{\tau^{max}_{CD} = 6.26\ ksi}$$