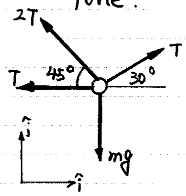
2.85, 2.91, 2.93, 2.78 5). 6)

2.85 Replace the force system by a single equivalent force.



Solution:

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$\vec{F}_1 = T(-\hat{i})$

$\vec{F}_2 = mg(-\hat{j})$

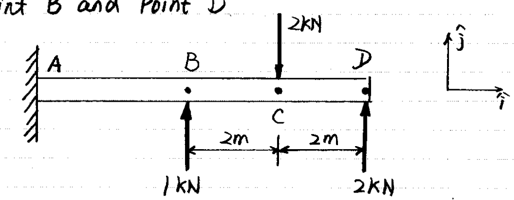
$\vec{F}_3 = T(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) = T(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j})$

$\vec{F}_4 = 2T(-\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) = 2T(-\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j})$

$$\vec{F}_{net} = -T\hat{i} - mg\hat{j} + \frac{T}{2}(\sqrt{3}\hat{i} + \hat{j}) + \sqrt{2}T(-\hat{i} + \hat{j})$$
$$= (-1 + \frac{\sqrt{3}}{2} - \sqrt{2})T\hat{i} + (\frac{T}{2} + \sqrt{2}T - mg)\hat{j}$$
$$= \boxed{-1.55T\hat{i} + (1.91T - mg)\hat{j}}$$

$\vec{M}_o = 0$

2.91. Find an equivalent force-couple system at point B and Point D



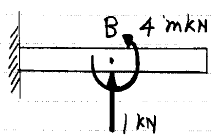
Solution:

a) at point B

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1\text{ kN}\hat{j} + 2\text{kN}\hat{j} - 2\text{kN}\hat{j} = 1\text{ kN}\hat{j}$$

$$\vec{M}_{Bnet} = \vec{r}_{BC} \times (-2\text{kN}\hat{j}) + \vec{r}_{BD} \times (2\text{kN}\hat{j})$$
$$= 2\hat{i} \times (-2\hat{j}) + 4\hat{i} \times 2\hat{j} \quad \text{mkN}$$
$$= -4\hat{k} + 8\hat{k} \quad \text{mkN}$$
$$= 4\hat{k} \quad \text{mkN}$$

$$\boxed{\begin{aligned}\vec{F}_{net} &= 1 \text{ kN}\hat{j} \\ \vec{M}_{Bnet} &= 4 \text{ mkN } \hat{k}\end{aligned}}$$



Cont. 2.91

b) at point D

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1\text{ kN}\hat{j} + 2\text{kN}\hat{j} - 2\text{kN}\hat{j} = 1\text{ kN}\hat{j}$$

$$\vec{M}_{Dnet} = \vec{r}_{DC} \times (-2\hat{j}) + \vec{r}_{DB} \times \hat{j} \quad \text{mkN}$$
$$= -2\hat{i} \times (-2\hat{j}) + (-4\hat{i}) \times \hat{j} \quad \text{mkN}$$
$$= 0 \quad \text{mkN } \hat{k}$$

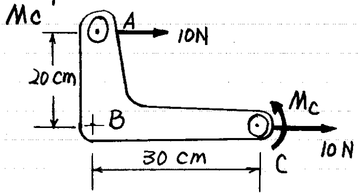
$$\boxed{\begin{aligned}\vec{F}_{net} &= 1 \text{ kN}\hat{j} \\ \vec{M}_{Dnet} &= 0 \text{ mkN } \hat{k}\end{aligned}}$$

Could also solve using the result from part (a)
$\underline{M}_{Dnet} = \underline{M}_{Bnet} + \underline{r}_{B/D} \times \underline{F}_{net} = 0$

2.93

Given: a force-moment system at point C. and an equivalent force = 10 N at A

Find: Mc



Solution:

$\vec{F}_{net} = 10 \text{ N } \hat{i}$

For the two force systems (the one at C, another at A) to be equivalent, the total moment about any point should be the same

Take point A:

$\vec{M}_{net1} = 0$ for 10N at A

$\vec{M}_{net2} = M_c\hat{k} + dF\hat{k} = (M_c + (0.2)(10))\hat{k}$ mN

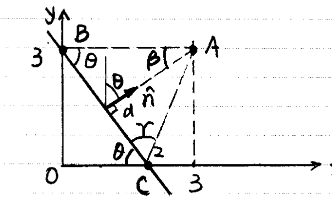
$\vec{M}_{net2} = \vec{M}_{net1} = 0$

$\Rightarrow M_c = 2$ mN

$\boxed{M_c = 2 \text{ mN}}$

$(\underline{M}_c = -2\text{mN}\,\underline{k})$

2.78. Find the distance from point A to line BC



Soln:

Method 1: by using cross product of $\vec{r}_{BA} \times \hat{\lambda}_{BC}$

$|\vec{r}_{BA} \times \hat{\lambda}_{BC}| = |\vec{r}_{BA}|\sin\theta = d$

$\vec{r}_{BA} = 3\hat{i}$

$\lambda_{BC} = \frac{\vec{r}_{BC}}{|\vec{r}_{BC}|}$

$\vec{r}_{BC} = \vec{r}_C - \vec{r}_B = 2\hat{i} - 3\hat{j}$

$|\vec{r}_{BC}| = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\Rightarrow \hat{\lambda}_{BC} = \frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

$$\Rightarrow d = |\vec{r}_{BA} \times \hat{\lambda}_{BC}| = \left|3\hat{i} \times \frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}\right| = \frac{1}{\sqrt{13}}|0 - 9\hat{k}| = \frac{9}{\sqrt{13}}$$

Method 2: Find $\hat{n}$ about line BC, using dot product

$|\vec{r}_{BA} \cdot \hat{n}| = |\vec{r}_{BA}|\cos\beta = d$

• Finding $\hat{n}$:

$$\hat{n} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j} = \frac{3}{\sqrt{2^2+3^2}}\hat{i} + \frac{2}{\sqrt{2^2+3^2}}\hat{j} = \frac{3}{\sqrt{13}}\hat{i} + \frac{2}{\sqrt{13}}\hat{j}$$

• Finding d:

$$d = 3\hat{i} \cdot \left(\frac{3}{\sqrt{13}}\hat{i} + \frac{2}{\sqrt{13}}\hat{j}\right) = \boxed{\frac{9}{\sqrt{13}}}$$

Cont. 2.87

Method 3: using $|\vec{r}_{CA} \times \hat{\lambda}_{BC}|$

$|\vec{r}_{CA} \times \hat{\lambda}_{BC}| = |\vec{r}_{CA}| \sin\gamma = d$

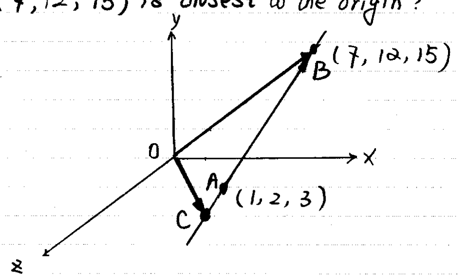
$\vec{r}_{CA} = \vec{r}_A - \vec{r}_C = 3\hat{i} + 3\hat{j} - 2\hat{i}$
$= \hat{i} + 3\hat{j}$

$\vec{r}_{CA} \times \hat{\lambda}_{BC} = (\hat{i} + 3\hat{j}) \times (2\hat{i} - 3\hat{j}) / \sqrt{13}$
$= \dfrac{-3\hat{k} - 6\hat{k}}{\sqrt{13}}$

$\Rightarrow d = |\vec{r}_{CA} \times \hat{\lambda}_{BC}| = \dfrac{9}{\sqrt{13}}$

There are many more methods....

5. what point on the line that goes through (1, 2, 3) and (7, 12, 15) is closest to the origin?



Solution:

The line goes from the origin to the point cocerned will be perpendicular to line AB.

let this point be C, then line OC ⊥ line AB the question now equals to find the coordinates of the point C, say, what's $\vec{r}_C$

From the figure:

- $\vec{r}_C = \vec{r}_B - \vec{r}_{CB}$

  $\vec{r}_B = 7\hat{i} + 12\hat{j} + 15\hat{k}$
- $\vec{r}_{CB} = (\vec{r}_B \cdot \hat{\lambda}_{AB})\hat{\lambda}_{AB}$
- $\hat{\lambda}_{AB} = \dfrac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \dfrac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|}$

Another method;

$\underline{n} = \dfrac{\underline{r}_{AB} \times (\underline{r}_{AB} \times \underline{r}_{OC})}{|\underline{r}_{AB} \times (\underline{r}_{AB} \times \underline{r}_{OC})|}$

$\underline{r}_{OA} = (\underline{r}_{OC} \cdot \underline{n})\,\underline{n}$

($\underline{n}$ is along dir of OA)

Cont. 5.

$\vec{r}_B - \vec{r}_A = (7\hat{i} + 12\hat{j} + 15\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$
$= 6\hat{i} + 10\hat{j} + 12\hat{k}$

$|\vec{r}_B - \vec{r}_A| = \sqrt{6^2 + 10^2 + 12^2}$
$= 2\sqrt{70}$

$\Rightarrow$

$\hat{\lambda}_{AB} = \dfrac{1}{2\sqrt{70}}(6\hat{i} + 10\hat{j} + 12\hat{k})$

$\Rightarrow$

$\vec{r}_{CB} = (\vec{r}_B \cdot \hat{\lambda}_{AB})\hat{\lambda}_{AB}$
$= (7\hat{i} + 12\hat{j} + 15\hat{k}) \cdot \dfrac{1}{2\sqrt{70}}(6\hat{i} + 10\hat{j} + 12\hat{k})\,\hat{\lambda}_{AB}$
$= \dfrac{1}{2\sqrt{70}}(42 + 120 + 180)\,\hat{\lambda}_{AB}$
$= \dfrac{342}{280}(6\hat{i} + 10\hat{j} + 12\hat{k})$

$\Rightarrow$

$\vec{r}_C = \vec{r}_B - \vec{r}_{CB}$
$= (7\hat{i} + 12\hat{j} + 15\hat{k}) - \dfrac{342}{280}(6\hat{i} + 10\hat{j} + 12\hat{k})$
$\doteq (7\hat{i} + 12\hat{j} + 15\hat{k}) - 1.221(6\hat{i} + 10\hat{j} + 12\hat{k})$
$= -.32\hat{i} - .2\hat{j} + .36\hat{k}$

$\boxed{\vec{r}_C = -.32\hat{i} - .2\hat{j} + .36\hat{k}}$

6. Find the distance BC and OA in a regular tetrahedron.

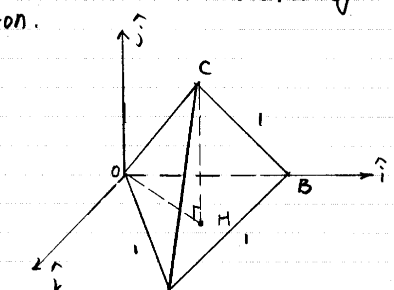


Figure a

Solution:

The distance between BC and OA will be the length of the line that's perpendicular to both lines.

Method 1: geometry only

From the figure, H is the centroid of the top, H' is the centroid of the bottom.

$\Rightarrow$ HH' ⊥ bottom & top (one of the properties of the cube)

$\Rightarrow$ HH' ⊥ BC
HH' ⊥ OA

$\Rightarrow$ HH' is the distance between line OA & line BC

$\Rightarrow$ HH' = EB (EB ∥ HH')
$= 1 \cdot \sin 45$
$= \boxed{\dfrac{\sqrt{2}}{2}}$

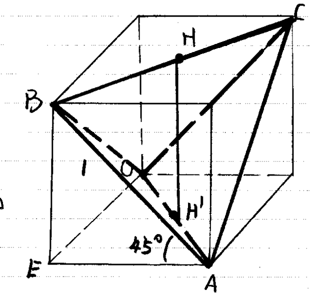


Figure b.

Method 2: geometry only also

From the figure:

CH ⊥ OB

$\Rightarrow$ $CH = 1 \cdot \sin 60° = \dfrac{\sqrt{3}}{2}$

H' is the center of AC

$\Rightarrow$ $CH' = \dfrac{1}{2}$

- can show:

  {HH' ⊥ AC
  {HH' ⊥ OB

- Proving HH' ⊥ AC

  H is the center of line OB

  $\Rightarrow$ AH ⊥ OB

  $\Rightarrow$ $AH = 1 \cdot \sin 60° = CH$
  H' is the center of AC

  $\Rightarrow$ HH' is the height of △AHC

  $\Rightarrow$ HH' ⊥ AC.

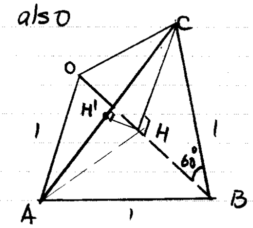


Figure c.

Cont. method 2. question 6

• Proving $HH' \perp OB$

Similarly:

HH' is the height of triangle $OH'B$

$\Rightarrow HH' \perp OB$

So, HH' is the distance between OB & AC

• Finding HH'

From triangle $HH'C$, $HH' \perp H'C$

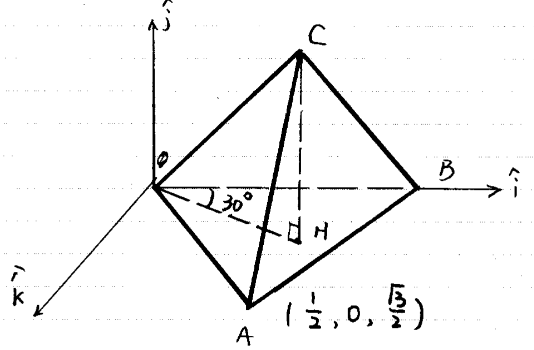
$$\Rightarrow HH' = \sqrt{HC^2 - H'C^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = \boxed{\frac{\sqrt{2}}{2}}$$

Method 3: triple product

• Finding the unit normal vector for both lines, OC & AB in figure

$$\hat{n} = \vec{r}_{OC} \times \vec{r}_{AB}$$

$$\vec{r}_{OC} = r_{cx}\hat{i} + r_{cy}\hat{j} + r_{cz}\hat{k}$$



$CH \perp$ to triangle OAB

$\Rightarrow CH \perp OH$

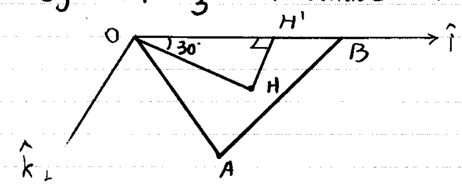
$$OH = \frac{2}{3} \cdot 1 \cdot \cos 30^\circ \quad (\text{property of equal side } \triangledown) = \sqrt{3}/3$$

Cont. Method 3. 6.

$$\Rightarrow CH = \sqrt{OC^2 - OH^2} = \sqrt{1 - \left(\frac{\sqrt{3}}{3}\right)^2} = \frac{\sqrt{6}}{3}$$

$\Rightarrow$

$C_y = CH = \frac{\sqrt{6}}{3}$ (because $CH \parallel \hat{j}$)



$HH' \perp OB$, $\angle H'OH = 30^\circ$

$$\Rightarrow C_x = OH' = OH \cdot \cos 30^\circ = \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{2} = \frac{1}{2}$$

$$C_z = HH' = OH \cdot \sin 30^\circ = \frac{\sqrt{3}}{6}$$

$$\Rightarrow \vec{r}_{OC} = \frac{1}{2}\hat{i} + \frac{\sqrt{6}}{3}\hat{j} + \frac{\sqrt{3}}{6}\hat{k}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = \hat{i} - \left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{k}\right) = \frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{k}$$

$$\Rightarrow \hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{6} \\ \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \end{vmatrix}$$

$$= \left(-\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{6}}{3}\right)\hat{i} - \left(-\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{12}\right)\hat{j} + \left(0 - \frac{\sqrt{6}}{6}\right)\hat{k}$$

$$= -\frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{3}}{3}\hat{j} - \frac{\sqrt{6}}{6}\hat{k}$$

• Choosing any vector from line OC to line AB

let's chose $\vec{r}_{OB} = \hat{i}$

$$|\vec{r}_{OB} \cdot \hat{n}| = d = \left|-\frac{\sqrt{2}}{2}\right| = \boxed{\frac{\sqrt{2}}{2}}$$