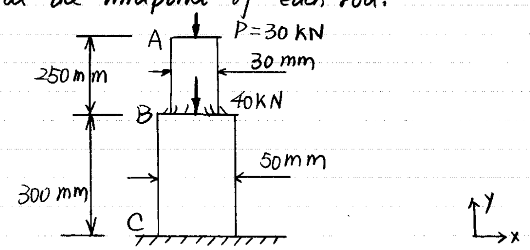
1.2, 1.20, 1.23, 1.36, 1.46, 2.2, 2.28

1.2 Two solid cylindrical rods are welded together at B, find the normal stress at the midpoint of each rod.

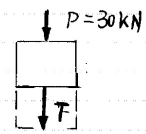


Soln:

Find the normal stress at the center of AB

$\Sigma F_y = 0$:

$T = -P = -30\,kN$



$\Rightarrow$

$\sigma = \frac{T}{A} = \frac{-30\ kN}{\pi(\frac{30}{2}mm)^2}$

F.B.D. of the upper half AB

$= \frac{-30\times10^3\ N}{\pi\ 225\times10^{-6}\ m}$

$\boxed{= -42\ \ MPa}$

Assume stresses is uniform across cross section

For bar BC, draw the F.B.D. of the upper half BC plus AC:

$\Sigma F_y = 0$:

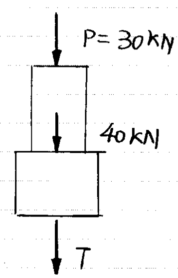
$-T = P + 40\ kN = 70\ kN$

$\Rightarrow$

$\sigma = \frac{T}{A} = -\frac{70\times10^3\ N}{\pi(\frac{50\ mm}{2})^2}$

$= -\frac{70\times10^3\ N}{\pi\ 625\times10^{-6}}$

$\boxed{= -35.6\ \ MPa}$



1.20 Find the normal stress in member AD, knowing the cross area of the member is $1500\,mm^2$

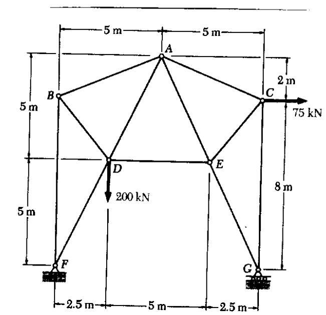
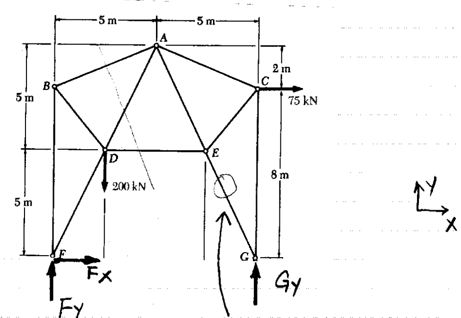


Fig. P1.19 and P1.20

Soln:

To find the normal stress in AD, we need to find the axial internal force in AD

- Finding the reactions at F first



the only zero-force member, Not useful! for this problem.

$+\circlearrowleft \Sigma M_G = 0 \Rightarrow$

$-F_y(10\,m) + 200\,kN\,(7.5\,m) - 75\,kN\,(8\,m) = 0$

$\Rightarrow$

$F_y = 90\ \ kN$

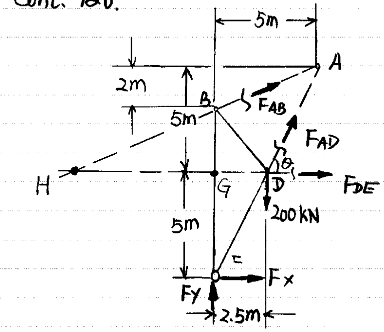
$(\Sigma \vec{F})\cdot\hat{i} = 0$

$\Rightarrow$

$F_x = -75\ kN$

- Cutting At AB, AD, DE and draw the FBD of BDF.

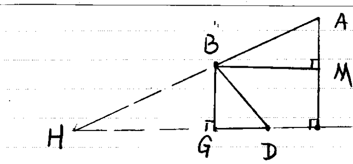
Cont. 1.20.



$\curvearrowleft \Sigma M_H = 0 \Rightarrow$

$F_y\cdot(HG) - F_x(5m) + F_{AD}\,(d) - 200\,kN\,(HD) = 0$

$d \rightarrow$ the distance from H to AD.



"is similar to"

$\Delta ABM \sim \Delta BHG$

$\Rightarrow$

$\frac{HG}{BM} = \frac{BG}{AM} = \frac{3m}{2m} = \frac{3}{2}$

$\Rightarrow$

$HG = \frac{3}{2}\times 5\,m = 7.5\ m$

$\Rightarrow$

$HD = HG + GD = 10\ m$

- $F_{AD}(d) = F_{ADy}\cdot HD = F_{AD}\sin\theta \cdot 10m$

$\sin\theta = \frac{5}{\sqrt{5^2+2.5^2}} = 0.89$

$\Rightarrow$

$+\circlearrowleft \Sigma M_H = F_y\cdot(HG) + F_x(5m) + F_{AD}\sin\theta(10m) - 200kN(HD)$
$= 90kN(7.5m) - 75kN(5m) + F_{AD}\,0.89(10m) - 200\,kN(10m)$

$\Rightarrow$

$F_{AD} = \frac{(2000 + 375 - 675)\ kN\cdot m}{0.89\times 10\ m}$

$= 191\ \ kN$

Cont. 1.20.

$$\sigma = \frac{F_{AD}}{A} = \frac{191\text{ kN}}{1200\text{ mm}^2} = \frac{191\times10^3\text{ N}}{1200\times10^{-6}\text{ m}^2}$$

$$\boxed{= 159\text{ MPa}}$$

1.23.

Given: The diameter of steel rod = 25 mm
DEF is a two identical arm and wheel units.
The weight of the tow bar AB = 2kN
G's the center of mass of the tow

Find : The normal stress in the rod.

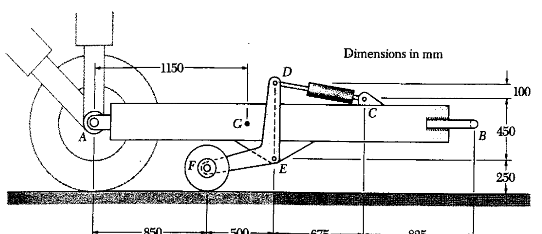
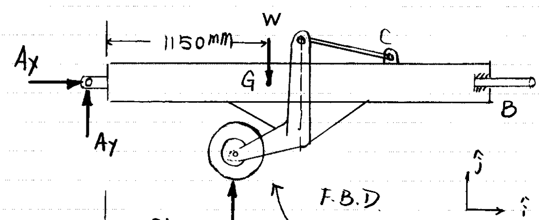


Fig. P1.23

Solution:

To find the normal stress in rod DC, we need to find the axial force in it.

- Study the bar and the units together at first



[assume ideal wheel so no tangential reac.]

$\curvearrowleft \Sigma M_A = 0 \Rightarrow$

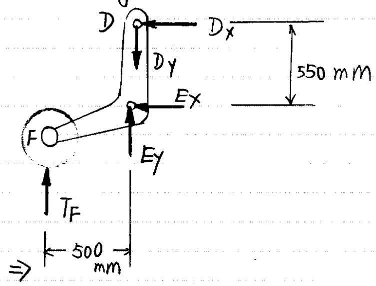
$T_F\cdot(850\text{ mm}) - W(1150\text{ mm}) = 0$

$\Rightarrow$

$T_F = 2.7\text{ kN}$

Cont. 1.23

- Study the unit only



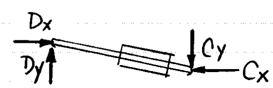
$\curvearrowleft \Sigma M_E = 0 \Rightarrow$

$-T_F(500\text{ mm}) + D_x(550\text{ mm}) = 0$

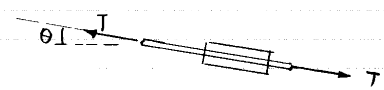
$\Rightarrow$

$D_x = 2.45\text{ kN}$

- Draw the F.B.D of the rod DC



rod DC is a two force member.



$\Rightarrow T\cos\theta = -D_x$

$$\cos\theta = \frac{675\text{ mm}}{\sqrt{675^2+100^2}\text{ mm}} = 0.99$$

$\Rightarrow T = -\dfrac{2.45}{0.99}\text{ kN} = -2.47\text{ kN}$ (compression)

$\Rightarrow$

$$\sigma = +\frac{T}{A} = -\frac{2.47\text{ kN}}{\pi\left(\frac{25\text{ mm}}{2}\right)^2} = \frac{-2.47\times10^3\text{ N}}{\pi\ 156.25\times10^{-6}\text{ m}^2}$$

$= -5.03\text{ MPa}$

$$\boxed{\sigma = -5.03\text{ MPa}}$$

1.36

Given: 3x5-in uniform rectangular cross sect. for two members.
$P = 800\text{ lb}$

Find: the normal and shearing stress in the glued joint.

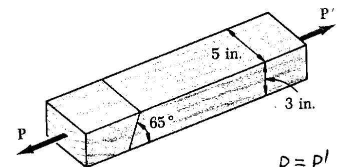
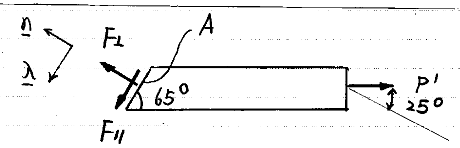


Fig. P1.36

$P = P'$, obviously

Solution: Draw the F.B.D of the right member



$\{\Sigma \underline{F} = 0\}\cdot \underline{n} \Rightarrow$

$F_\perp - P'\cos25° = 0$

$\Rightarrow F_\perp = P'\cos25°$
$= P\cos25°$

$\Rightarrow$

$$\sigma = \frac{F_\perp}{A} = \frac{P\cos25°}{A_0/\sin65°} = \frac{(800\text{ lb})\cos25°\sin65°}{3\times5\text{ in}^2}$$

$= 43.8\text{ psi}$

$\{\Sigma \underline{F} = 0\}\cdot \underline{\lambda} \Rightarrow$

$F_{//} - P\sin25° = 0$

$\Rightarrow$

$F_{//} = +P\sin25°$

$$\tau = \frac{F_{//}}{A} = \frac{P\sin25°}{A_0/\sin65°}$$

$$= \frac{(800\text{ lb})\sin25°\sin65°}{3\times5\text{ in}^2}$$

$= 20.4\text{ psi}$

$= 20.4\text{ psi}$

Cont. 1.36

$\boxed{\begin{array}{l} \tau = 20.4 \text{ psi} \\ \sigma = 43.8 \text{ psi} \end{array}}$

1.46 The wooden members are joined by plywood splice plates which're fully glued on the surface in contact. $\tau_{ult} = 2.5$ MPa in the glued joint.

Find: The factor of safety when $L = 180$ mm

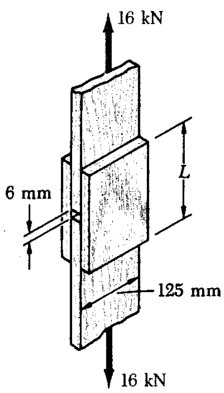
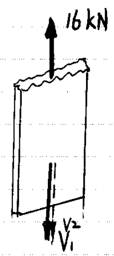


Fig. P1.45



F.B.D

V_1 is on the outer side of the plate.
V_2 is on the inner side of the plate.

Since: $F.S = \dfrac{\tau_{ult}}{\tau_{all}}$

we need to find $\tau_{all} = \dfrac{V}{A}$

from the F.B.D of the upper plate: $\Sigma F_y = 0 \Rightarrow$

$$V_1 = V_2 = V = \frac{16 \text{ kN}}{2} = 8 \text{ kN}$$

$$\Rightarrow \tau_{all} = \frac{8 \text{ kN}}{A}$$

$$A = (125 \text{ mm}) \frac{(L - 6 \text{ mm})}{2} = 10875 \text{ mm}^2$$

$$\Rightarrow F.S = \frac{\tau_{ult}}{\tau_{all}} = \frac{2.5 \text{ MPa}}{\left(\frac{8 \times 10^3 \text{ N}}{10875 \times 10^{-6} \text{ m}^2}\right)}$$

$$= \frac{2.5 \text{ MPa}}{0.736 \text{ MPa}} = 3.4 \quad \boxed{F.S = 3.4}$$

2.2.

Given: when $T = 6$ kN, $L = 60$ m for steel wire
$\Delta L \leq 48$ mm
$E = 200$ GPa

Find: (a) the smallest diameter for the wire
(b) the value of the nomal stress

Solution:
(a).

$\Delta L = \dfrac{PL}{EA}$ for axial loading

when $\Delta L = 48$ mm $\Rightarrow$

$$A = \frac{PL}{E\Delta L} \quad \text{will be smallest}$$

$$= \frac{(6 \text{ kN})(60 \text{ m})}{(200 \text{ GPa})(48 \times 10^{-3}) \text{ m}}$$

$$= \frac{7.5 \times 10^6 \text{ N}}{200 \times 10^9 \ (\text{N/m}^2)}$$

$$= 37.5 \text{ mm}^2$$

$$\Rightarrow d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{37.5 \text{ mm}^2}{\pi}} = 6.91 \text{ mm}$$

$$\boxed{d = 6.91 \text{ mm}}$$

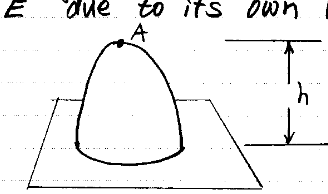
(b). when $d = 6.91$ mm

$$\sigma = \frac{T}{A} = \frac{6 \text{ kN}}{37.5 \text{ mm}^2} = 160 \text{ MPa}$$

$$\boxed{\sigma = 160 \text{ MPa}}$$

2.28.

Determine the deflextion of the apex A of the homogeneous paraboid with height h density ρ E due to its own weight.

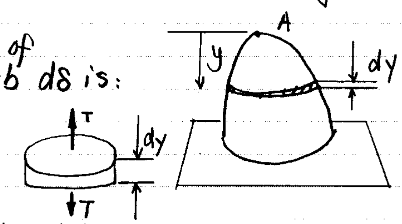


Cont. 2.28.

This's a problem regarding non-uniform A

Take a small slab of thickness dy

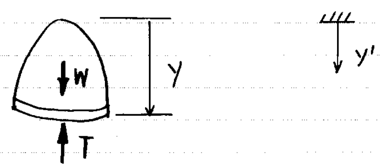
$\Rightarrow$ the deflection of this small slab $d\delta$ is:



$$d\delta = \frac{T\,dy}{EA}$$

because now the slab can be viewed as a circular plate with $L = dy$

P will be the weight above the slab according to the F.B.D of the upper part of the slab



$$V = \int_0^Y A(y')\,dy'$$
$$= \int_0^Y \pi\, r^2(y')\,dy'$$
$$= \pi \int_0^Y [r(y')]^2 dy$$

Eqn. of volume for paraboloid from top to y.

For paraboloid, we have

$r(y') = C\sqrt{y'}$ C: is for how fat the paraboloid is.

$\Rightarrow$

$$V = \pi \int_0^Y (C\sqrt{y'})^2 dy'$$
$$= \pi C^2 \int_0^Y y'\,dy'$$
$$= \frac{1}{2}\pi C^2 Y^2$$

$$\Rightarrow T = -W = -\rho g V = -\rho g \tfrac{1}{2}\pi C^2 Y^2$$

$$\Rightarrow d\delta = \frac{T\,dy}{EA} = \frac{-\rho g \frac{1}{2}\pi C^2 Y^2}{E\,\pi (C\sqrt{Y})^2}$$

Cont. 228.

where A at $y = \pi (C\sqrt{y'})^2$, because now $y' = y$

$\Rightarrow$

$$d\delta = -\frac{\rho g Y}{2E}\,dy$$

$\Rightarrow$ The deflection of point A is

$$\delta = \int d\delta = -\int_0^h \frac{\rho g Y}{2E}\,dy$$
$$= -\frac{\rho g}{2E}\left(\tfrac{1}{2}Y^2\right)\Big|_0^h$$
$$= -\frac{\rho g}{4E} h^2$$

(Note: C drops out! why? Two paraboloids side by side are like one fatter paraboloid.)

$$\boxed{\delta = -\frac{\rho g h^2}{4E}}$$

negative δ means the paraboloid gets shorter