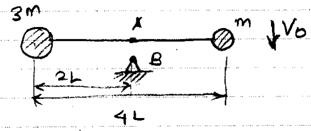
Solution by Basant Sharma April 25 '02

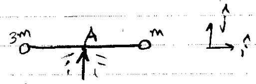
① $\boxed{8.56}$

Point A in the center of the bar shown below strikes point B and sticks to it. find $\dot{\theta}$ just after the collision?

no gravity

FBD of bar (collision)





Since no torque is acting on the bar before or during or after the points A and B meet, by AMB:

$\dot{\underset{\sim}{H}}_{/A} = 0$ or $\underset{\sim}{H}_{/A} = \text{constant}.$

angular velocity just after collision

$\Leftrightarrow \quad \underset{\sim}{H}_{/A,\text{ before collision}} = \underset{\sim}{H}_{/A,\text{ after collision}}$

$\Leftrightarrow (-2L\hat{i})\times(-3mV_0\hat{j}) + (2L\hat{i})\times(-mV_0\hat{j}) = \left[3m(2L)^2 + m(2L)^2\right]\dot{\theta}\hat{k}$

So $\{6mV_0L\hat{k} - 2mV_0L\hat{k} = 16mL^2\dot{\theta}\hat{k}\}$

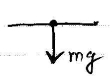
$\{\}\cdot\hat{k} \Rightarrow \quad 4mV_0L = 16mL^2\dot{\theta} \Rightarrow \boxed{\dot{\theta} = \dfrac{V_0}{4L}}$

② $\boxed{8.60}$ (acrobat)

An acrobat modeled as a uniform rigid rod of mass m and length ℓ. Initially she falls w/o rotation from height h (from state of rest) and grabs a bar with firm but slippery grip. What is h so that final position is stationary handstand.

0. Initial state.
1. Just before grip.
2. Just after grip.
3. Final state.

FBD (before gripping the bar)



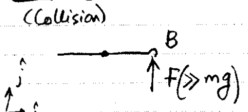
Since Energy is conserved

$E_{k_0} + E_{P_0} = E_{k_1} + E_{P_1}$

$\Leftrightarrow \quad 0 + mgh = \frac{1}{2}mV_{cm}^2 + 0$

$\Rightarrow \quad V_{cm} = \sqrt{2gh}$

FBD (1-2) (Collision)



$\underset{\sim}{H}_{/B}$ conserved as $\underbrace{\sum \underset{\sim}{M}_{/B}}_{0} = \dot{\underset{\sim}{H}}_{/B}$.

$\underset{\sim}{H}_{/B,1} = -\frac{L}{2}\hat{i}\times m\sqrt{2gh}(-\hat{j})$

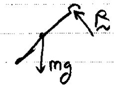
$= \frac{mL}{2}\sqrt{2gh}\,\hat{k}$

$\underset{\sim}{H}_{/B,2} = I_{zz}^{B}\,\omega\hat{k}$

$= \frac{mL^2}{3}\omega\hat{k}$

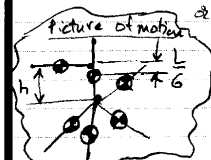
$\therefore \{\underset{\sim}{H}_{/B,1} = \underset{\sim}{H}_{/B,2}\}\cdot\hat{k} \Rightarrow \quad \omega = \dfrac{3\sqrt{2gh}}{2L}$

FBD (2-3)



R the reaction force does no work and the gravitational force is conservative, so the energy is conserved.

$\Rightarrow \quad E_{k_2} + E_{P_2} = E_{k_3} + E_{P_3}$



& $\frac{1}{2}I_{zz}^{B}\omega^2 + 0 = 0 + mg\frac{L}{2}$

$\therefore \frac{1}{2}\left(\frac{mL^2}{3}\right)\left(\frac{9}{4}\cdot\frac{2gh}{L^2}\right) = mg\frac{L}{2}$

$\Rightarrow \quad \boxed{h = \frac{2}{3}L}$

$\Rightarrow$ Lost energy in collision $= \frac{1}{6}mgL$!!!

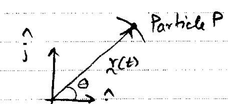
③ $\boxed{9.3}$

The particle follows a path given by $\theta = br$.

So $\underset{\sim}{r}(t) = r\hat{e}_r$

$\underset{\sim}{V}(t) = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$= \frac{\dot{\theta}}{b}\hat{e}_r + \frac{\theta\dot{\theta}}{b}\hat{e}_\theta$



$\underset{\sim}{a}(t) = \frac{\ddot{\theta}}{b}\hat{e}_r + \frac{\dot{\theta}}{b}(\dot{\theta}\hat{e}_\theta) + \frac{\dot{\theta}^2}{b}\hat{e}_\theta + \frac{\theta\ddot{\theta}}{b}\hat{e}_\theta - \frac{\theta\dot{\theta}^2}{b}\hat{e}_r$

$= \left(\frac{\ddot{\theta}}{b} - \frac{\theta\dot{\theta}^2}{b}\right)\hat{e}_r + \left(\frac{2\dot{\theta}^2}{b} + \frac{\theta\ddot{\theta}}{b}\hat{e}_\theta\right)$

Or using the formula $\underset{\sim}{a}(t) = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$

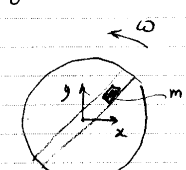
we get $\boxed{\underset{\sim}{a}(t) = \left(\frac{\ddot{\theta}}{b} - \frac{\theta}{b}\dot{\theta}^2\right)\hat{e}_r + \left(\frac{2\dot{\theta}^2}{b} + \frac{\theta}{b}\ddot{\theta}\right)\hat{e}_\theta}$

if $\ddot{\theta} \equiv 0$ then $\boxed{\underset{\sim}{a}(t) = -\frac{\theta}{b}\dot{\theta}^2\hat{e}_r + \frac{2\dot{\theta}^2}{b}\hat{e}_\theta}$

④ $\boxed{9.5}$

The velocity of the mass or bug is given by

$\underset{\sim}{V}(t) = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$



Case 1: $\dot{R} = 0$

In this case the bug does not move in the radial direction, so it sticks at a particular point on the disk like a fixed point of disk $\Rightarrow \underset{\sim}{V} = R\dot{\theta}\hat{e}_\theta$

Case 2: $R\dot{\theta} = 0$

(If $R = 0$ we get $\dot{R} = 0$ too so we rule that out)

if $\dot{\theta} = 0$ that means $\omega = 0$, ie we are not rotating the table; in this case bug can move in the slot freely without any rotations imposed on it $\Rightarrow \underset{\sim}{V} = \dot{R}\hat{e}_R$.

⑤ $\boxed{9.7}$

Given $\ddot{R} - R\dot{\theta}^2 = 0$

$2\dot{R}\dot{\theta} + R\ddot{\theta} = 0$

has the solution $R = \sqrt{d^2 + (v(t-t_0))^2}$ —— ①

$\theta = \theta_0 + \tan^{-1}(v(t-t_0)/d)$ —— ②

for θ_0, d, t_0, v arbitrary constants.

Shape of the curve:

from ②, $\dfrac{v(t-t_0)}{d} = \tan(\theta - \theta_0)$ —— ③

Using ③ in ①, $R = \sqrt{d^2 + (d\tan(\theta-\theta_0))^2}$

$= d\sqrt{1 + \tan^2(\theta-\theta_0)}$

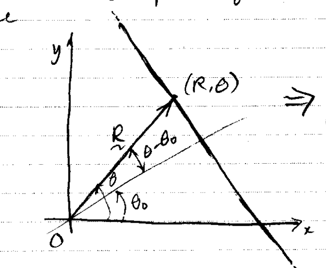
(Using $\sec^2\theta = 1 + \tan^2\theta$)

$= d\sec(\theta - \theta_0)$

So $R\cos(\theta - \theta_0) = d$

ie. if (R,θ) gives the position of the point in plane then $R\cos(\theta-\theta_0) = \text{const}$.

So the following picture gives the path of the particle

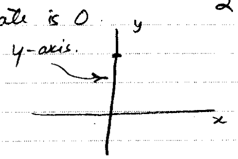


$\Rightarrow$ Curve is a straight line.

Check: if $\theta_0 = 0$, $d = 0$ then $R\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$,

ie. x-coordinate is 0

ie. the path is y-axis.



Alternative reasoning:

$\underset{\sim}{F}(t) = m\underset{\sim}{a}(t) = m\left((\underbrace{\ddot{R} - R\dot{\theta}^2}_{0})\hat{e}_r + (\underbrace{2\dot{R}\dot{\theta} + R\ddot{\theta}}_{0})\hat{e}_\theta\right)$

Given

$\equiv 0$

So the force acting on the particle of mass m is zero. By LMB: $\underset{\sim}{V} = \underset{\sim}{V}_0 = \text{constant}$

$\Rightarrow \underbrace{\underset{\sim}{r} = \underset{\sim}{r}_0 + \underset{\sim}{v}_0 t}_{\text{parametric equation of st. line.}}$

⑥ $\boxed{9.9a}$ (tedious!!!)

Given: $\underset{\sim}{r} = t^2 \frac{m}{s^2}\hat{i} + e^{t/s} m\hat{j} = \left(\left(\frac{t}{s}\right)^2\hat{i} + e^{t/s}\hat{j}\right)m$

$\Rightarrow \quad \underset{\sim}{v} = \frac{d}{dt}r(t)$

$= 2t\frac{m}{s^2}\hat{i} + \frac{1}{s}e^{t/s}m\hat{j} = \left(\frac{2t}{s}\hat{i} + e^{t/s}\hat{j}\right)\frac{m}{s}$

$\Rightarrow \quad \underset{\sim}{a} = 2\frac{m}{s^2}\hat{i} + \frac{1}{s^2}e^{t/s}m\hat{j} = \left(2\hat{i} + e^{t/s}\hat{j}\right)\frac{m}{s^2}$

So

$$\hat{e}_t = \frac{\underset{\sim}{v}}{|v|} = \frac{\frac{2t}{s}\hat{i} + e^{t/s}\hat{j}}{\sqrt{\left(\left(\frac{2t}{s}\right)^2 + e^{2t/s}\right)}} = \frac{\frac{2t}{s}\hat{i} + e^{t/s}\hat{j}}{\sqrt{\frac{4t^2}{s^2} + e^{2t/s}}}$$

$$a_t = a \cdot e_t$$

$$\underset{\sim}{a}_t = \frac{\frac{4t}{s} + e^{2t/s}}{\sqrt{\frac{4t^2}{s^2} + e^{2t/s}}}\left(\frac{m}{s^2}\right)\left(\frac{\frac{2t}{s}\hat{i} + e^{t/s}\hat{j}}{\sqrt{\frac{4t^2}{s^2} + e^{2t/s}}}\right)$$

$$\underset{\sim}{a}_n = \underset{\sim}{a} - a_t\hat{e}_t$$

$$= \left(2\hat{i} + e^{t/s}\hat{j}\right)\frac{m}{s^2} - \frac{\frac{4t}{s} + e^{2t/s}}{\left(\sqrt{\frac{4t^2}{s^2} + e^{2t/s}}\right)^2}\left(\frac{2t}{s}\hat{i} + e^{t/s}\hat{j}\right)\frac{m}{s^2}$$

$$= \left[\left\{2 - \frac{\left(\frac{4t}{s} + e^{2t/s}\right)\frac{2t}{s}}{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)}\right\}\hat{i} + \left\{e^{t/s} - \frac{\left(\frac{4t}{s} + e^{2t/s}\right)e^{t/s}}{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)}\right\}\hat{j}\right]\frac{m}{s^2}$$

$$= \left[\left\{\frac{2e^{2t/s} - \frac{2t}{s}e^{2t/s}}{\frac{4t^2}{s^2} + e^{2t/s}}\right\}\hat{i} + \left\{\frac{\frac{4t^2}{s^2}e^{t/s} + \cancel{e^{3t/s}} - \frac{4t}{s}e^{t/s} - \cancel{e^{3t/s}}}{\frac{4t^2}{s^2} + e^{2t/s}}\right\}\hat{j}\right]\frac{m}{s^2}$$

$$\therefore \underset{\sim}{a}_n = \left[\frac{\left(1 - \frac{t}{s}\right)2e^{2t/s}}{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)}\hat{i} + \frac{\frac{4t}{s}e^{t/s}\left(\frac{t}{s} - 1\right)}{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)}\hat{j}\right]\frac{m}{s^2}$$

$$\hat{e}_n = \frac{a_n}{|a_n|} = \frac{\left(1 - \frac{t}{s}\right)2e^{2t/s}\hat{i} + \frac{4t}{s}e^{t/s}\left(\frac{t}{s} - 1\right)\hat{j}}{\sqrt{\left(1 - \frac{t}{s}\right)^2 4e^{4t/s} + \frac{16t^2}{s^2}e^{2t/s}\left(\frac{t}{s} - 1\right)^2}}$$

Now $\rho = \frac{v^2}{a_n} = \frac{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)^2}{\sqrt{\left(1 - \frac{t}{s}\right)^2 4e^{4t/s} + \frac{16t^2}{s^2}e^{2t/s}\left(\frac{t}{s} - 1\right)^2}}\, m$

Final solution:

$$\boxed{\hat{e}_t = \frac{2t/s\,\hat{i} + e^{t/s}\hat{j}}{\sqrt{4t^2/s^2 + e^{2t/s}}}, \quad a_t = \frac{4t/s + e^{2t/s}}{\sqrt{4t^2/s^2 + e^{2t/s}}}\frac{m}{s^2} \qquad a_n = \frac{2e^{t/s}}{\left(\frac{4t^2}{s^2} + e^{2t/s}\right)}\left|1 - \frac{t}{s}\right|\sqrt{e^{2t/s} + 4\frac{t^2}{s^2}}\,\frac{m}{s^2}}$$

$$\boxed{\hat{e}_n = \frac{\left(1 - \frac{t}{s}\right)}{\left|1 - \frac{t}{s}\right|}\left(e^{t/s}\hat{i} - \frac{2t}{s}\hat{j}\right) \qquad \rho = \frac{\left(4t^2/s^2 + e^{2t/s}\right)^2}{2e^{t/s}\left|1 - \frac{t}{s}\right|\sqrt{e^{2t/s} + 4t^2/s^2}}\, m}$$