Basant Sharma Spring

(1) The function iknowthis (t,z) generates a first-order system of linear differential equations

$$\dot{x} = y$$
$$\dot{y} = -x$$

This is equivalent to the second order differential equation

$$\ddot{x} + x = 0$$

(for an easy to see this, take the time derivative of $\dot{x} = y$ and use it with $\dot{y} = -x$)

The characteristic equation of this second order linear differential equation is

$$r^2 + 1 = 0 \Leftrightarrow r^2 = -1 \Leftrightarrow r = \pm i$$

So the general solution is

$$x(t) = A\cos(t) + B\sin(t)$$

The initial conditions are

$$x(0) = 1$$
$$y(0) = \dot{x}(0) = 0$$

So we obtain $A = x(0) = 1$

and since $\dot{x}(t) = -A\sin(t) + B\cos(t)$,

$$B = \dot{x}(0) = 0$$

Thus

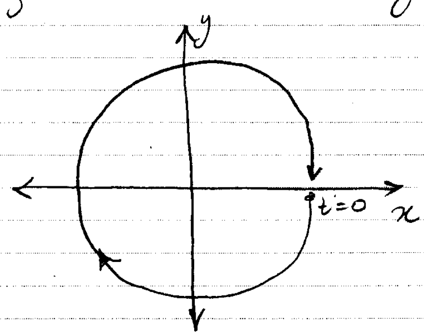
$$x(t) = \cos(t)$$

and $y(t) = \dot{x}(t) = -\sin(t)$.

The plot of $x(t)$ vs $y(t)$ will be a parametric curve (time t is the parameter) with

$$\left.\begin{aligned} x(t) &= \cos(t) \\ y(t) &= -\sin(t) \end{aligned}\right\} \text{for } t = 0 \text{ to } 2\pi$$

This will create a circle of radius 1 starting at (1,0) and moving clockwise...

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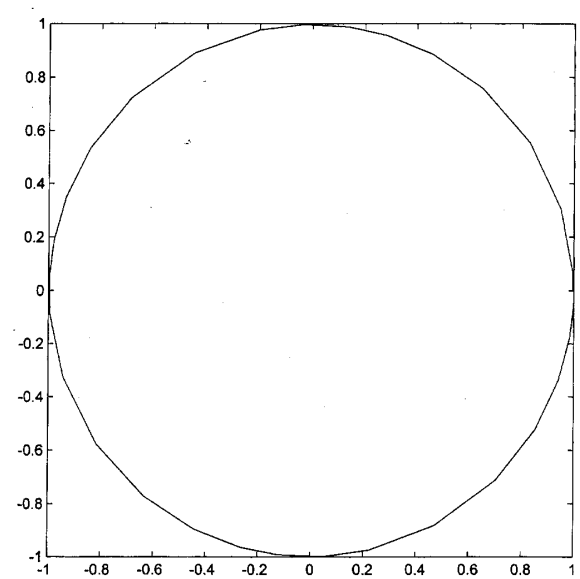
The following are the two files you were asked to run
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%These 4 lines are in a script file
[t,z]=ODE23('iknowthis', [0 2*pi], [1 0]);
plot(z(:,1),  z(:,2))
axis('square')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function zdot=iknowthis(t,z)
% These seven lines are in the file iknowthis.m
z1 = z(1);
z2 = z(2);

z1dot=  z2;
z2dot= -z1;

zdot=[z1dot, z2dot]';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

This results in the figure shown below...



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%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%These 4 lines are in a script file
options=odeset('RelTol',1e-8,'AbsTol',1e-8);
[t,z]=ODE23('iknowthis', [-10 50], [-2 -1]);
plot(z(:,1),  z(:,2))
axis('square')
xlabel('x');
ylabel('y');
title('Basant Sharma, HW1, Problem 1(iii)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function zdot=iknowthis(t,z)
% These seven lines are in the file iknowthis.m
z1 = z(1);
z2 = z(2);

z1dot=  1+z2+sin(z1)+cos(z2);
z2dot= -2*z1*sin(2*z1);
zdot=[z1dot, z2dot]';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

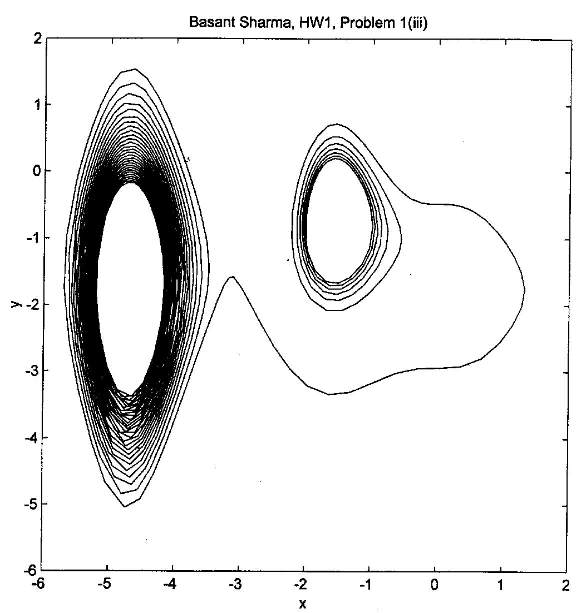

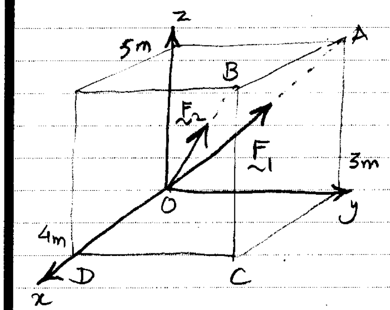

Given

$\underset{\sim}{F}_1 = 5N$ along OA

$\underset{\sim}{F}_2 = 7N$ along OB

a) $\underset{\sim}{n}_{OB}$ = unit vector along OB

$= \dfrac{4\hat{i}+3\hat{j}+5\hat{k}\ m}{\sqrt{4^2+3^2+5^2}\ m}$

$\boxed{= \dfrac{1}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})}$

b) $\underset{\sim}{n}_{OA} = \dfrac{\underset{\sim}{OA}}{|\underset{\sim}{OA}|} = \dfrac{3\hat{j}+5\hat{k}\ m}{\sqrt{3^2+5^2}\ m} = \boxed{\dfrac{1}{\sqrt{34}}(3\hat{j}+5\hat{k})}$

c) $\underset{\sim}{F}_1 = |\underset{\sim}{F}_1|\,\underset{\sim}{n}_{OA} = \dfrac{5}{\sqrt{34}}(3\hat{j}+5\hat{k})\ N$;

$\underset{\sim}{F}_2 = |\underset{\sim}{F}_2|\,\underset{\sim}{n}_{OB} = \dfrac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})\ N$;

d) $\cos(\angle AOB) = \underset{\sim}{n}_{OA}\cdot\underset{\sim}{n}_{OB} = \dfrac{1}{10\sqrt{17}}(3\times3+5\times5)$

$= \dfrac{1}{10\sqrt{17}}\times 34$

$\therefore\ \angle AOB = \boxed{\cos^{-1}\dfrac{34}{10\sqrt{17}}} \approx 34.45^\circ$

e) Component of $\underset{\sim}{F}_1$ in x-direction

$F_{1x} = \underset{\sim}{F}_1\cdot\hat{i} = \dfrac{5}{\sqrt{34}}(3\hat{j}+5\hat{k})\cdot\hat{i}\ N$

$\boxed{= 0\ N}$

f) Moment of $\underset{\sim}{F}_1$ about point D

$\underset{\sim}{r}_{DO}\times\underset{\sim}{F}_1 = \underset{\sim}{DO}\times\underset{\sim}{F}_1 = -4\hat{i}\,m\times\dfrac{5}{\sqrt{34}}(3\hat{j}+5\hat{k})N$

$= -\dfrac{20}{\sqrt{34}}(3\hat{k}-\hat{j})\ Nm$

$\boxed{= \dfrac{20}{\sqrt{34}}(5\hat{j}-3\hat{k})\ Nm}$

g) Moment of $\underset{\sim}{F}_2$ about the axis DC $= \hat{j}\cdot(\underset{\sim}{r}_{DO}\times\underset{\sim}{F}_2)$

$= \hat{j}\cdot\left(-4\hat{i}\,m\times\dfrac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})N\right) = \hat{j}\cdot\left(\dfrac{-28}{5\sqrt{2}}\right)(3\hat{k}-5\hat{j})\ Nm$

$\boxed{= 14\sqrt{2}\ Nm}$

h) $\hat{j}\cdot(\underset{\sim}{x}_{CO}\times\underset{\sim}{F}_2) = \hat{j}\cdot\left[(-4\hat{i}-3\hat{j})m\times\dfrac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})N\right]$

$= \hat{j}\cdot\dfrac{7}{5\sqrt{2}}(\cancel{-12\hat{k}}+20\hat{j}+\cancel{12\hat{k}}-15\hat{i})\ Nm$

$\boxed{= 14\sqrt{2}\ Nm}$ as before

Also, $\hat{j}\cdot(\underset{\sim}{r}_{CB}\times\underset{\sim}{F}_2) = \hat{j}\cdot\left[5\hat{k}\,m\times\dfrac{7}{5\sqrt{2}}(4\hat{i}+3\hat{j}+5\hat{k})N\right]$

$= \dfrac{7}{\sqrt{2}}\hat{j}\cdot(4\hat{j}-3\hat{i})\ Nm\ \boxed{= 14\sqrt{2}\ Nm}$