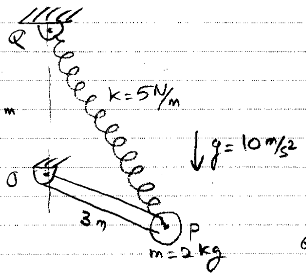
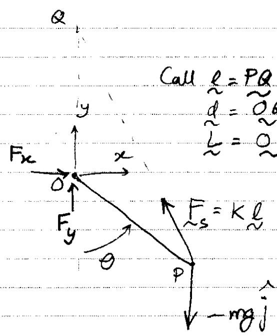
Basant Sharma Spring

① **4.54**



FBD of Pendulum :

Call $\underset{\sim}{\ell} = \underset{\sim}{PQ}$, $\underset{\sim}{d} = \underset{\sim}{OQ}$, $\underset{\sim}{L} = \underset{\sim}{OP}$



$\sum \underset{\sim}{M}_{/O} = 0:$

$\underset{\sim}{L} \times (\underset{\sim}{F}_s - mg\hat{j}) = 0$

But $\underset{\sim}{F}_s = k\underset{\sim}{\ell} = k(\underset{\sim}{PQ})$
$= k(\underset{\sim}{OQ} - \underset{\sim}{OP})$
$= k(\underset{\sim}{d} - \underset{\sim}{L})$

$\therefore \quad \underset{\sim}{L} \times (k\underset{\sim}{d} - k\underset{\sim}{L} - mg\hat{j}) = 0$

$\Rightarrow \quad k\underset{\sim}{L} \times \underset{\sim}{d} - mg\, \underset{\sim}{L} \times \hat{j} = 0$

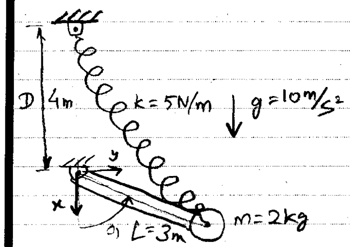
$\Rightarrow \quad kLd\sin\theta - mgL\sin\theta = 0$

Since $kd - mg = 4 \times 5 - 2 \times 10 = 0$

So $\sum \underset{\sim}{M}_{/O} = 0$ holds for arbitrary θ.
Hence at any angle given we achieve equilibrium and the other two equations give the reaction forces F_x and F_y.

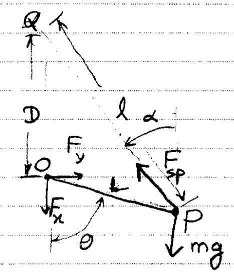
(for an alternative method see next page)

① **4.54** (alternative long approach)



Given : zero length spring
Find : θ for static equilibrium.

FBD of the pendulum :



$\sum M_{/O} = 0: \; F_{sp}\cos\alpha\, L\sin\theta$
$- F_{sp}\sin\alpha\, L\cos\theta - mgL\sin\theta = 0$

From geometry :
$\cos\alpha = \dfrac{L\cos\theta + D}{\ell}$

$\sin\alpha = \dfrac{L\sin\theta}{\ell}$

So we have :
$F_{sp}\left(\dfrac{L\cos\theta + D}{\ell}\right)L\sin\theta - F_{sp}\left(\dfrac{L\sin\theta}{\ell}\right)L\cos\theta - mgL\sin\theta = 0$

Using the given information about the spring
$F_{sp} = k\ell \iff \dfrac{F_{sp}}{\ell} = k$

So $k(L\cos\theta + D)L\sin\theta - k(L\sin\theta)L\cos\theta - mgL\sin\theta = 0$

$\iff \quad kDL\sin\theta - mgL\sin\theta = 0$

$\iff \quad (kD - mg)L\sin\theta = 0$

Since $L \neq 0$, $(kD - mg)\sin\theta = 0$
Substituting $k = 5$, $D = 4$, $m = 2$, $g = 10$, we obtain $0.\sin\theta = 0$
Thus Moment about the origin is balanced for any θ.

$\sum M_{/P} = 0: \; -F_y \cdot L\cos\theta + F_x L\sin\theta = 0$
$\iff \quad F_y\cos\theta = F_x\sin\theta$ ——— (1)

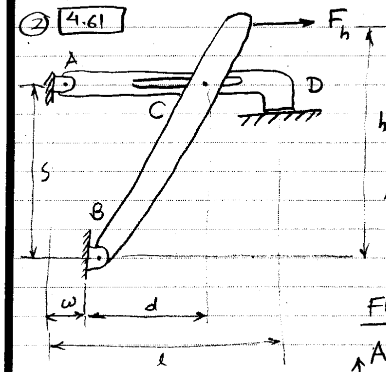
$\sum M_{/Q} = 0: \; F_y \cdot D - mgL\sin\theta = 0$
$\iff \quad F_y = \dfrac{mgL}{D}\sin\theta$

(1) $\Rightarrow \quad \sin\theta F_x = \dfrac{mgL}{D}\sin\theta\cos\theta$

if $\theta \neq 0$, $\quad F_x = \dfrac{mgL}{D}\cos\theta$, $F_y = \dfrac{mgL}{D}\sin\theta$
if $\theta = 0$, $\quad F_y = 0$ and F_x is undetermined.

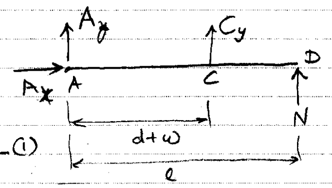
Thus we have seen that any θ yields appropriate reaction forces and satisfies all equations of equilibrium.

② **4.61**



Given :
– no gravity
– frictionless hinges A, B and smooth roller at C.
Find : Reaction at D

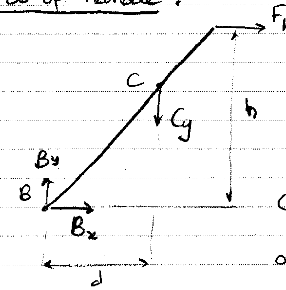
FBD of Stamp arm :



$\sum M_{/A} = 0: (w+d)C_y + \ell N = 0$

So $C_y = -\dfrac{\ell N}{(w+d)}$ ——— (1)

FBD of handle :



$\sum M_{/B} = 0: -C_y d - F_h \cdot h = 0$

So $C_y = -\dfrac{F_h \cdot h}{d}$ ——— (2)

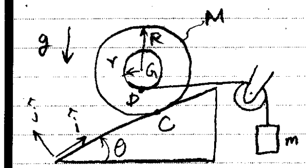
(1) and (2) $\Rightarrow \dfrac{\ell N}{w+d} = \dfrac{F_h h}{d}$

or $N = F_h \cdot \dfrac{h}{d}\dfrac{(w+d)}{\ell}$

$$\boxed{N = F_h \cdot \frac{h}{\ell} \cdot \left(1 + \frac{w}{d}\right)}$$

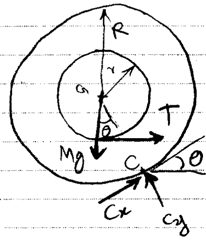
Note : we didnot use the force balance equations but if the reaction forces at B and A and C were of interest we would have used them surely.

(3) 4.65



Given: string is massless and inextensible.
$r = \frac{1}{2}R$.
no slip at C.

a) FBD of the reel :



$\Sigma F_x = 0: \; C_x + T\cos\theta - Mg\sin\theta = 0$ — (1)

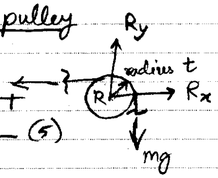
$\Sigma F_y = 0: \; C_y - T\sin\theta - Mg\cos\theta = 0$ — (2)

$\Sigma M_{/C} = 0: \; Mg R\sin\theta - T(R\cos\theta - r) = 0$ — (3)

Using the equation (3)

$$T = \frac{Mg R\sin\theta}{(R\cos\theta - r)} \quad \text{— (4)}$$

FBD of the string and pulley



Since we need only T, consider

$\Sigma M_{/R} = 0: \; t\,T = mg\,t$

$\Rightarrow \quad T = mg$ — (5)

(4),(5) $\Rightarrow \quad mg = \dfrac{Mg R\sin\theta}{(R\cos\theta - r)}$

or $\dfrac{m}{M} = \dfrac{R\sin\theta}{(R\cos\theta - r)}$

or $\boxed{\dfrac{m}{M} = \dfrac{\sin\theta}{\cos\theta - \frac{r}{R}}}$ here $r = \frac{1}{2}R$ So $\boxed{\dfrac{m}{M} = \dfrac{\sin\theta}{\cos\theta - \frac{1}{2}}}$

b) $\boxed{T = mg}$ see part a) equation (5)

c) We have from the equations (1) and (2)

$C_x = Mg\sin\theta - T\cos\theta$
$\quad = Mg\sin\theta - mg\cos\theta$

and $C_y = T\sin\theta - Mg\cos\theta$
$\quad = mg\sin\theta + Mg\cos\theta$

So $\underset{\sim}{F_C} = C_x\hat{i} + C_y\hat{j}$
$\quad = Mg\sin\theta\,\hat{i} + Mg\cos\theta\,\hat{j}$
$\quad + (-mg\cos\theta)\,\hat{i} + mg\sin\theta\,\hat{j}$

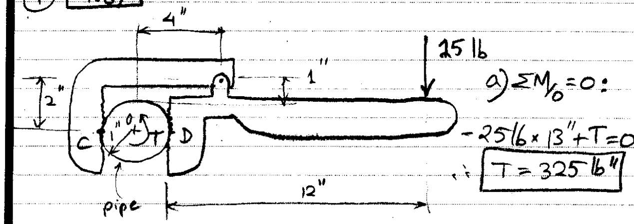
$$\boxed{\underset{\sim}{F_C} = Mg\left(\sin\theta - \frac{m}{M}\cos\theta\right)\hat{i} + Mg\left(\cos\theta + \frac{m}{M}\sin\theta\right)\hat{j}}$$

where $\dfrac{m}{M} = \dfrac{\sin\theta}{\cos\theta - \frac{1}{2}}$

Check: $\theta = 0$ gives $\frac{m}{M} = 0$, $\underset{\sim}{F_c} = 0\hat{i} + Mg\hat{j} = Mg\hat{j}$

$\theta = \frac{\pi}{2}$ gives $\frac{m}{M} = -2$, $\underset{\sim}{F_c} = Mg\,\hat{i} - 2Mg\,\hat{j} = Mg(\hat{i} - 2\hat{j})$

(4) 4.67

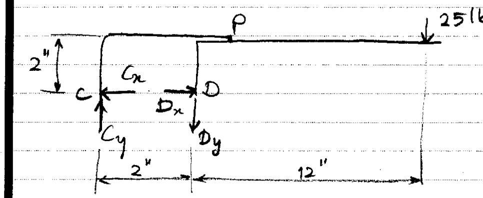


a) $\Sigma M_{/0} = 0$:

$-25\,lb \times 13'' + T = 0$

$\therefore \boxed{T = 325\ lb''}$

b) FBD of the wrench without pipe :



$\Sigma F_x = 0: \; -C_x + D_x = 0$

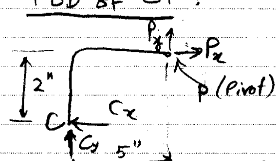
$\Sigma F_y = 0: \; C_y - D_y - 25\,lb = 0$

$\Sigma M_{/C} = 0: \; -D_y 2'' - 25\,lb \cdot 14'' = 0 \iff \boxed{D_y = -175\ lb}$

$\Rightarrow \quad C_y = D_y + 25\,lb$

or $\boxed{C_y = -150\ lb}$

FBD of CP :

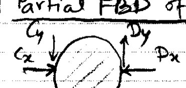


Since we are interested only in C_x and C_y, we take moments about P as:

$\Sigma M_{/c} = 0: \; -2C_x - 5C_y = 0$

So $C_x = -\frac{5}{2}C_y = -\frac{5}{2}\times(-150\,lb)$

"Partial" FBD of pipe



So: $\boxed{C_x = 375\ lb}$

and $\boxed{D_x = C_x = 375\ lb}$

So $\underset{\sim}{F_C} = +C_x\hat{i} - C_y\hat{j} = (375\,\hat{i} + 150\,\hat{j})\,lb$
$\quad \underset{\sim}{F_D} = -D_x\hat{i} + D_y\hat{j} = (-375\hat{i} - 175\hat{j})\,lb$

c) we need $|C_y| \le \mu_c |C_x|$, $|D_y| \le \mu_D |D_x|$

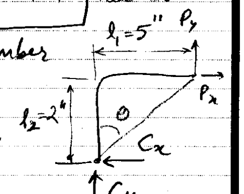
i.e. $\boxed{\mu_c \ge 0.4}$, $\boxed{\mu_D \ge 0.467}$

if $\mu_c = \mu_D$ then $\boxed{\mu_c = \mu_D \ge 0.467}$ needed.

d) Since C-P is a 2-force member

$\mu_c \ge \dfrac{|C_y|}{|C_x|} = \dfrac{l_2}{l_1}$

So by decreasing l_2 or increasing l_1, $(\mu_c)_{reqd.}$ decreases.



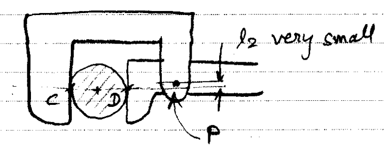
4.67 Contd...

c) If l_1 is increased, a small rotation at pivot point, P, amplifies to large displacement of the clamp C w.r.t. D. This is not a good design.

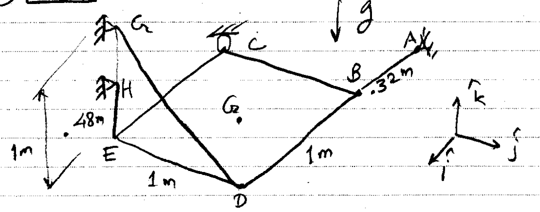
If l_2 is decreased to a very small value then

(i) As soon as load is applied on the handle of wrench, a huge crushing force (C_x and D_x) is generated! which could damage the pipe.

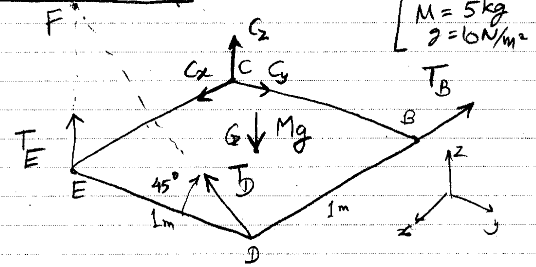
(ii) A small displacement of pt. P, due say to small deformation of pipe, would cause P to go below the line CD. In this case you would calculate a tension at C and P. That is, the wrench opens rather than closes on the pipe.



(5) **4.87**



a) FBD of the shelf :



$\left[\begin{array}{l} M = 5kg \\ g = 10 N/m^2 \end{array}\right.$

b) Taking Moment about CE : $T_D \times \frac{1}{\sqrt{2}} \times 1m - Mg \times 0.5m = 0$

So $\boxed{T_D = \frac{Mg}{\sqrt{2}} = \frac{50}{\sqrt{2}} N}$

Taking Moment about CD : $T_E \times \frac{1}{\sqrt{2}} m = 0$

So $\boxed{T_E = 0\ N}$

Taking Moment about CF :

$\frac{1}{\sqrt{2}}(\hat{i}+\hat{k}).(0.5\hat{i}+0.5\hat{j}) \times (-Mg)\hat{k}\ m$

$\frac{1}{\sqrt{2}}(\hat{i}+\hat{k}).(T_B\hat{k})m = 0$

$\Rightarrow -\frac{Mg}{2\sqrt{2}} + \frac{T_B}{\sqrt{2}} = 0 \Leftrightarrow \boxed{T_B = \frac{Mg}{2} = 25N}$

Taking Moment about ED : $-Mg \times 0.5m + C_z \times 1m = 0$

So $\boxed{C_z = \frac{1}{2} \times 50N = 25N}$

Taking Moment about x-axis through F :

$C_y \times 1m - Mg \times 0.5m = 0$

$\Rightarrow \boxed{C_y = \frac{Mg}{2} = 25N}$

Thus we obtained T_B, T_E, T_B and C_y, C_z by using one equation in one unknown only.

I havenot figured out a way to find C_x by such method.

c) $\sum \underset{\sim}{F} = 0$:

$(C_x - T_B)\hat{i} + (C_y - \frac{T_D}{\sqrt{2}})\hat{j} + (-Mg + C_z + T_E + \frac{T_D}{\sqrt{2}})\hat{k} = \underset{\sim}{0}$

d) $\sum \underset{\sim}{M}_{/G} = 0$: $\underset{\sim}{GC} \times (C_x\hat{i} + C_y\hat{j} + C_z\hat{k})$

$+ \underset{\sim}{GE} \times T_E\hat{k} + \underset{\sim}{GD} \times (-\frac{T_D}{\sqrt{2}}\hat{j} + \frac{T_D}{\sqrt{2}}\hat{k})$

$+ \underset{\sim}{GB} \times (-T_B\hat{i}) = \underset{\sim}{0}$

where $-\underset{\sim}{GD} = \underset{\sim}{GC} = -\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$, $\underset{\sim}{GE} = -\underset{\sim}{GB} = \frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

e) $\sum F_x = 0$: $C_x - T_B = 0$ —— (1)

$\sum F_y = 0$: $C_y - \frac{T_D}{\sqrt{2}} = 0$ —— (2)

$\sum F_z = 0$: $-Mg + C_z + T_E + \frac{T_D}{\sqrt{2}} = 0$ —— (3)

From moment balance :

$\frac{1}{\sqrt{2}}(\hat{i}+\hat{j}) \times \left((-C_x)\hat{i} + (-C_y - \frac{T_D}{\sqrt{2}})\hat{j} + (-C_z + \frac{T_D}{\sqrt{2}})\hat{k} \right)$

$+ \frac{1}{\sqrt{2}}(\hat{i}-\hat{j}) \times (+T_B\hat{i} + T_E\hat{k}) = \underset{\sim}{0}$

So $+\frac{1}{\sqrt{2}}(C_z + \frac{T_D}{\sqrt{2}}) - \frac{T_E}{\sqrt{2}} = 0$ —— (4)

$-\frac{1}{\sqrt{2}}(C_z + \frac{T_D}{\sqrt{2}}) - \frac{T_E}{\sqrt{2}} = 0$ —— (5)

$\frac{1}{\sqrt{2}}(C_x - C_y - \frac{T_D}{\sqrt{2}}) + \frac{T_B}{\sqrt{2}} = 0$ —— (6)

f) Solving (1) to (6) we obtain

(4) & (5) $\Rightarrow T_E = 0$, $C_z = \frac{T_D}{\sqrt{2}}$

(3) $\Rightarrow 2C_z = Mg$ or $C_z = \frac{Mg}{2}$ so $T_D = \frac{Mg}{\sqrt{2}}$

(2) $\Rightarrow C_y = \frac{T_D}{\sqrt{2}} = \frac{Mg}{2}$

(6) $\Rightarrow \frac{1}{\sqrt{2}}(C_x - 2C_y) + \frac{T_B}{\sqrt{2}} = 0$ So $C_x + T_B = Mg$

(1) $\Rightarrow C_x = T_B = \frac{Mg}{2}$

So
$$\boxed{\begin{array}{l} C_x = C_y = C_z = T_B = \frac{Mg}{2} = 25N \\ T_D = \frac{Mg}{\sqrt{2}} \approx 35.35\ N \\ T_E = 0 \end{array}}$$

(as before)

h) see part b)


```
% Hanging Plate Problem 4.187
% Basant Sharma's solution, Feb 9 2002.

rt2=1/sqrt(2)

%     Cx Cy Cz TB TD TE
A = [1 0 0 -1 0 0
     0 1 0 0 -rt2 0
     0 0 1 0 rt2 1
     0 0 -1 0 rt2 -1
     0 0 1 0 -rt2 -1
     1 -1 0 1 -rt2 0]
b = [0 0 1 0 0 0]'

x = A\b
```

```
plate

2 =

  0.7071

=

  1.0000        0        0  -1.0000        0        0
       0   1.0000        0        0  -0.7071        0
       0        0   1.0000        0   0.7071   1.0000
       0        0  -1.0000        0   0.7071  -1.0000
       0        0   1.0000        0  -0.7071  -1.0000
  1.0000  -1.0000        0   1.0000  -0.7071        0

=

  0
  0
  1
  0
  0
  0

=

  0.5000
  0.5000
  0.5000
  0.5000
  0.7071
       0
```

C_x, C_y, C_z, T_B, T_D, T_E — times mg