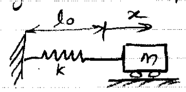
Basant Sharma.

① $\boxed{5.28}$



Mass released from rest with spring

$m = 2kg$, $k = 5N/m$, $x(0) = 0.5m$

$\dot{x}(0) = 0$

Since mass moves on a frictionless flat surface its gravitational potential Energy remains constant.

a) $E_K(0) = ?$, $E_P(0) = ?$

Initially, the potential energy $= \frac{1}{2} k\,(\text{stretch in spring})^2$

$= \frac{1}{2}\, 5N/m \times (0.5m)^2 = \frac{5}{8}\, Nm = 0.625\, Nm.$

the kinetic energy $= \frac{1}{2} m\,(\text{speed of mass})^2$

$= \frac{1}{2} m\,(\dot{x}(0))^2 = \frac{1}{2}\, 2\,kg \cdot (0\, m/s)^2 = 0$

b) when $x = 0$, $E_P = ?$, $E_K = ?$

As the mass passes through the static equilibrium the potential energy $= \frac{1}{2} k\,(\text{stretch in spring})^2$

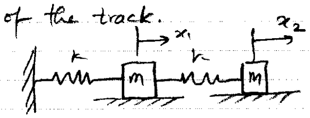
$= \frac{1}{2}\, 5N/m\,(0m)^2 = 0$

the kinetic energy $= \frac{1}{2} m v^2$, where v is the speed of the mass.

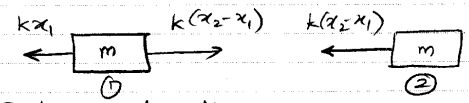
Since total energy is conserved (no damping in system), the kinetic energy $= 0.625\, Nm$.

② $\boxed{5.59}$

Two equal masses on an air track connected by a spring to the end of the track.



a) FBDs



b) Find eqns of motion ?

By Newton's second law for ①, $\left[m\underline{a}\cdot\hat{i} = \Sigma \underline{F}\cdot\hat{i} \right]$

$m\ddot{x}_1 = -kx_1 + k(x_2 - x_1)$ ——— (1)

By Newton's second law for ②

$m\ddot{x}_2 = -k(x_2 - x_1)$ ——— (2)

$$(1) \iff \boxed{\begin{aligned} \ddot{x}_1 &= -\frac{2k}{m} x_1 + \frac{k}{m} x_2 \\ \ddot{x}_2 &= \frac{k}{m} x_1 - \frac{k}{m} x_2 \end{aligned}} \iff (2)$$

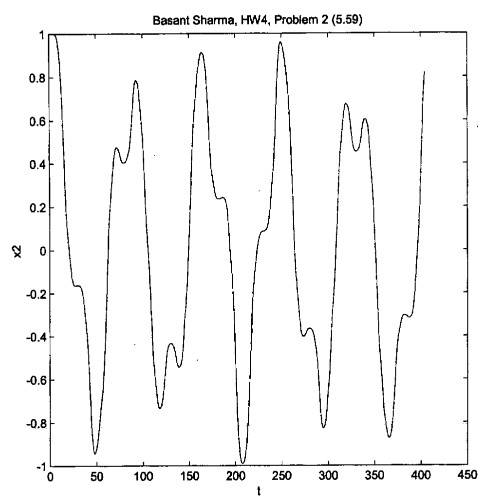
c) Find ODE form ?

Let $v_1 = \dot{x}_1$, $v_2 = \dot{x}_2$

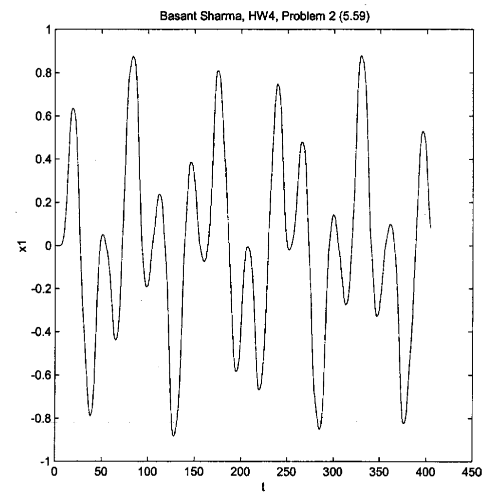
$$\boxed{\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 \\ \dot{v}_1 &= -\frac{2k}{m} x_1 + \frac{k}{m} x_2 \\ \dot{v}_2 &= \frac{k}{m} x_1 - \frac{k}{m} x_2. \end{aligned}}$$

d) pick $k = 1$ and $m = 1$, and $x_1(0) = 0$, $x_2(0) = 1$

$\dot{x}_1(0) = 0$, $\dot{x}_2(0) = 0$.




```
% Two equal masses on air track: Homework Problem 2 (5.59)
% Numerical Solution of the ODE involved
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[t,z]=ODE23('twomass', [0 50], [0.5 1.5 0 0]);
plot(z(:,1));
axis('square');
xlabel('t');
ylabel('x1');
title('Basant Sharma, HW4, Problem 2 (5.59)');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Equations of Motion 2mass.m
function zdot = twomass(t,z)
x1 = z(1);
x2 = z(2);
v1 = z(3);
v2 = z(4);
% mass and spring constant
m = 1;
k = 1;
% the ratio k/m
f = k/m;
x1dot = v1;
x2dot = v2;
v1dot = f*(-2*x1+x2);
v2dot = f*(x1-x2);
zdot = [x1dot, x2dot, v1dot, v2dot]';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

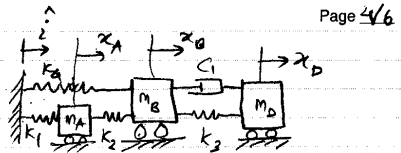


e) Based on the initial conditions, x_1 should increase and x_2 should decrease as verified by the two plots. The solutions are bounded and not periodic (could have been periodic if initial conditions are chosen appropriately, One can think about a case when certain symmetry exists between the motion of both masses).

(3) $\boxed{5.64}$

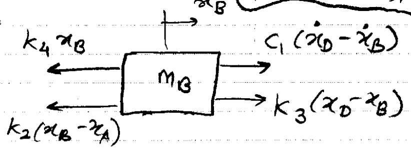
$a_B = ?$

Three masses connected by springs to a wall and a damper between two masses.



x_A, x_B, x_D measured from relaxed state

FBD of m_B:



By Newton's second law:

$$\sum \underset{\sim}{F} \cdot \hat{i} = m \underset{\sim}{a} \cdot \hat{i} : \quad -k_4 x_B - k_2(x_B - x_A) + c_1(\dot{x}_D - \dot{x}_B) + k_3(x_D - x_B) = m_B \ddot{x}_B$$

$$\Rightarrow \boxed{\underset{\sim}{a} = \ddot{x}_B \hat{i} = \frac{1}{m_B}\left[k_2 x_A - (k_2 + k_3 + k_4) x_B + k_3 x_D - c_1 \dot{x}_B + c_1 \dot{x}_D\right]\hat{i}}$$

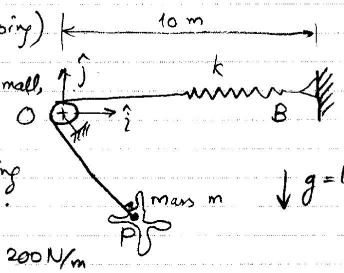
(4) $\boxed{5.120}$

accelerating mass pulled by three strings.

```
m=3; kg
a=[1 2 3]'; m/s^2
rAB=[2 3 5]';
rAC=[-3 4 2]';  } in m
rAD=[1 1 1]';

uAB=rAB/norm(rAB);
uAC=rAC/norm(rAC);
uAD=rAD/norm(rAD);

T=[uAB uAC uAD]\(m*a) <- by LMB or Newton's Second law.

» hw4_4   Solution

T =

TAB = 10.4024
TAC =  1.0097   } in Newtons
TAD =  0.3248
```


(5) (Bungy Jumping)

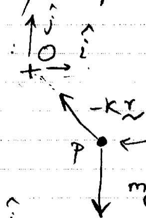
Pulley: massless, small, and frictionless. The person sits at O if spring is unstretched.



$m = 100\,kg$, $k = 200\,N/m$

a) $\underset{\sim}{r} = x\hat{i} + y\hat{j}$, $\underset{\sim}{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$, give the position and velocity respectively.

FBD of person:



located at $\underset{\sim}{r}$ w.r.t O or $\underset{\sim}{OP} = \underset{\sim}{r}$.

By second law

$m\underset{\sim}{a} = \sum \underset{\sim}{F}$

So $m\underset{\sim}{a} = -k\underset{\sim}{r} - mg\hat{j}$

$$\Leftrightarrow \underset{\sim}{a} = -\frac{k}{m}\underset{\sim}{r} - g\hat{j} \quad \text{or} \quad \boxed{\underset{\sim}{a} = \left(-2\left(\tfrac{1}{s^2}\right)\right)\underset{\sim}{r} - 10\tfrac{m}{s^2}\hat{j}}$$

b) In component form, from a), the acceleration of the person is

$$\left.\begin{aligned}\ddot{x}(t) &= -2x(t)\\ \ddot{y}(t) &= -2y(t) - 10\end{aligned}\right\} (*) \quad \text{(where } x \text{ and } y \text{ are in m.)}$$

Using initial condition $\underset{\sim}{r}_0 = x(0)\hat{i} + y(0)\hat{j} = 1m\,\hat{i} - 5m\,\hat{j}$

$\underset{\sim}{v}_0 = \dot{x}(0)\hat{i} + \dot{y}(0)\hat{j} = \underset{\sim}{0}$,

we can write (*) as a system of 4 first order ODEs:

Let $V_x = \dot{x}$, $V_y = \dot{y}$,

So

$$\dot{x}(t) = V_x(t)$$
$$\dot{y}(t) = V_y(t)$$
$$\dot{V}_x(t) = -2x(t)$$
$$\dot{V}_y(t) = -2y(t) - 10$$

with Initial Condition: $x(0) = 1$, $y(0) = -5$, $v_x(0) = V_y(0) = 0$.

((b) Contd...)

MATLAB commands to find position at $t = \frac{\pi}{\sqrt{2}}s$.

```
% Equations of Motion bungy.m
function zdot = bungy(t,z)
x = z(1);
y = z(2);
vx = z(3);
vy = z(4);
xdot = vx;
ydot = vy;
vxdot = -2*x;
vydot = -2*y-10;
zdot = [xdot, ydot, vxdot, vydot]';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

» [t,z]=ode23('bungy',[0 pi/sqrt(2)],[1 -5 0 0]);
» z(end,1)

ans =

   -0.9984
» z(end,2)

ans =

   -5
```

c) Solving (*)

$x(t) = A\cos\sqrt{2}t + B\sin\sqrt{2}t$

$y(t) = -5 + C\cos\sqrt{2}t + D\sin\sqrt{2}t$

Using the initial conditions

$x(0) = 1$, $y(0) = -5$

$\dot{x}(0) = 0$, $\dot{y}(0) = 0$

we get

$$\boxed{\begin{aligned}x(t) &= \cos\sqrt{2}t\\ y(t) &= -5\end{aligned}}$$

So at $t_0 = \frac{\pi}{\sqrt{2}}$, $x(t_0) = -1$, $y(t_0) = -5$

$$\text{or} \quad \boxed{\underset{\sim}{r}(t_0) = -\hat{i} - 5\hat{j}}$$