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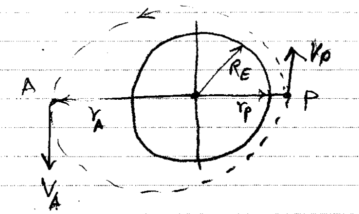
(1) $\boxed{5.141}$

A satellite put into an orbit around the earth.



Total energy at P = Potential energy + kinetic energy

$$= -m\frac{gR^2}{r_p} + \frac{1}{2}mv_p^2$$

Total energy at A $= -m\frac{gR^2}{r_A} + \frac{1}{2}mV_A^2$

By the conservation of energy

$$-\frac{mgR^2}{r_p} + \frac{1}{2}mV_p^2 = -\frac{mgR^2}{r_A} + \frac{1}{2}mV_A^2$$

$$\Leftrightarrow \quad (V_A^2 - V_p^2) = 2gR^2\left(\frac{1}{r_A} - \frac{1}{r_p}\right)$$

$$\text{or} \quad \left(\frac{V_A}{V_p}\right)^2 - 1 = \frac{2gR^2}{V_p^2 r_p}\left(\frac{r_p}{r_A} - 1\right) \quad \text{—(1)}$$

By the conservation of angular momentum of the satellite about the center of earth

$$r_p m V_p = r_A m V_A$$

$$\text{or} \quad \frac{r_p}{r_A} = \frac{V_A}{V_p} \quad \text{—(2)}$$

Using (2) in (1)

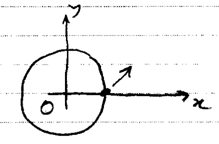
$$\left(\frac{V_A}{V_p}\right)^2 - 1 = \frac{2gR^2}{V_p^2 r_p}\left(\frac{V_A}{V_p} - 1\right)$$

$$\Rightarrow \quad \frac{V_A}{V_p} + 1 = \frac{2gR^2}{V_p^2 r_p} \quad (\text{Since } V_A \neq V_p \text{ as } r_A \neq r_p)$$

So

$$\boxed{V_A = V_p\left(\frac{2gR^2}{V_p^2 r_p} - 1\right)}$$

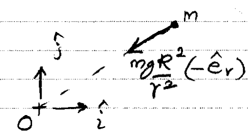
(2) $\boxed{5.142}$



A missile launched from earth at an angle of 45°. (Ignore rotation of earth) Gravitational force $F = mgR^2/r^2$ towards the center of earth.

a) FBD of missile :

LMB : $m\ddot{\underset{\sim}{r}} = -\frac{mgR^2}{r^2}\hat{e}_r$



In components

$$\left.\begin{aligned} m\ddot{x} + \frac{mgR^2\,x}{(x^2+y^2)^{3/2}} &= 0 \\ m\ddot{y} + \frac{mgR^2\,y}{(x^2+y^2)^{3/2}} &= 0 \end{aligned}\right\} (*)$$

Let $v_x = \dot{x}$, $v_y = \dot{y}$ then

$$(*) \Rightarrow \boxed{\begin{aligned} \dot{x} &= v_x \\ \dot{y} &= v_y \\ \dot{v}_x &= -gR^2 x/(x^2+y^2)^{3/2} \\ \dot{v}_y &= -gR^2 y/(x^2+y^2)^{3/2} \end{aligned}}$$

with initial conditions

$$x(0) = R = 6.4\times10^6\,m$$
$$y(0) = 0$$
$$v_x(0) = \underset{\sim}{v}_0\cdot\hat{i}$$
$$v_y(0) = \underset{\sim}{v}_0\cdot\hat{j}$$

where $\underset{\sim}{v}_0 = v_0\left(\cos\frac{\pi}{4}\,\hat{i} + \sin\frac{\pi}{4}\,\hat{j}\right)$

$$= \frac{v_0}{\sqrt{2}}(\hat{i}+\hat{j})$$

Also with given data : $g = 10\,m/s^2$, $v_0 = 9000\,m/s$

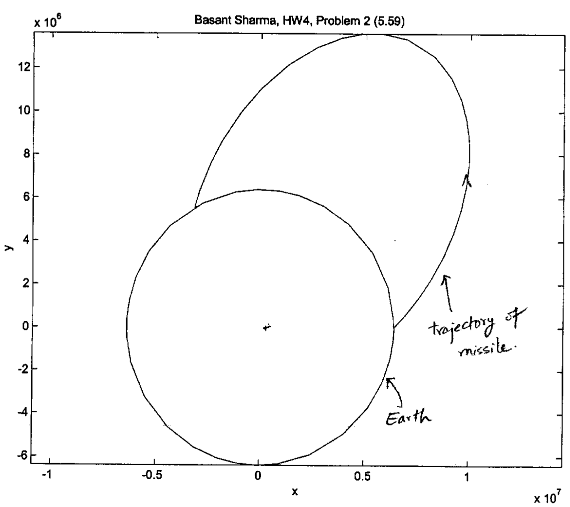
b)

```
% Nuclear Warhead: Homework Problem 2 (5.142)
% Numerical Solution of the ODE involved
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% earth's radius
R = 6400000;
v0 = 9000;
theta = pi/4;
v0x = v0*cos(theta);
v0y = v0*sin(theta);
Tf = 6670;

options=odeset('RelTol',1e-6,'AbsTol',1e-6);
[t,z]=ODE23('missile', [0 Tf], [R 0 v0x v0y], options);
plot(z(:,1),z(:,2));
axis('square');
xlabel('x');
ylabel('y');
title('Basant Sharma, HW4, Problem 2 (5.59)');
hold on;
[t,z]=ODE23('iknowthis', [0 2*pi], [6400000 0]);
plot(z(:,1),z(:,2));
axis equal;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

  % Equations of Motion missile.m
  function zdot = missile(t,z)
  x = z(1);
  y = z(2);
  vx = z(3);
  vy = z(4);
  % acceleration due to gravity on earth
  g = 10;
  R = 6400000;
  K = g*R^2;

  xdot = vx;
  ydot = vy;
  vxdot = -K*x/(x^2+y^2)^1.5;
  vydot = -K*y/(x^2+y^2)^1.5;
  zdot = [xdot, ydot, vxdot, vydot]';
  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```



(3) $\boxed{5.143}$

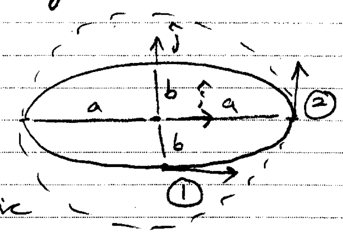
A particle moves in xy-plane under a central force.

Given $m = 2kg$

force $\underset{\sim}{F} = -k\underset{\sim}{r}$

$k = 200 N/m$

and $\underset{\sim}{r}$ is the position of the particle relative to center of force.



a) By Newton's second law

$$m\underset{\sim}{a} = -k\underset{\sim}{r} \quad \text{where } \underset{\sim}{a} = \ddot{\underset{\sim}{r}}.$$

In components

$$\ddot{x} = -\frac{k}{m}x \ , \ \ddot{y} = -\frac{k}{m}y$$

Solving these equations, call $\omega = \sqrt{\frac{k}{m}}$

$$x(t) = A\cos\omega t + B\sin\omega t$$
$$y(t) = D\cos\omega t + E\sin\omega t$$

Choosing $A = E = r_0$, $B = D = 0$

or $A = E = 0$, $B = D = r_0$

we get the equation of a circle.

For example: $x(t) = r_0\cos\omega t$

$y(t) = r_0\sin\omega t$ where $\omega = \sqrt{\frac{k}{m}}$

In our case $\omega = \sqrt{\frac{200}{2} \cdot \frac{1}{s}} = 10\ s^{-1}$.

Infact whenever $\sqrt{A^2+B^2} = \sqrt{D^2+E^2} = r_0$, we get the equation of a circle as (ϕ arbitrary)

$$\underset{\sim}{r}(t) = r_0\left(\cos(\omega t-\phi)\,\hat{i} + \sin(\omega t-\phi)\,\hat{j}\right)$$
$$\Rightarrow \dot{\underset{\sim}{r}}(t) = \underset{\sim}{v}(t) = r_0\omega\left(-\sin(\omega t-\phi)\,\hat{i} + \cos(\omega t-\phi)\,\hat{j}\right)$$

So $v = |\underset{\sim}{v}| = r_0\omega = r_0\sqrt{\frac{k}{m}}$

In our case, $\boxed{V = 10 r_0\ s^{-1}}$ must hold on a circular trajectory.

b)

From part a)

The speed required for the particle to be in circular trajectory through (2) is

$$V^* = 10 \cdot a\ s^{-1} \quad \text{—— (1)}$$

Since $a = 1\,m$, we get $v^* = 10\,ms^{-1}$.

If the particle moves from (1) to (2) in an elliptical trajectory then by the balance of angular momentum

$$\underset{\sim}{r}_{(1)} \times m\underset{\sim}{v}_{(1)} = \underset{\sim}{r}_{(2)} \times m\underset{\sim}{v}_{(2)}$$

Since $\underset{\sim}{v}_{(1)}$ and $\underset{\sim}{v}_{(2)}$ are perpendicular to $\underset{\sim}{r}_{(1)}$ and $\underset{\sim}{r}_{(2)}$ respectively,

$$b v_1 = a v_2$$

So $$v_2 = \frac{b}{a}v_1 \quad \text{—— (2)}$$

We need to supply the speed increment ΔV so that (2) corresponds to circular trajectory rather than elliptical from (1),

So $v_2 + \Delta V = V^*$

from (1) and (2), $\boxed{\Delta V = \frac{10}{s}a - \frac{b}{a}v_1}$

Using $a = 1\,m$, $b = 0.8\,m$,

$$\boxed{\Delta V = 10\frac{m}{s} - 0.8\,v_1}$$